Lecture 9

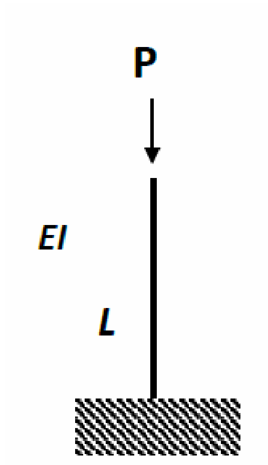
Stability of Elastic Structures

Lecture 10

Advanced Topic in Column Buckling

Problem 9-1:

A clamped-free column is loaded at its tip by a load P. The issue here is to find the critical buckling load.

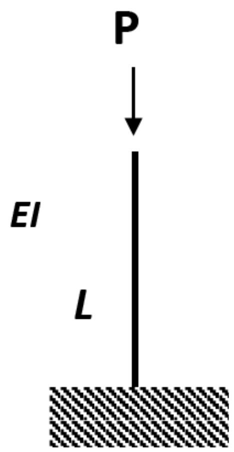


a) Suggest a simple form of the buckled of the column, satisfying kinematic boundary conditions.
b) Use the Rayleigh-Ritz quotient to find the approximate value of the buckling load.
c) Come up with another buckling shape which would give you a lower value for the buckling load.
d) Find the exact solution of the problem and show the convergence of the approximate solution to the exact solution.

Follow the example of a pin-pin column, which is presented in the notes of Lecture 9.

Problem 9-2:

Consider a clamped-free column loaded by a compressive force at the free end.



a) Determine the critical slenderness ratio β_{crit} distinguishing between the elastic and plastic buckling response. What is the buckling stress and strain?

b) Calculate the critical plastic buckling load for $\beta = 0.5\beta_{crit}$ and the corresponding stress and strain.

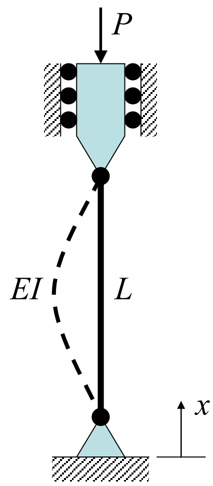
c) Calculate the critical elastic buckling load for $\beta = 2\beta_{crit}$ and the corresponding stress and strain.

d) Compare all three results.

Problem 9-3: Consider the pin-pin column.

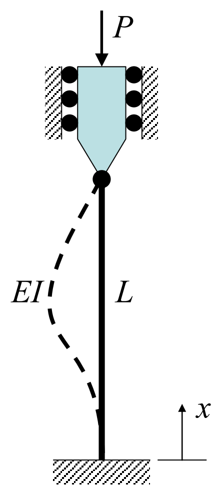
a) Suggest a polynomial buckling shape function $\phi(x)$ to improve the approximate solution derived in lecture note. Note that the one used in class was the parabolic shape.

b) Determine the accuracy relative to the exact solution.



Problem 9-4:

Present a step-by-step derivation of the buckling solution of the pin-clamped column from the local equilibrium equation.



Problem 9-5:

a) Derive the solution for an imperfect clamped-free column (like that considered in problem 9-1, following a similar derivation given in the notes for a pin-pin column in the notes.

b) Find the ratio of current deflection amplitude to the amplitude of the initial imperfection such that the resulting load is 80% of the theoretical buckling load of a perfect column.

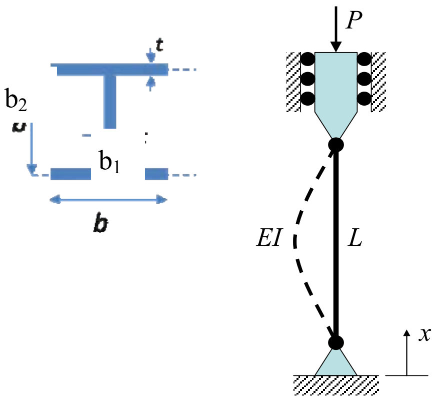
Problem 9-6:

The pin-pin elastic column of length L (shown below) is an "I" section can buckle in either plane.

a) Determine the buckling load in terms of L, b_1,b_2, t and E. Assume that $t \ll b$.

b) What should the ratio of b_1/b_2 be in order for the probability of buckling in either of the buckling planes to be the same?

Bonus: What could happen for very large width to thickness ratio?



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