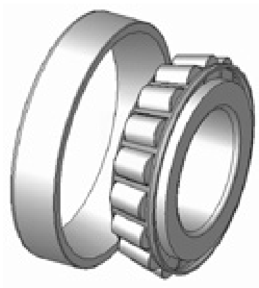
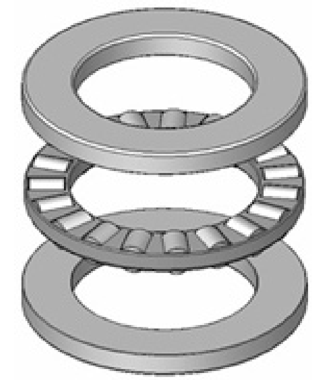
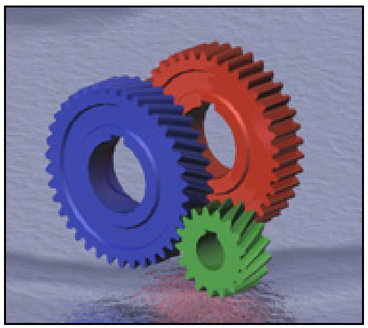
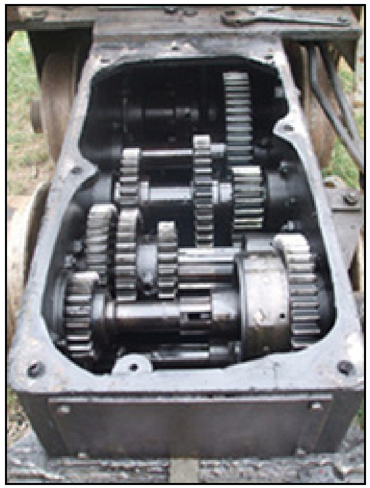
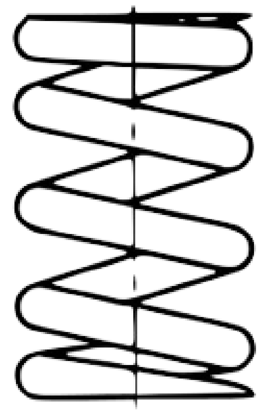
Topics in Machine Elements

