

## Lecture 4

# Development of Constitutive Equations for Continuum, Beams and Plates

### Problem 4-1: Summation Convention

Hooke's law, a constitutive equation for a linear, elastic material, can be written in general form as:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \quad \text{where } \lambda \text{ and } \mu \text{ are Lamé constants.}$$

- Expand Hooke's Law. How many independent equations are there?
- Express  $\lambda$  and  $\mu$  in terms of Young's Modulus,  $E$ , and Poisson's ratio,  $\nu$ .
- Where does the symmetry of the stress strain tensor come from?
- Re-write the Hooke's law in terms of  $E$  and  $\nu$ .
- Specify the constitutive equations to the case of plane stress.

### Problem 4-2: Inverting constitutive equations

The original form of the constitutive equation is to express stress  $\sigma_{ij}$  in terms of strain. Invert the 3D constitutive equation and the 2D (plane stress) constitutive equation, meaning that strain  $\varepsilon_{ij}$  will be expressed in terms of stresses. The starting point of this problem is Eq. 4.19 for the 3D case in the printed lecture notes. For the 2D case you can use Eq. 4.32 as a starting point.

### Problem 4-3: Stress and strain deviator

Defining the stress deviator  $s_{ij}$

$$s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$

and the strain deviator  $e_{ij}$

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij}$$

Convert the constitutive equation into two separate equations, one for the spherical part and other for the distortional part. The spherical part gives a relation between the hydrostatic pressure and the change of volume (Eq. 4.21). For the distortional part the equation was not given in the notes, so we are asking you to find it.

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