2.06 Equation Sheet for Quiz 2 (Spring 2013)

Conservation Relations for Open Systems (using ϑ for velocity)

Mass conservation

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$

Linear momentum conservation

$$\frac{d(M\vec{\vartheta})_{CV}}{dt} = \sum \dot{m}_{in}\vec{\vartheta}_{in} - \sum \dot{m}_{out}\vec{\vartheta}_{out} + \sum \vec{F}_{ext}$$

Bernoulli Equation (using ϑ for velocity)

For steady, frictionless flows,

$$\frac{P_1}{\rho} + \frac{\vartheta_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\vartheta_2^2}{2} + gz_2$$

Differential Form

Mass conservation: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{\vartheta}) = 0$

(for incompressible) $\nabla \cdot \vec{\vartheta} = 0$

Navier-Stokes equation: $\rho \frac{D\vec{\vartheta}}{Dt} = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{\vartheta}$

Total Derivative operator: $\frac{D}{Dt} = \left[\frac{\partial}{\partial t} + \vec{\vartheta}.\nabla \right]$

Vector Operators

Cartesian Coordinates (x, y, z)

$$\nabla \psi = \frac{\partial \psi}{\partial x}\hat{i} + \frac{\partial \psi}{\partial y}\hat{j} + \frac{\partial \psi}{\partial z}\hat{k}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

Cartesian coordinates:

Mass conservation (continuity equation): $$\frac{\partial \vartheta_x}{\partial x}+\frac{\partial \vartheta_y}{\partial y}+\frac{\partial \vartheta_z}{\partial z}=0$$

Navier-Stokes equations:

$$\rho\left(\frac{\partial \vartheta_x}{\partial t}+\vartheta_x\frac{\partial \vartheta_x}{\partial x}+\vartheta_y\frac{\partial \vartheta_x}{\partial y}+\vartheta_z\frac{\partial \vartheta_x}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 \vartheta_x}{\partial x^2}+\frac{\partial^2 \vartheta_x}{\partial y^2}+\frac{\partial^2 \vartheta_x}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial \vartheta_y}{\partial t}+\vartheta_x\frac{\partial \vartheta_y}{\partial x}+\vartheta_y\frac{\partial \vartheta_y}{\partial y}+\vartheta_z\frac{\partial \vartheta_y}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 \vartheta_y}{\partial x^2}+\frac{\partial^2 \vartheta_y}{\partial y^2}+\frac{\partial^2 \vartheta_y}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial \vartheta_z}{\partial t}+\vartheta_x\frac{\partial \vartheta_z}{\partial x}+\vartheta_y\frac{\partial \vartheta_z}{\partial y}+\vartheta_z\frac{\partial \vartheta_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 \vartheta_z}{\partial x^2}+\frac{\partial^2 \vartheta_z}{\partial y^2}+\frac{\partial^2 \vartheta_z}{\partial z^2}\right)$$

Newtonian viscous shear stresses :

$$\tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial \vartheta_x}{\partial y}+\frac{\partial \vartheta_y}{\partial x}\right)\quad \tau_{xz}=\tau_{zx}=\mu\left(\frac{\partial \vartheta_x}{\partial z}+\frac{\partial \vartheta_z}{\partial x}\right)\quad \tau_{yz}=\tau_{zy}=\mu\left(\frac{\partial \vartheta_y}{\partial z}+\frac{\partial \vartheta_z}{\partial y}\right)$$

Reduced Equations

Above equations reduce to much simpler 2D form when the thickness of the fluid film is much smaller its length. When flow direction is *x* and thickness direction is *y*

Mass Conservation

$$\frac{\partial \vartheta}{\partial x}=0$$

Navier-Stokes Equation

$$-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 \vartheta}{\partial y}\right)=0$$

2.06 Fluid Dynamics
Spring 2013