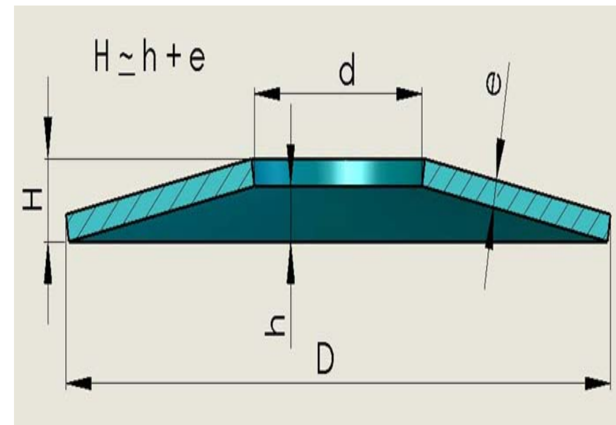


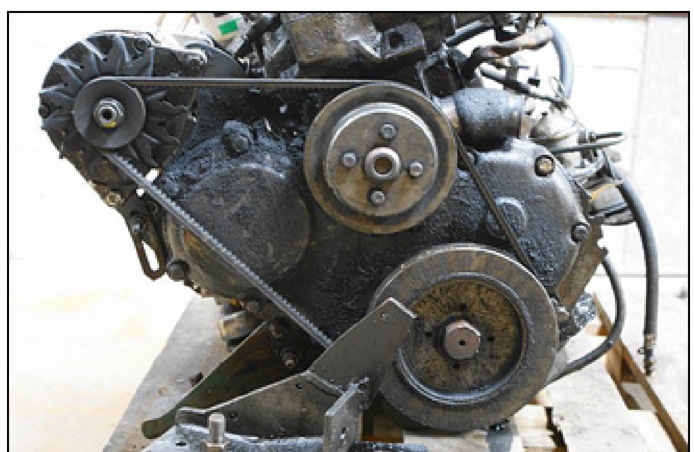




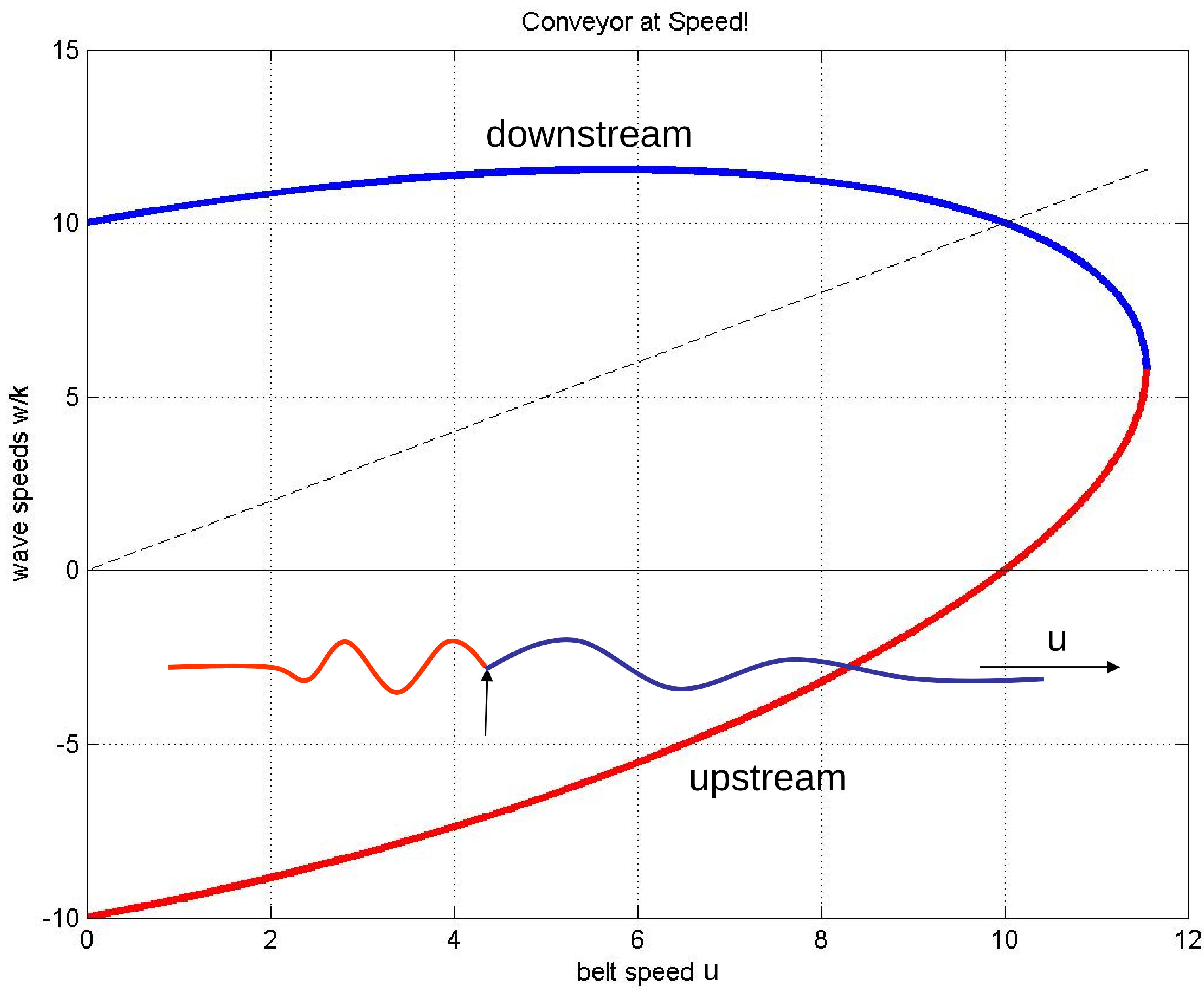




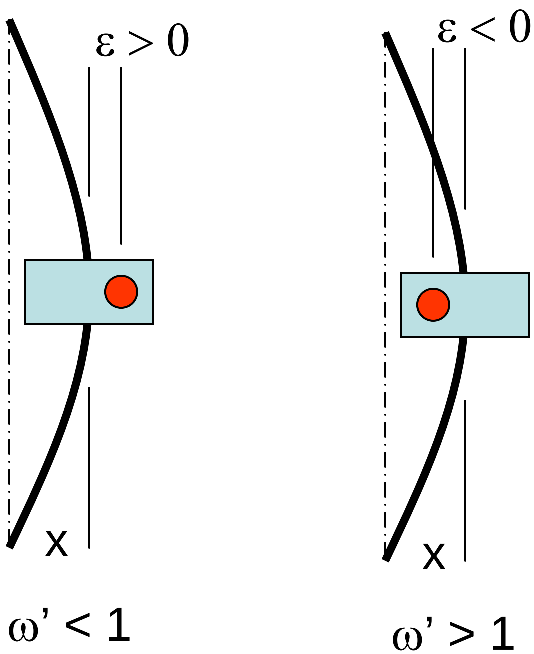


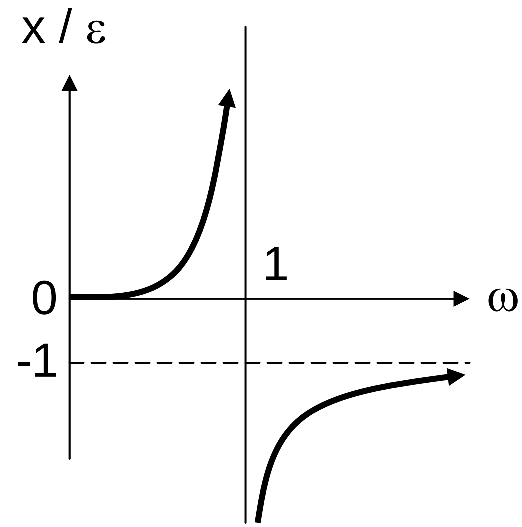


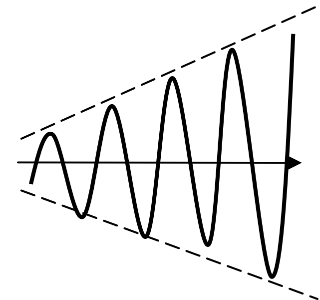
Images by Silberwolf, Red Rooster, ruizo, and Aspectomat at Wikimedia Commons, and i am indisposed and David on LRM on Flickr.


Conveyor at Speed!
downstream
upstream
u
wave speeds w/k
belt speed u
15
10
5
0
-5
-10
0
2
4
6
8
10
12

Critical speed of a shaft with a mass – synchronous case (slowest)







At $\omega' = 1$, no steady-state configuration; deflection grows linearly with time!

Shaft has stiffness k at the location of the flywheel,
ε is eccentricity

In steady-state,

$$kx = m\omega^2(x+\varepsilon) \rightarrow$$

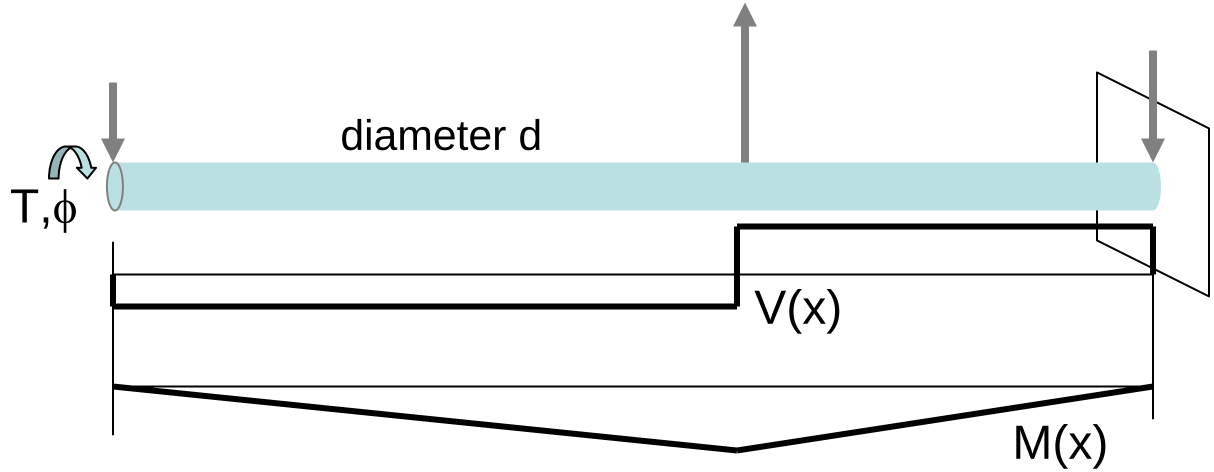
$$(k - m\omega^2)x = m\omega^2\varepsilon \rightarrow$$

