

**Quiz 3**

December 11, 2013

**Problem 1:**

Use your knowledge of the plastic material membrane and the material testing lab in the following problem:

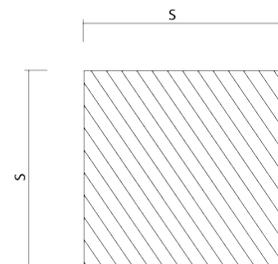
- Re-derive the solution for the load displacement relation of a circular thin clamped plastic membrane loaded by a concentrated force at the middle using the principle of virtual velocity and assume the material to be rigid-plastic. Make clear assumptions about out-of-plane displacement  $w(r)$  and in-plane displacement  $u(r)$
- Calculate all components of the in-plane strain tensor
- Define the component of the through-thickness strain,  $\epsilon_{zz}$ , using the engineering definition of strain, and determine the current thickness from the condition of plastic incompressibility.
- Compare the solution for the load displacement for constant thickness and thickness varying with strain. Find the correction factor when the variable thickness is included in the solution.

**Extra Credit:**

Calculate the coordinates of the point of instability (maximum force) for both linear and logarithmic definition of strain

**Problem 2:**

Consider an elastic-perfectly plastic material of a square cross-section of length  $s$ . What is the moment capacity of the cross section when the curvature is twice the maximum elastic curvature?



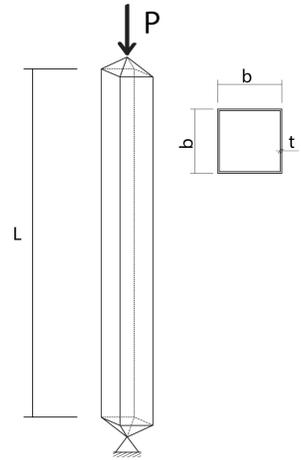
$$M = M_p \left( 1 - \left( \frac{\kappa}{\kappa_e} \right)^2 \right)$$

Where

$$\begin{aligned} \kappa &= 2\kappa_e \\ M_p &= \frac{\sigma_y s^3}{4} \\ M &= \frac{11}{12} M_p = \frac{11\sigma_y s^3}{48} \end{aligned}$$

**Problem 3:**

Consider a geometrically perfect square thin-walled elastic box column of the length  $L$ , width  $b$ , thickness  $t$ , and material properties:  $\sigma_y, E, \nu$ . The column is simply supported at the ends and subjected to a compressive load  $P$ . What is the expression for the thickness, “ $t$ ”, in terms of the input parameters such that the global Euler buckling load is equal to the ultimate strength of the buckled plates (effective width theory).



**Extra Credit:**

Given the geometry that satisfies Problem 3, which failure mode will occur first if the column is restrained at the mid-point (shown below)



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