

1.138J/2.062J/18.376J, WAVE PROPAGATION

Fall, 2006 MIT

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HW 5: Due Nov 30. Take-home final will be handed out on Dec 4

1. Capillary-gravity waves.

Consider a free surface with capillarity in water of infinite depth. Solve for the transient two-dimensional response to a localized initial elevation on the free surface

$$\zeta(x, 0) = \frac{b}{\pi} \frac{1}{x^2 + b^2}$$

- Deduce by Fourier transform the formal solution.
- Deduce the asymptotic result for $\zeta(x, t)$ for large t but fixed x/t . and describe the physics. Sketch first the curve of C_g vs k and observe that for some x/t there are two stationary points.
- Examine the case where the two stationary points coalesce to one where

$$\omega''(k) = 0, \quad \text{but} \quad \omega'''(k) \neq 0.$$

Suggestion : You have to perform Taylor expansion of the phase function by keeping the term involving $\omega'''(k)$.