

Quiz 4 will take place on Wednesday, 5 December, starting promptly at 7:30 PM, in Rooms 1-190 and 3-270. Students with last names beginning “A” through “L” should report to Room 1-190; students with last names beginning “M” through “Z” should report to Room 3-270. You will be given 90 minutes to complete the quiz.

Quiz 4 will be “open book”: you may refer to the textbook and other class materials as well as your own notes and scripts. You will not need a calculator for Quiz 4; however you can use a calculator, if you like, in order to check simple arithmetic operations. As always, laptops, tablets, and smartphones are not permitted. The quiz is entirely multiple-choice questions.

Quiz 4 is focused on the material of Unit V. In preparation for Quiz 4 you should find the study guide/summary below useful. You should also review the Unit V textbook reading, your lecture notes, and the Matlab Exercises Rec10.11. You are also encouraged to take the Unit V Quiz Question Sampler “practice quiz.”

#### MATLAB:

The main new material is creation, inspection, and manipulation of sparse matrices: the notion of “declared sparse”; MATLAB built-in functions such as `spalloc`, `sparse`, `issparse`, `nnz`, `spy`, and `full`; MATLAB sparse matrix storage; sparse matrix and vector operations; sparse backslash (for sparse Gaussian Elimination and Back Substitution). You should understand the effect on operation counts *both* of the number of nonzero entries in a matrix *and* of whether the matrix is declared sparse or not. Also, understand the use of `tic` and `toc` for timing operations. Finally, recall from Unit III the MATLAB built-in function `inv` for the evil inverse.

#### Math & Numerics:

Existence and uniqueness of solutions to linear systems of  $n$  equations in  $n$  unknowns: row view; column view; the three possibilities (the solution exists and is unique; the solution exists but is not unique; the solution does not exist); the various conditions confirmation of which assure a unique solution; infinite families of solutions in the non-unique case.

You should be prepared to do “by hand” calculations, including verification of general solutions in the non-unique case, for  $n = 2$  (two equations in two unknowns) and  $n = 3$  (three equations in three unknowns). Note for the case of  $n = 3$  we will restrict attention to matrices  $A$  with relatively simple columns and hence correspondingly simple calculations.

Spring-mass problems: Hooke’s law; derivation of equilibrium (force balance) equations; matrix formulation of the equilibrium equations as  $Ku = f$ ; the origins of sparsity in “local interactions” between nearest neighbors.

Symmetric Positive Definite (SPD) matrices: definition of S (Symmetry) and PD (Positive Definiteness); relation of the quadratic form  $(1/2)u^T Ku$  to the potential (elastic) energy in a spring system; implications for Gaussian Elimination (see below).

Gaussian Elimination and Back Substitution for  $Au = f$ : pivots and row operations; construction of  $U$  and  $\hat{f}$  from  $A$  and  $f$ ; back substitution to obtain  $u$  from  $U$  and  $\hat{f}$ ; operation

counts for general matrices. Note you should be comfortable both with the general concepts and with “by hand” calculation for small systems (i.e.,  $n = 2$  or  $n = 3$ ).

Sparse storage and operation counts for matrix and vector operations, such as the matrix vector product.

Gaussian Elimination and Back Substitution for sparse matrices: sparsity and fill-in (in  $U$  during the Gaussian Elimination process); storage requirements; operation counts (and hence timings); the special cases of tridiagonal and pentadiagonal matrices; cyclic springs and related systems.

You should understand that row permutations (“pivoting” or “partial pivoting”) are sometimes required for numerical stability, and that column permutations (reordering of unknowns) can be electively pursued to reduce fill-in and improve computational performance. For SPD systems stability is ensured and no pivoting is required; however, even for SPD systems, often column permutations are performed to improve efficiency — and then similar row permutations are also affected to preserve symmetry (and positive definiteness) and hence stability. Note on the quiz, we will always give you SPD matrices and furthermore we will never consider either row permutations or column permutations — Gaussian Elimination and Back Substitution will always be performed (by you or MATLAB) on the matrix “as given.”

The Evil Inverse (of  $A$ ): definition of the matrix inverse; physical interpretation of the columns of  $A^{-1}$ ; non-sparsity of  $A^{-1}$ ; solution of  $Au = f$  as  $u = A^{-1}f$ ; operation counts and timings (in particular, comparisons with sparse Gaussian Elimination and Back Substitution for solution of sparse linear systems).

MIT OpenCourseWare  
<http://ocw.mit.edu>

2.086 Numerical Computation for Mechanical Engineers  
Fall 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.