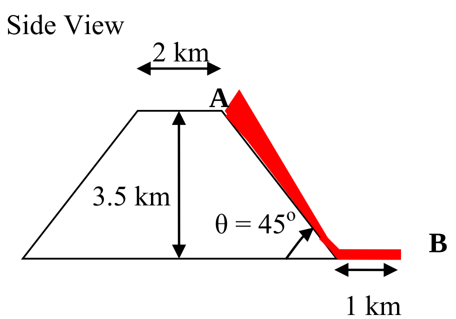
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

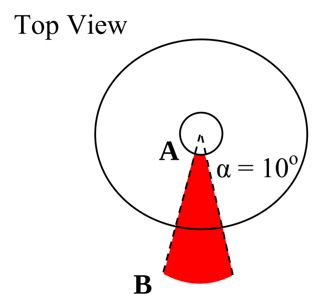
2.06 Fluid Dynamics

Practice Problems for Quiz 2, Spring Term 2013

Problem 1

A volcano erupts. A constant-density lava flows down the volcano's slope at an angle $\theta = 45^{o}$. The steady volumetric flow rate is known to be 300 m3/s at point A (elevation: 3.5 km). The lava spreads as it flows down the slope, covering an angle α=10o, as shown on the figure. In steady-state, what is the average velocity vB of the lava at point B if the height of the lava flow at B is 10 cm?

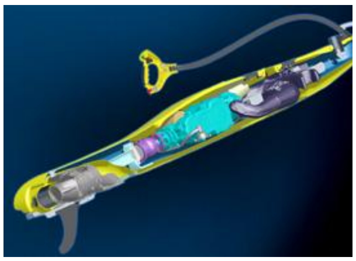


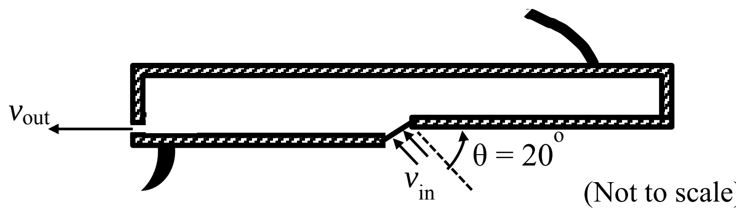


$v_B =$

Problem 2: Surfboard

In a motorized surfboard design, water (ρ= 1000 kg/m3) enters the horizontal board at average velocity v_{in}=5 m/s at a nominal angle θ=20^o and through an area (perpendicular to vin) of A_{in}=0.01 m2. The water is pumped through the board and ultimately exists at high speed v_{out}. What is the required mass flow rate and constant average velocity v_{out} of the exit jet if the total drag forces along the board is to be F=1500 N?





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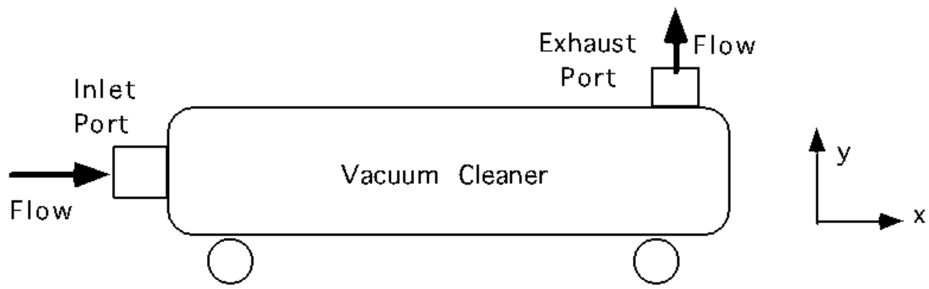
Problem 3: Vacuum Cleaner

A vacuum cleaner is operated as shown in the diagram. The diameter of the vacuum cleaner inlet port is d_{in} = 3 cm. The velocity of the flow at the inlet port is v_{in} = 30 m/s. The density of air is 1.2 kg/m^3.



a) Sketch the streamlines into the input port of the vacuum cleaner.

b) Estimate the force in the x-direction on the body of the vacuum cleaner necessary to keep it stationary.

