MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

2.087 Spring 2014 **Quiz 2** **March 11, 2014**

Answer all of the questions in the booklet provided. **Be sure your name is marked BOTH on the cover of your booklet and on this exam.** Partial credit will be awarded so be sure and show all your work – this includes drawing *clear* and *well-labeled* plots. You may bring one double-sided sheets of notes; NO CALCULATORS.

NAME: ____

	Score
Problem 1 (/4 pts)	
Problem 2a (/3 pts)	
Problem 2b (/4 pts)	
Problem 2c (/5 pts)	
Problem 2d (/3 pts)	
Problem 2e (/2 pts)	
Problem 2f (/3 pts)	
Problem 2g (/9 pts)	
Problem 2h (/3 pts)	
Problem 3 (/13 pts)	
Total (/49 pts)	

Problem 1. Find the solution to the following IVP:

$$y'' + 6y' + 9y = 0 \qquad\qquad y(0) = 2, \quad y'(0) = 1$$

Problem 2.

a. Write the general solution to the following ODE in both complex and real form:

$$y'' + 2y' + 5y = 0$$

b. Find the particular solution for the initial conditions $y(0) = 1$ and $y'(0) = 0$ and sketch the solution for $y(t)$.

c. We now add a forcing term, $F(t) = \cos(2t) + 3$. Find the general solution to

$$y'' + 2y' + 5y = \cos(2t) + 3.$$

d. Sketch the long-time behavior of the solution to (c), again with initial conditions $y(0) = 1$ and $y'(0) = 0$. You do not need to get the details of the initial transients correct, but your sketch should accurately reflect the behavior of the system as $t \to \infty$ in terms of the frequency and amplitude of any oscillatory behavior.

e. Consider our original homogeneous equation: $y'' + 2y' + 5y = 0$. Rewrite this as a system of first order coupled equations.

f. Calculate the trace, p, and determinant, q, of your system matrix $\mathbf{A}$ in (e). On the basis of these, classify the critical point for the homogeneous system.

g. Write down the general solution to the system you derived in part (e) in both complex and real form.

h. Sketch the relevant phase portrait for the general solution in part (g).

Problem 3. Consider the following six differential equations (you do *not* need to solve these if you do not need the full solution to answer the questions below):

1. $y'' - 3y' - 3y = 0$
2. $\sec(t)y' = 1/y, \qquad y(0) = 1$
3. $y'' + 4y = 0$
4. $y'' + 8y' + 15y = 0$
5. $y' + 4t^3 y = e^{-t^4}$
6. $y'' + 8y' + 15y = \cos(3t)$

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a. Which (if any) of these have periodically oscillating solutions as $t \to \infty$?

b. Which (if any) of these have solutions that decay to zero as $t \to \infty$?

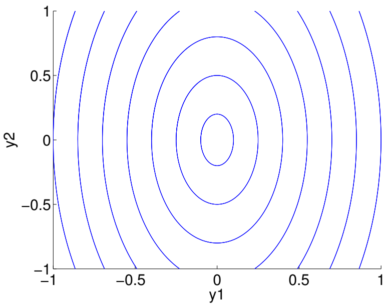
c. Which (if any) of these have solutions that blow up (i.e. become infinitely large) as $t \to \infty$?

d. For which (if any) of the *oscillating* solutions does the amplitude of oscillation depend on the initial conditions as $t \to \infty$?

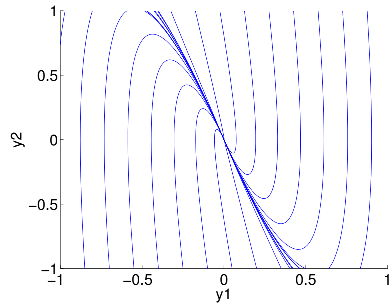
e. Which (if any) of the second order systems have an *unstable* critical point?

f. Which (if any) of these equations are *nonlinear*?

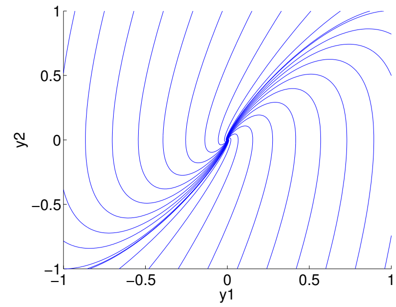
g. Match the phase portraits below with the relevant equations from the list above.



A



B



C

2.087 Engineering Math: Differential Equations and Linear Algebra
Fall 2014