$$x / \varepsilon = m\omega^2 / (k - m\omega^2)$$

$$= \boldsymbol{\omega'^2 / (1 - \omega'^2)} \qquad \text{where } \omega' = \omega/\omega_m$$

ADDITIONAL formulas available for multiple masses on a shaft

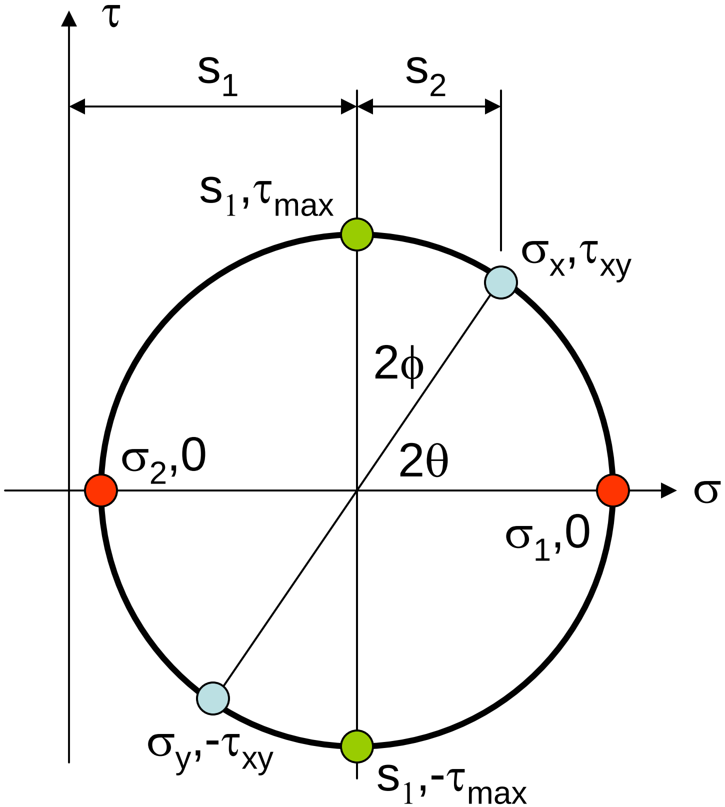
Circular shafts in **combined** loading

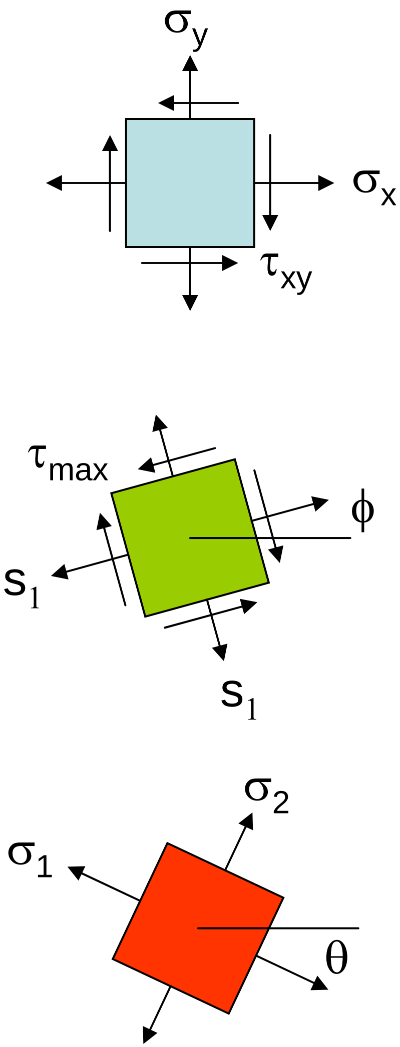


Polar moment of inertia:	$J = \pi d^4 / 32$
Bending moment of inertia:	$I = \pi d^4 / 64 \quad (= J / 2)$
Torque:	T
Angular deflection	$\phi = T L / J G$
Bending shear stress:	$V(x)$
Bending moment:	$dM(x)/dx = V(x)$
Max shear stress (at shaft surface):	$\tau_{max} = Tr/J = 16T / \pi d^3$
Max bending stress (at top/bottom surface):	$\sigma_b (x) = M(x)r / I = 32 M(x) / \pi d^3$
Transverse shear stress (on u/d centerline):	$\sigma_t (x) \sim 16 V(x) / 3 \pi d^2$

… and don't forget stress concentration!

Just what you need – Mohr stress!





$s_1 = (\sigma_x + \sigma_y) / 2$
$s_2 = (\sigma_x - \sigma_y) / 2$

$\tau_{max} = \mathrm{sqrt}(s_2^2 + \tau_{xy}^2)$

$\tan 2\theta = \tau_{xy} / s_2$
$\tan 2\phi = - s_2 / \tau_{xy}$

$\sigma_1 = s_1 + \tau_{max}$
$\sigma_2 = s_1 - \tau_{max}$

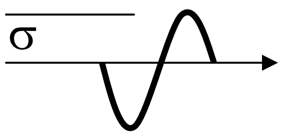
principal stresses

Fatigue of Engineering Materials

Basquin's equation: $\mathbf{L = k_b \sigma^{-b}}$ where L is the lifetime (cycles) at stress level σ

