

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
2.06 Fluid Dynamics, Spring 2013

Quiz 1

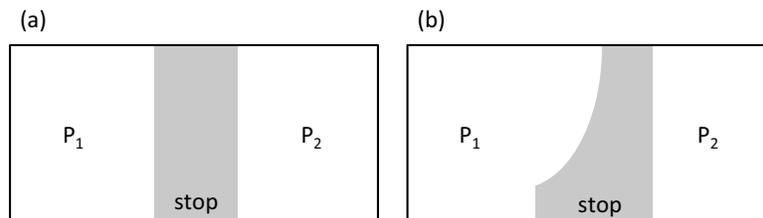
April 25, 2013

Instructions: *Closed Book, no crib sheets, no Phones, no Laptops, no Ipads. You are allowed a calculator and a writing instrument for this quiz. Please show your work steps. State your assumptions and justify your equations. Partial credit will be awarded on careful and clear arguments. Wrong unsupported numerical answers can only receive zero credit.*

Problem 1 (6 points): *Concept Questions:* Indicate which of the following statements are TRUE and which are FALSE. You do NOT need to explain why each statement is true or false, just whether it is true or false.

a) (2 points) Consider an object suspended in a liquid. The density of the object and liquid are identical. If container holding the liquid and object moves to the right with constant acceleration, the object will move to the left.

b) (2 points) A stop is placed to separate two gas chambers at uniform pressures P_1 and P_2 as shown below. The shape of the stop is different in two scenarios (a) and (b) sketched below. The horizontal pressure force acting on the stops (a) and (b) are identical.

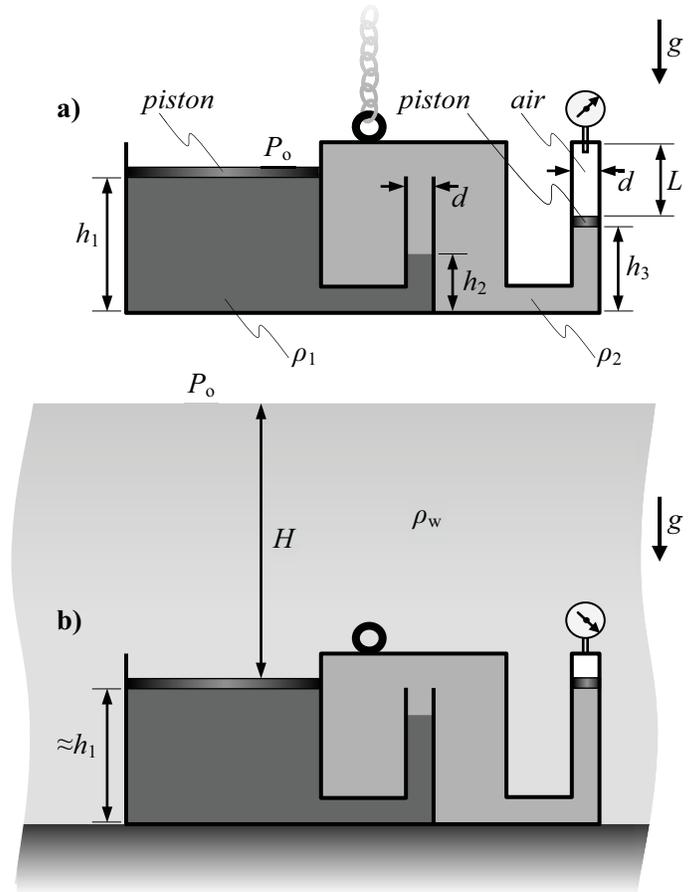


c) (2 points) Assuming that the interfacial tensions of water-air and water-oil interface are identical, the maximum size of a stable water drop in air is smaller than that in oil.

Problem 2 (15 points)

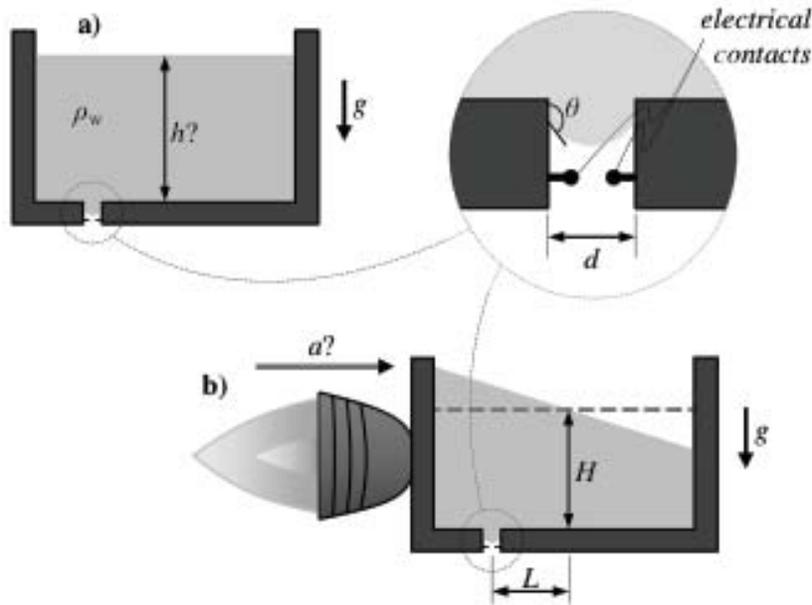
Consider the system comprising two liquid columns (densities ρ_1 and ρ_2), two pistons, and an air column as shown in the figure to the right. Both the liquids are incompressible. The two pistons are ideal, i.e., massless and frictionless. Air can be modeled as an ideal gas (i.e., $PV = mRT$)

