

2.086 NUMERICAL COMPUTATION FOR MECHANICAL ENGINEERS

MINI-QUIZ 3

Fall 2014

You may refer to the textbook, lecture notes, MATLAB[®] tutorials, and other class materials as well as your own notes and scripts.

You may use a calculator (for simple arithmetic operations and function evaluations). However, laptops, tablets, and smartphones are not permitted.

You have 30 minutes of recitation to complete the mini quiz. When you are finished, you can hand in your quiz and start working on your assignment.

NAME _____

There are a total of 100 points: four questions, each worth 25 points.

All questions are multiple choice; *circle one and only one answer.*

We provide two blank pages at the end of the quiz which you may use for any derivations, but note that we will *only grade your multiple choice selections.*

This (same) quiz will be administered in all recitation sections. *You may not discuss this quiz with other people until the graded quizzes are returned to the class.*

Question 1 (25 points). This question refers to using Monte Carlo to estimate an area denoted A_D , using a bounding rectangle of area A_R and a large number, n , of random variates (drawn from the appropriate distributions).

Which of the following statements is incorrect?

- (a) It is good practice to use the smallest bounding rectangle (smallest area A_R) possible, because this minimizes the relative error (relative statistical *a priori* error bound) for a given (fixed) n .
- (b) For large bounding rectangles ($A_R \gg A_D$), n needs to increase proportionally with A_R to keep the relative error (relative statistical *a priori* error bound) constant.
- (c) The slope of the relative statistical *a priori* error bound versus n on a log-log plot increases as A_R increases.
- (d) Two bounding rectangles of the same area yield the same relative error (relative statistical *a priori* error bound).

Question 2 (25 points). A student needs to calculate the area of a segment of a unit circle of height $h = 0.5$, shown shaded in Figure 1. He decides to do this by a Monte Carlo procedure and generates 20 points that are uniformly distributed on the half-unit circle ($x^2 + y^2 \leq 1$, $-1 \leq x \leq 1$, $0 \leq y \leq 1$); the points are given in Table 1 below in terms of their x and y coordinates based on the coordinate system shown in Figure 1.

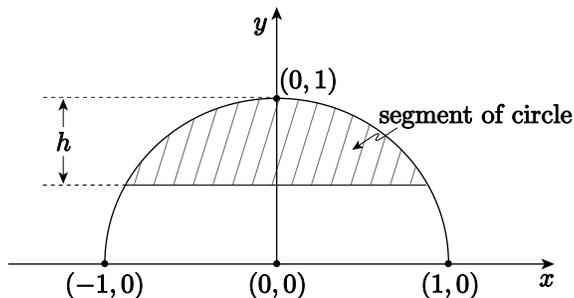


Figure 1: The segment of height $h = 0.5$ shown shaded.

Note that points are generated only in the half-unit circle, $x^2 + y^2 \leq 1$, $-1 \leq x \leq 1$, $0 \leq y \leq 1$.

Using these points, an *estimate* for the area of the segment is

- (a) $2/5$
- (b) $\pi/4$
- (c) $1/4$
- (d) $\pi/3$
- (e) $\pi/5$

x	y
-0.1543	0.5479
0.8855	0.4177
0.4022	0.6663
0.0783	0.6981
0.3331	0.1781
-0.6578	0.0326
0.1224	0.8819
0.3384	0.1904
-0.2622	0.4607
0.9633	0.1564
0.7110	0.6448
-0.2475	0.1909
-0.1435	0.4820
-0.7588	0.5895
-0.5476	0.3846
0.1660	0.2518
-0.4191	0.6171
-0.4694	0.8244
-0.3122	0.5841
-0.4785	0.5944

Question 3 (25 points). A student decides to calculate the integral

$$I = \int_0^1 \exp(x) dx$$

using the Monte Carlo sample-mean approach. Using MATLAB, she generates a sequence of n (pseudo)random variates representing a uniform random variable in the interval $(0, 1)$, denoted here by $x_i, i = 1, \dots, n$. The sample-mean estimate for I, \hat{I}_n , is given by

$$(a) \hat{I}_n = \sum_{i=1}^{n-1} \frac{\exp(x_i) + \exp(x_{i+1})}{2}$$

$$(b) \hat{I}_n = \frac{1}{n} \sum_{i=1}^n \exp(x_i)$$

$$(c) \hat{I}_n = \exp\left(\frac{1}{n} \sum_{i=1}^n x_i\right)$$

$$(d) \hat{I}_n = \frac{1}{n-1} \sum_{i=1}^{n-1} (x_{i+1} - x_i) \cdot \exp(x_i)$$

Question 4 (25 points). This question concerns the calculation of the integral

$$I = \int_0^3 \exp(x) dx$$

via the Monte Carlo Hit or Miss approach, using n Bernoulli variates, defined by

$$b_i = \begin{cases} 0 & y_i > \exp(x_i) \\ 1 & y_i \leq \exp(x_i) \end{cases} .$$

Here, x_i and y_i are uniformly distributed (pseudo)random variates over the intervals $(0, 3)$ and $(0, r)$, respectively.

Which of the following statements is not true?

- (a) For n realizations of b_i , the number of (pseudo)random variates that need to be generated from a one-dimensional uniform distribution is $2n$.
- (b) The Hit and Miss procedure will provide the correct answer in the limit $n \rightarrow \infty$ for any $r \geq \exp(3)$.
- (c) In the limit that $n \rightarrow \infty$, for any $r \geq \exp(3)$, $\frac{1}{n} \sum_{i=1}^n b_i$ will approach $\frac{I}{3 \exp(3)}$.
- (d) I can be more efficiently estimated by writing

$$I = \int_0^3 (\exp(x) - 1) dx + 3$$

and estimating the integral

$$\hat{I} = \int_0^3 (\exp(x) - 1) dx$$

using the Bernoulli variable

$$\hat{b}_i = \begin{cases} 0 & \hat{y}_i > \exp(\hat{x}_i) - 1 \\ 1 & \hat{y}_i \leq \exp(\hat{x}_i) - 1 \end{cases} ,$$

where \hat{x}_i and \hat{y}_i are uniformly distributed (pseudo)random variates over the intervals $(0, 3)$ and $(0, \exp(3) - 1)$, respectively.

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