k_b, b are determined from test data

Curve fits data for cycles at a given, constant stress level



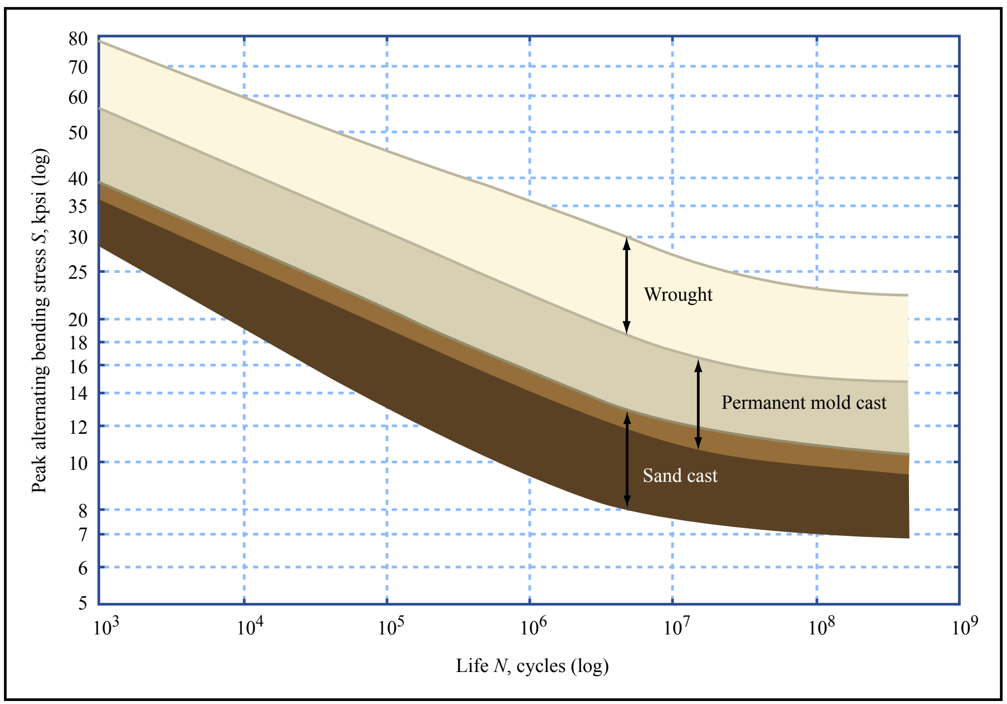


Figure by MIT OpenCourseWare. Adapted from Fig. 6-11 in Shigley & Mischke.

For loading at multiple levels, Consider Miner's equation:

