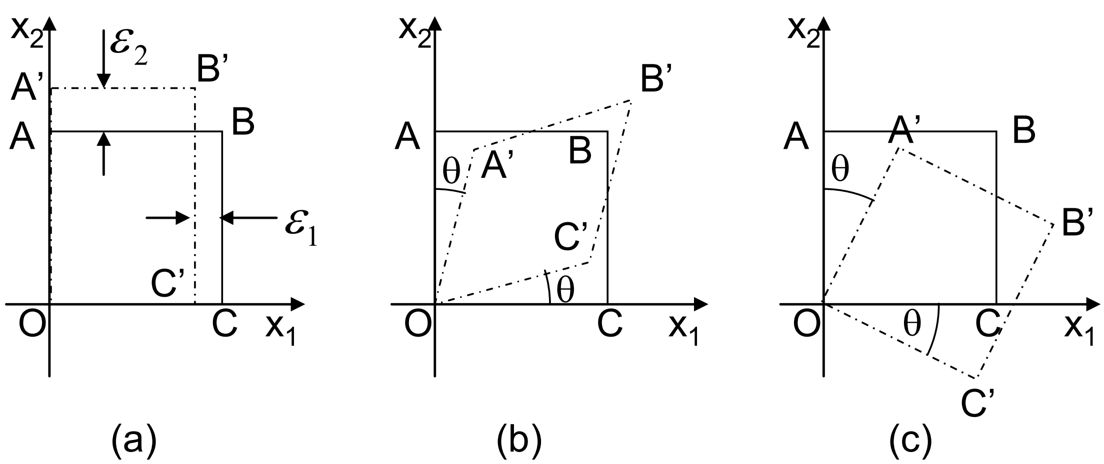
Lecture 2

The Concept of Strain

Problem 2-1: A thin- walled steel pipe of length 60 cm, diameter 6 cm, and wall thickness 0.12 cm is stretched 0.01 cm axially, expanded 0.001 cm in diameter, and twisted through 1^o. Determine the strain components of the pipe. Note that the shell with a diameter to thickness ratio of 50 is predominately in the membrane state. For plane stress there are only 3 components of the strain tensor.

Problem 2-2: Derive an expression for the change in volume of a unit volume element subjected to an arbitrary small strain tensor.

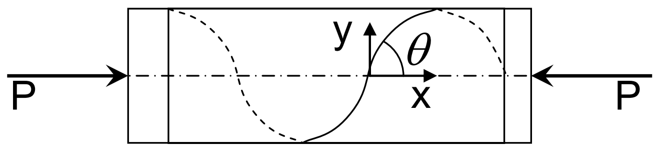
Problem 2-3: A unit square *OABC* is distorted to *OA'B'C'* in three ways, as shown in the figure below. In each case write down the displacement field (u_1, u_2) of every point in the square as a function of the location of that point (x_1, x_2) and the strain components ε_{ij}.



Problem 2-4: The strain (plane strain) in a given point of a body is described by the 2x2 matrix below. Find components of the strain tensor $\varepsilon_{x'y'}$ in a new coordinate system rotated by the angle θ. Consider four cases $\theta = 45^o, -45^o, 60^o$ *and* -60^o.

$$\varepsilon_{xy} = \begin{bmatrix} 0 & 0.05 \\ 0.05 & 0 \end{bmatrix}$$

Problem 2-5: A cylindrical pipe of 160-mm outside diameter and 10-mm thickness, spirally welded at an angle of $\theta=40^o$ with the axial (x) direction, is subjected to an axial compressive load of P=150 kN through the rigid end plates (see below). Determine the normal force $\sigma_{x'}$ and shearing stresses $\tau_{x'y'}$ acting simultaneously in the plane of the weld.



Problem 2-6: A displacement field in a body is given by:

$$u = c(x^2 + 10)$$
$$v = 2cyz$$
$$w = c(-xy + z^2)$$

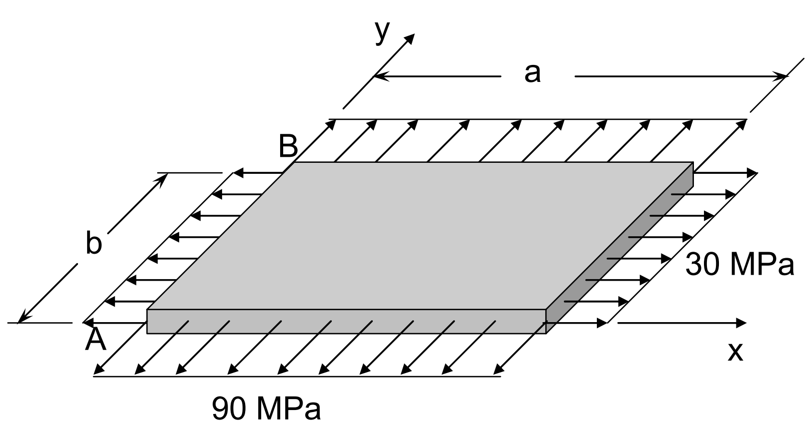
where $c=10^{-4}$. Determine the state of strain on an element positioned at $(0, 2, 1)$.

Problem 2-7: The distribution of stress in an aluminum machine component is given by:

$$\sigma_x = d(cy + 2z^2), \qquad \tau_{xy} = 3dz^2$$
$$\sigma_y = d(cx + cz), \qquad \tau_{yz} = dx^2$$
$$\sigma_z = d(3cx + cy), \qquad \tau_{xz} = 2dy^2$$

where c= 1 mm and d=1MPa/mm^2. Calculate the state of strain of a point positioned at $(1, 2, 4)$ mm. Use E=70 GPa and ν=0.3.

Problem 2-8: An aluminum alloy plate (E=70 GPa, ν =1/3) of dimensions a=300 mm, b=400 mm, and thickness t=10 mm is subjected to biaxial stresses as shown below. Calculate the change in (a) the length AB; (b) the volume of the plate.



<u>Problem 2-9:</u> Consider the general definition of the strain tensor in the 3D continuum. The three Euler-Bernoulli hypotheses of the elementary beam theory state:

1. Plane remain plane
2. Normal remain normal
3. Transverse deflections are only a function of the length coordinate x.

Proof that under the above assumptions the state in the beam is uni-axial, meaning that the only surviving component of the strain is in the length, x-direction. The proof is sketched in the lecture notes, but we want you to redo the step by step derivation.