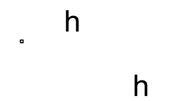
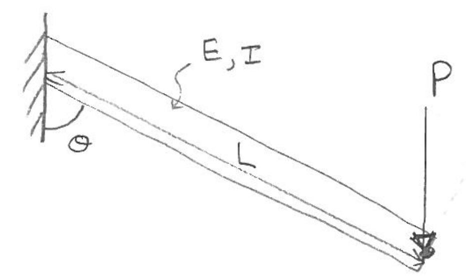
Recitation 4

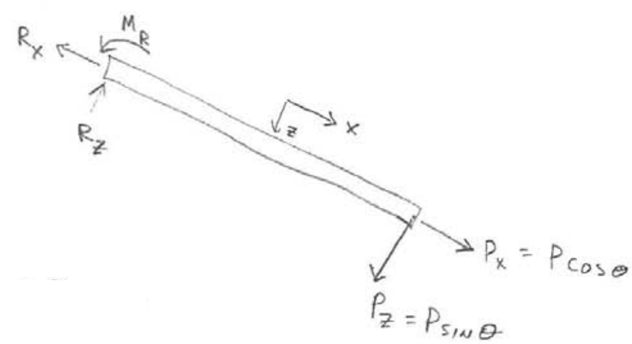
4.1 Find the max. P that this beam can support without yielding.

Cross-section

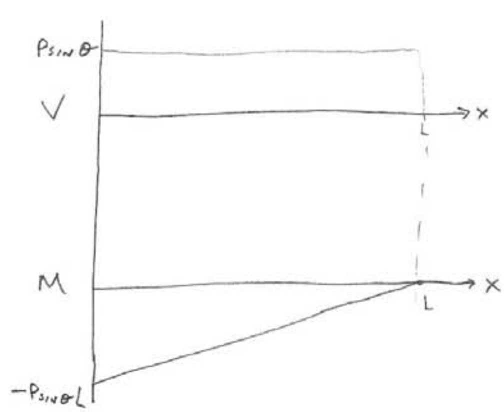




FBD:



$$
\begin{aligned}
R_x &= P\cos\theta \\
R_z &= P\sin\theta \\
M_R &= P\sin\theta L
\end{aligned}
\tag{4.1}
$$



$$
M(x) = -P\sin\theta L(1-\frac{x}{l}) \tag{4.2}
$$

In general, $\sigma_{xx} = \underbrace{\frac{N}{A}}_{\text{axial force}} + \underbrace{\frac{Mz}{I}}_{\text{bending}}$ (eqn. 4.47).

In this case:

$$\begin{aligned} N &= P\cos\theta \\ A &= h^2 \\ I &= \frac{h^4}{12} \\ M(x) &= -P\sin\theta L(1-\frac{x}{L}) \end{aligned} \tag{4.3}$$

Axial component is *constant* along length and height, $= \frac{P\cos\theta}{h^2}$ (tension).

Bending component is maximum tension at $x = 0$, $z = -\frac{h}{2}$:

$$\sigma_b = -\frac{P\sin\theta L(-\frac{h}{2})}{\frac{h^4}{12}} = \frac{6P\sin\theta L}{h^3} \tag{4.4}$$

$$\Rightarrow \sigma_{xx,max} = \frac{P\cos\theta}{h^2} + \frac{6P\sin\theta L}{h^3} \tag{4.5}$$

First yield occurs when $\sigma_{xx,max} = \sigma_y$: $\frac{P}{h^2}(\cos\theta + 6\sin\theta\frac{L}{h}) = \sigma_y$.

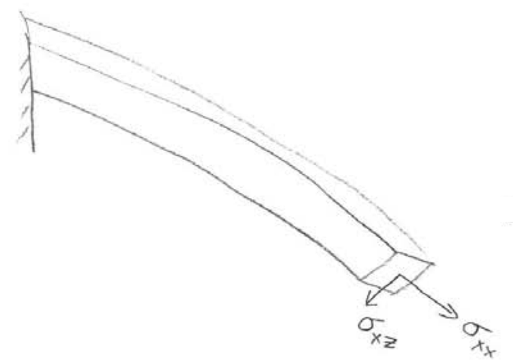
$$\boxed{P = \frac{\sigma_y h^2}{\cos\theta + 6\sin\theta(L/h)}} \tag{4.6}$$

* What is the ratio of the 2 stress components?

$$\frac{\sigma_{\text{axial}}}{\sigma_{\text{bending}}} = \frac{\frac{P\cos\theta}{h^2}}{\frac{6P\sin\theta L}{h^3}} = (\frac{1}{6})(\frac{h}{L})(\frac{\cos\theta}{\sin\theta}) \tag{4.7}$$

Example: if $\theta = 45°$, $L = 20h \rightarrow \frac{\sigma_{\text{axial}}}{\sigma_{\text{bending}}} = \frac{1}{120}$

4.2 Compare the max. shear stress in this beam to the max. tensile stress.



Recall that shear stress σ_{xz} is a parabolic function of z (eqn. 4.55)

$$\sigma_{xz}(z) = \frac{3V}{2A}\left[1 - \frac{z^2}{(h/2)^2}\right] \tag{4.8}$$

In our case, $V = P\sin\theta$

$$\sigma_{xz} \text{ is max at } z = 0, \text{ constant W.R.T. } x \;\rightarrow\; \sigma_{xz,max} = \frac{3}{2}\frac{P\sin\theta}{h^2}$$

$$\sigma_{xx,max} = \frac{P}{h^2}(\cos\theta + 6\sin\theta\frac{L}{h})$$

$$\frac{\sigma_{xz}}{\sigma_{xx}} = \frac{\frac{3}{2}\frac{P\sin\theta}{h^2}}{\frac{P}{h^2}(\cos\theta + 6\sin\theta\frac{L}{h})} = \boxed{\frac{3\sin\theta}{2(\cos\theta + 6\sin\theta L/h)}} \tag{4.9}$$

Same example: $\frac{\sigma_{xz}}{\sigma_{xx}} = \frac{3}{2(1+120)} \simeq 0.012$

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