$$\mathbf{N_1/L_1 + N_2/L_2 + \ldots = 1}$$

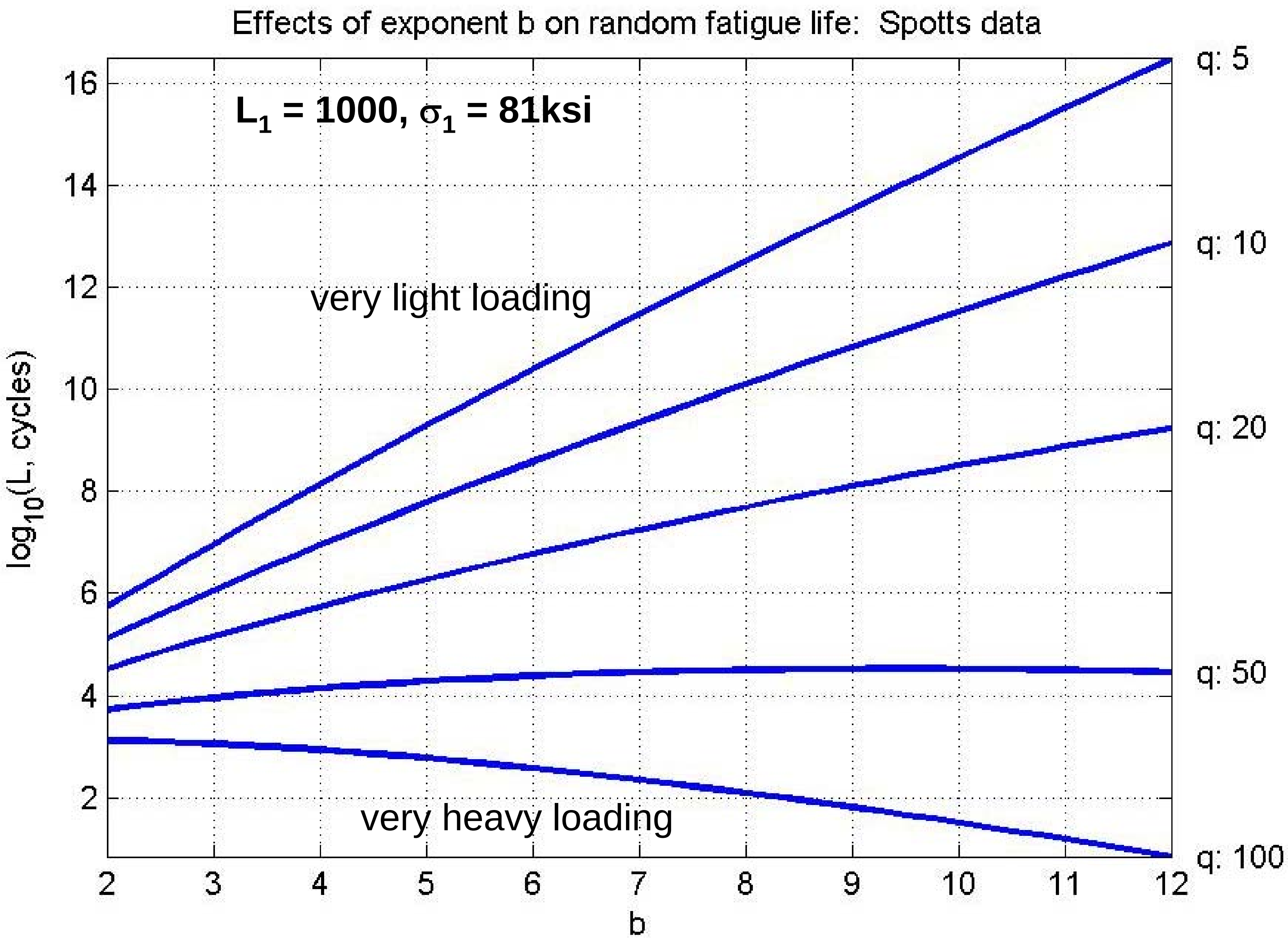
where

L_i is the lifetime at stress level σ_i

N_i is the number of cycles actually executed at σ_i

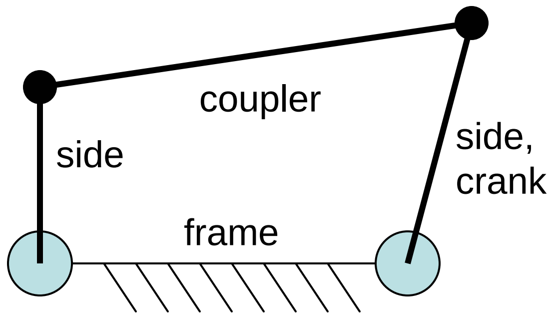
Concept: **Accumulation of damage**

Simplified approach does not take into account the sequence of loading levels



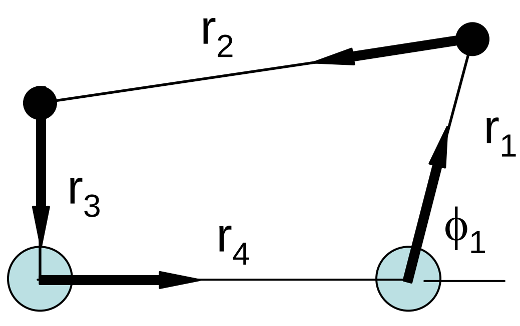
q: twice the standard deviation of the underlying Gaussian stress

Four-Bar Linkage Kinematics



l = length of longest link
s = length of shortest link
p,q = lengths of intermediate links

Grashof's theorem:
If s+l <= p + q, then at least one link will revolve.
If s+l > p + q, then all three links are rocking.



Categories

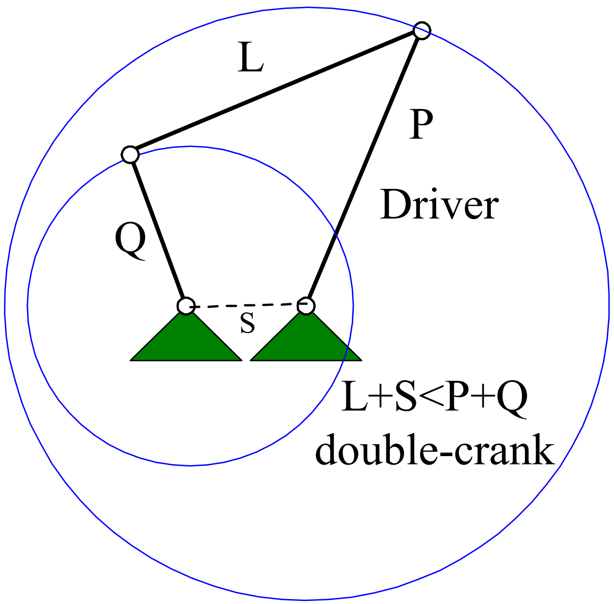
l + s < p + q	*double-crank, if s is the frame*
l + s < p + q	*rocker-crank, if s is one side*
l + s < p + q	*double rocker, if s is coupler*
l + s = p + q	*change point*
l + s > p + q	*triple-rocker*

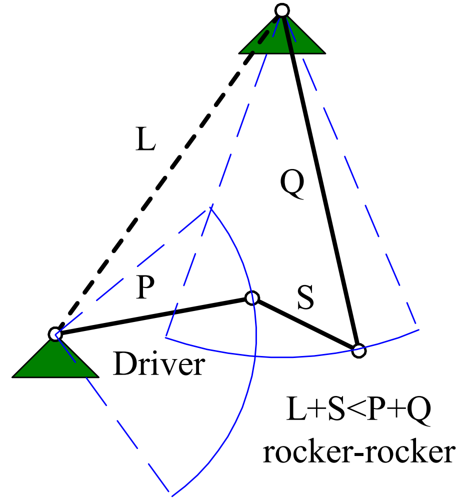
Let ϕ_i be the absolute angle of link r_i vector as shown
The chain satisfies:

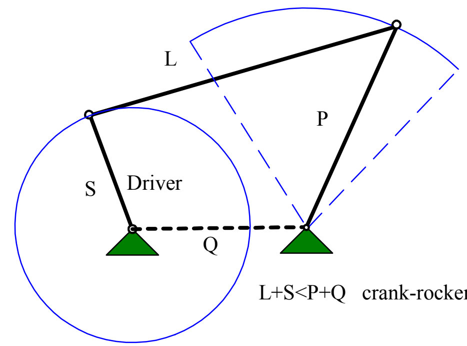
X-loop: $r_1\cos\phi_1 + r_2\cos\phi_2 + r_3\cos\phi_3 + r_4 = 0$ (note $\phi_4 = 0$)

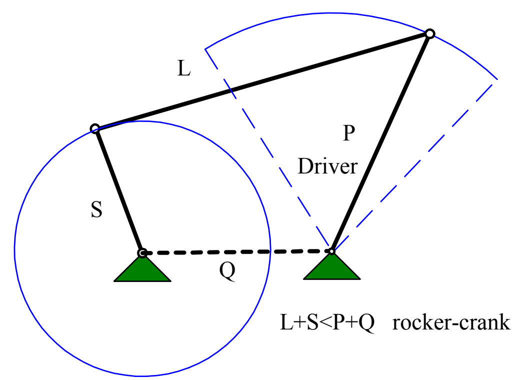
Y-loop: $r_1\sin\phi_1 + r_2\sin\phi_2 + r_3\sin\phi_3 = 0$

Two equations, two unknowns $[\phi_2, \phi_3]$ if ϕ_1 given – use a nonlinear solver