The vacuum cleaner is run in reverse so that it blows at $v_{r,out}$ = 30 m/s out its inlet port.

c) Sketch the streamlines into the "inlet" port of the vacuum cleaner.

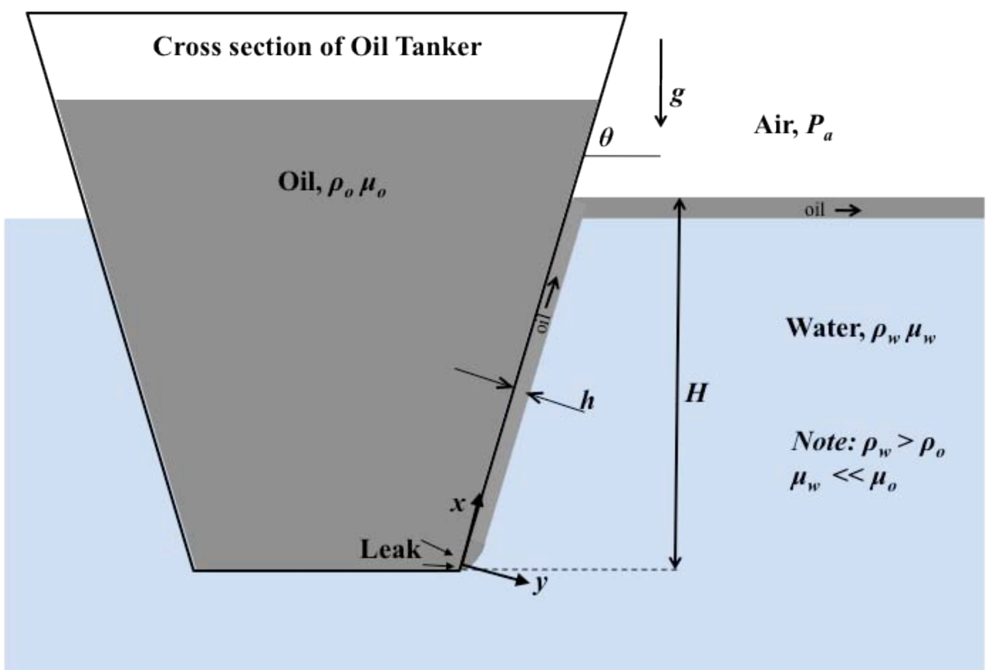
d) Estimate the force necessary in the x-direction to keep the vacuum cleaner stationary.

e) Are the forces calculated above equal and opposite? If so why? If not why not?

Problem 4: Oil Spill from an Oil Tanker

Accidents involving breach of the structure of oil tankers can result in oil spills that can have catastrophic effects on sea life and require expensive cleanup operations. Some recent examples include spills near New Zealand and South Korea from tanker accidents as shown here. The goal of the problem is to determine the flow rate of the leaking oil in terms of measurable parameters of the oil films that form.

Consider oil leaking out of a crack in an oil tanker at sea as shown in the cross section schematic below. The crack has a width *w* into the page and is located at a depth *H* from the surface. The density of the seawater ρ_w is greater than that of the oil $\rho_{o,}$ causing the oil to rise along the wall of the tanker, up to the ocean's surface. The leaked oil forms a thin film along the wall of the tanker, which is inclined at an angle θ from the horizontal. The thickness of the oil film is h ($<< H$) and the viscosity of the oil is μ_o. Assume that the viscosity of the sea water μ_w is much smaller than that of the oil, i.e. $\mu_w << \mu_o$. You can also assume that the flow in the oil film is fully-developed. Note that the velocity at the oil-water interface is not zero.



Considering the steady state oil flow in the inclined film of thickness h,

a). Provide the velocity and pressure boundary conditions for the oil flow in the inclined film, from the crack to the free surface (*Hint: four boundary conditions in total*).

b). Determine the pressure gradient $\dfrac{\partial p}{\partial x}$ for this oil flow in the inclined film, in terms of the parameters of the problem.

c). Derive an expression for the velocity of this oil flow and sketch the velocity profile, in terms of the parameters of the problem.

d). Derive an expression for the volumetric oil flow rate Q per unit length of the ship and in terms of the parameters of the problem.

e). What is the velocity of the water next to the oil film, i.e. at $y = h$, in terms of the parameters of the problem?

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