- a) (5 points) Consider the system in state (a) surrounded by ambient pressure P_o . Determine the air pressure P_a in terms of the densities and heights of the liquids (h_1 , h_2 and h_3 as shown on the figure).
- b) (10 points) Now the entire system is slowly lowered into water to a depth of H as shown in part (b) of the figure. Assume temperature remains constant throughout the process. Derive a relation between the depth H and the air pressure P_a . (Hint: the liquid columns will change in height. Since the diameter of the first column is large, neglect changes to h_1)



Problem 3 (12 points)

Consider a trough with a small cylindrical hole of diameter d at the bottom. The hole is hydrophobic, i.e., the three-phase contact angle $\theta > 90^\circ$ as shown in the zoomed-in view in the figure. Two electrical contacts are placed under the meniscus.

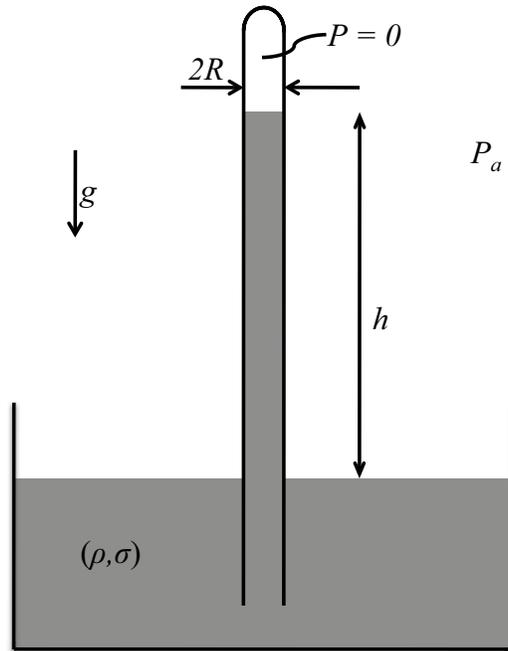


(a) (5 points) Determine the minimum height h of the liquid column at which the air-water interface within the cylindrical hole becomes unstable and moves down shorting the electrical contacts.

(b) (7 points) Now consider another situation (case b in the figure) in which the trough has a static liquid column height of H . The cylindrical hole of diameter d is a distance L from the trough's center. What is the minimum acceleration a at which the air-water interface within the cylindrical hole becomes unstable and moves down shorting the electrical contacts? Please express your answer in terms of the given quantities.

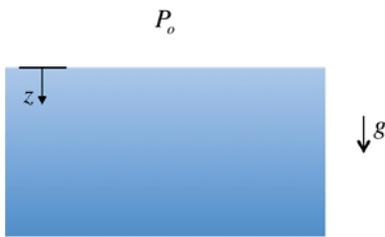
Problem 4 (17 points)

Consider a barometer shown in figure below. The density of the liquid is ρ and the acceleration due to gravity is g . The ambient pressure is P_a and the pressure above the liquid column in the tube is zero.



1. (2 points) What is the relation between the ambient pressure P_a and height of the liquid column h , when surface tension effects are neglected?
2. (7 points) Now consider a case when the radius R of the barometer tube is small and surface tension effects become important. The three phase contact angle is θ and the surface tension of the liquid is σ . Consider a case when $\theta > 90^\circ$ (hydrophobic). What is the expression for P_a under this regime?
3. (5 points) Under what conditions can we neglect surface tension effects? Please express your answer in terms of the given quantities.
4. Now consider the case when the entire system is accelerated with an acceleration a to the right. Will the height of the liquid column in the tube above the free surface be greater than, equal to, or smaller than the height of the liquid column for a stationary barometer for the following cases:
 - (a) (1 point) when the barometer tube is in the middle (as above)
 - (b) (1 point) when the barometer tube is to the left
 - (c) (1 point) when the barometer tube is to the right

Fluid statics

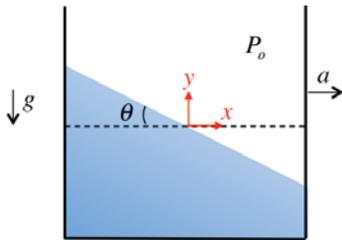


$$-\nabla P + \rho \bar{g} = \rho \bar{a}$$

with gravity, $\bar{g} = g\hat{k}$ & no acceleration

$$P(z) = P_o + \rho gz$$

Acceleration



with gravity, $\bar{g} = -g\hat{j}$ & an acceleration, $\bar{a} = a\hat{i}$

$$P(x, y) = -\rho ax - \rho gy + P_o$$

$$\tan \theta = \frac{a}{g}$$

Buoyancy

$$F_B = \rho g V$$

where ρ is the density of a displaced liquid, and V is the volume of a displaced liquid.

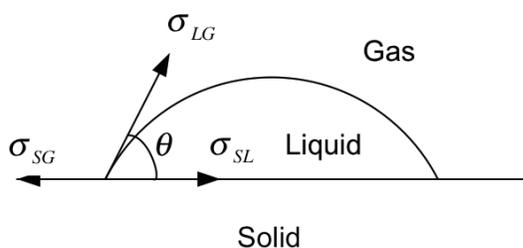
Surface Tension

Young-Laplace equation

For a curved liquid-air interface, defined by two radii of curvature, R_1 & R_2

$$P_i - P_o = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Contact angle, θ



$$\cos \theta = \frac{\sigma_{SG} - \sigma_{SL}}{\sigma_{LG}}$$

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