B. Paul, Kinematics and dynamics of planar machinery, 1984.

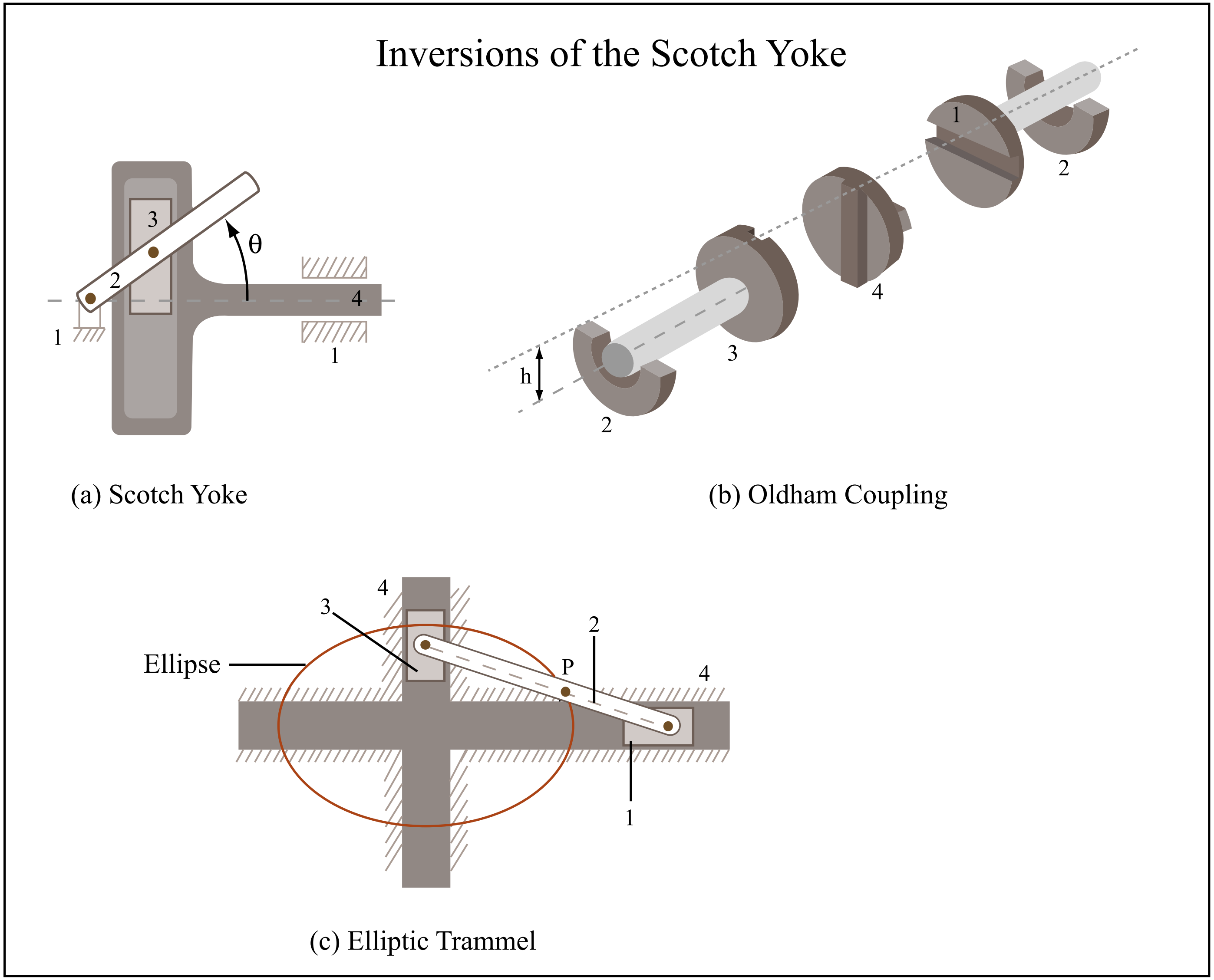
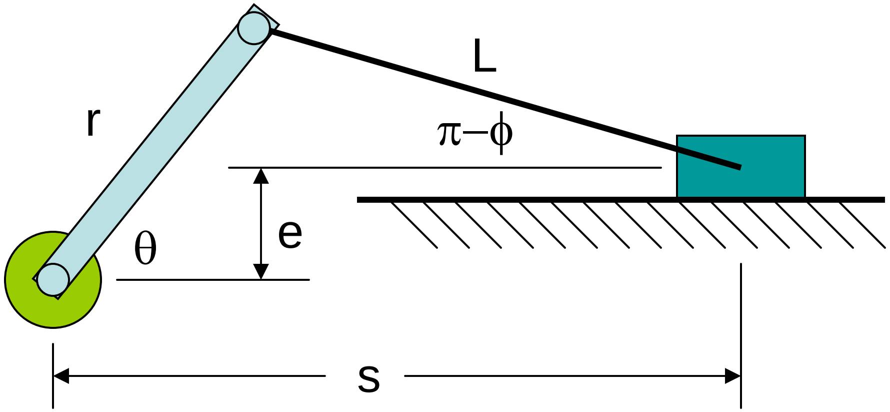


Figure by MIT OpenCourseWare. Adapted from Fig. 1.51-1 in Paul, Burton. *Kinematics and Dynamics of Planar Machinery*. Englewood Cliffs, NJ: Prentice-Hall, 1979.

Slider-Crank Kinematics



X-loop: **$r \cos\theta - L \cos\phi - s = 0$**

Y-loop: **$r \sin\theta - L \sin\phi - e = 0$**

Two equations, two unknowns $[s,\phi]$ if θ is given

$s_{max} = s_1 = \text{sqrt}\,[\,(L + r)^2 - e^2\,]$

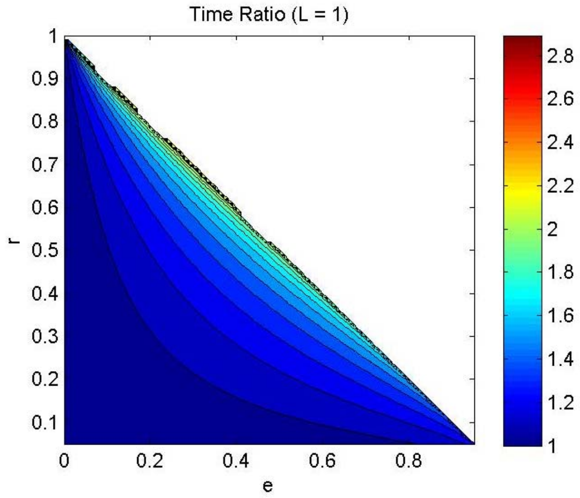
$s_{min} = s_2 = \text{sqrt}\,[\,(L - r)^2 - e^2\,]$

θ at $s_{max} = \theta_1 = \arcsin(\,e / (L + r)\,)$

θ at $s_{min} = \theta_2 = \pi + \arcsin(\,e / (L - r)\,)$

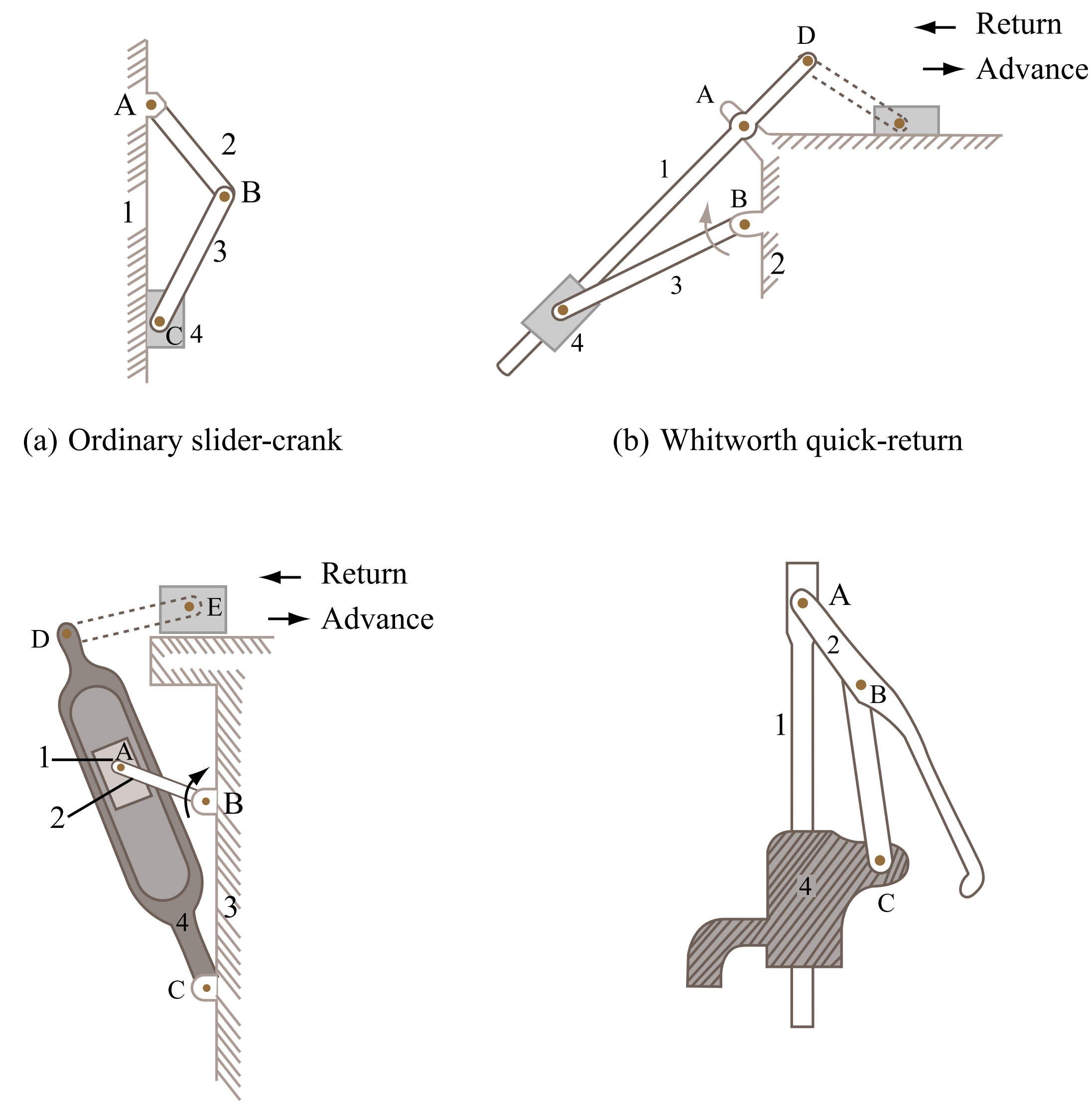
Slider moves to the right $s_{min} \rightarrow s_{max} : \theta_2 \rightarrow \theta_1$

Slider moves to the left $s_{max} \rightarrow s_{min} : \theta_1 \rightarrow \theta_2$



So time ratio TR = $(\theta_2 - \theta_1) / (2\pi - \theta_2 + \theta_1)$: captures "quick-return" characteristic

Inversions of Slider-Crank Mechanism



(a) Ordinary slider-crank

(b) Whitworth quick-return

(c) Crank-shaper (ordinary quick-return)

(d) Hand pump

Figure by MIT OpenCourseWare. Adapted from Fig. 1.42-1 in Paul, Burton. *Kinematics and Dynamics of Planar Machinery.* Englewood Cliffs, NJ: Prentice-Hall, 1979.

Radial Ball Bearings

Ball Bearings in radial loading

- Load rating is based on fatigue:
  - Basic Rating Load C causes failure in 10% of bearings at 1 million cycles
- Hardness and finish of balls and rollers is critical!
  - Use e.g., high-carbon chromium steel 52100, min 58 Rockwell.
  - Finish balls to 50nm typical, races to 150nm typical
  - Quality indexed by ABEC rating: 1 to 9
- Examples of Ratings:
  - *#102: 15mm bore, 9x32mm dia: 4.3 kN C 2.4 kN static*
  - *#108: 40mm bore, 15x68mm dia: 13.6 kN C 10.9 kN static*
  - *#314: 70mm bore, 20x110mm dia: 80 kN C 59 kN static*
- Note static load rating < dynamic load rating!

- Scaling: <u>life goes as load cubed</u>
  - Decreasing the load by ½ will increase expected life by 8-fold, etc.

Effect of Axial Loading on Radial Bearings: Equivalent radial load

$$\mathbf{Max(1.2P_r\,,\ 1.2XP_r + YP_a)}$$

where P_r and P_a are axial and radial loads, and X,Y →

P_a/ZiD^2	X	Y
25	0.56	2.3
50	0.56	2.0
100	0.56	1.7
200	0.56	1.5
500	0.56	1.2
1000	0.56	1.0

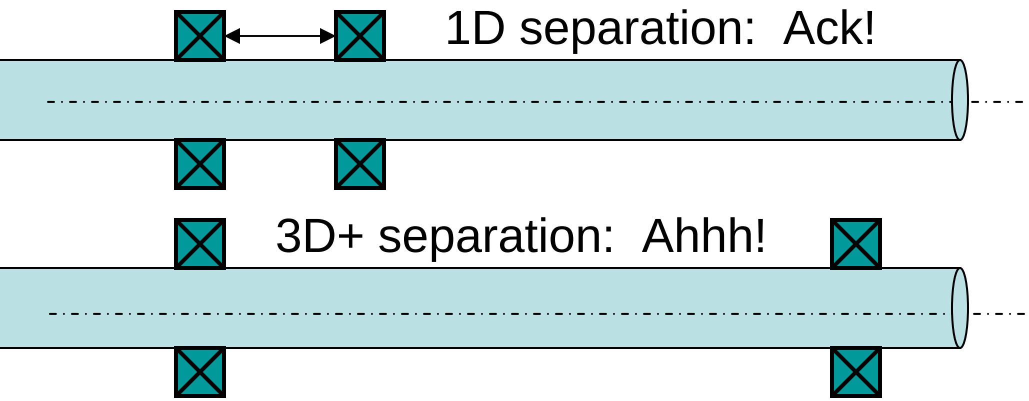
Z = number of balls
i = number of rings
D = ball diameter

Service factor C_1 = [1 - 3+] to account for shock loads:

$$\mathbf{Max(1.2C_1P_r\,,\ 1.2C_1XP_r + C_1YP_a)}$$

Concept of <u>accumulated damage</u> (Miner's equation) applies

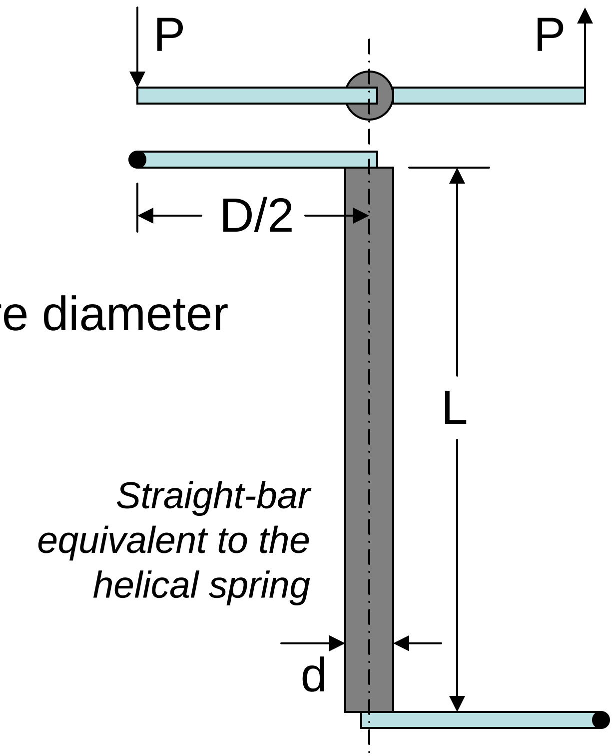
Use tapered roller bearings for large combined loads
OR
Radial bearings and thrust bearings separately



Confidence levels adjustment to lifetime:

90%	**1.0**
95%	**0.62**
99%	**0.21**

Helical Springs



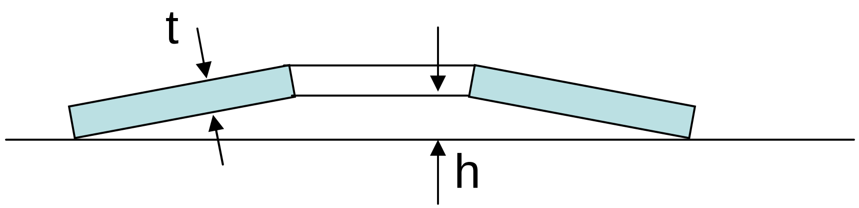
Yes, you <u>can</u> derive the stiffness in a helical spring!

Let	$c = D/d$ = coil diameter / wire diameter
Number of coils	N
Wire length	$L \sim \pi D N$
Wire area	$A = \pi d^2 / 4$
Rotary MOI of wire	$J = \pi d^4 / 32$
Axial load	P
Wire torsion from load	$T = P D / 2$

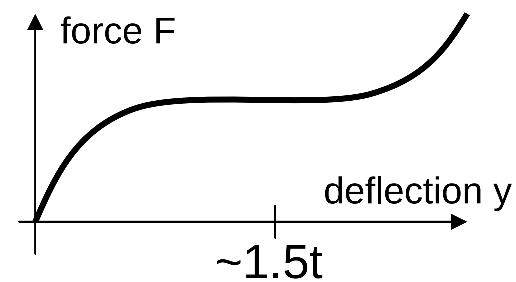
Torsional shear at wire surface	$\tau_T = T d / 2 J = 8 P D / \pi d^3$, and
Transverse shear at mid-line	$\tau_t = 1.23 P / A = (0.615/c) \times \tau_t$, so
Total shear stress	$\tau = \tau_t + \tau_T = (1 + 0.615/c) \times \tau_t$
	(but 0.615/c is small if c is big)

Differential angle	$\delta\phi = T \delta L / J G = 16 P c^2 \delta N / d^2 G$
Differential deflection	$\delta x = \delta\phi D / 2$ (90 degrees away) $\sim 8 P c^3 \delta N / d G$
Integrated deflection	$x = 8 P c^3 N / d G$
Stiffness	$\mathbf{k = P / x = G d / 8 c^3 N}$

Belleville Spring



For the case h/t ~ 1.5 →



Useful in assembly operations…

Spur Gears

Generation of Involute Teeth

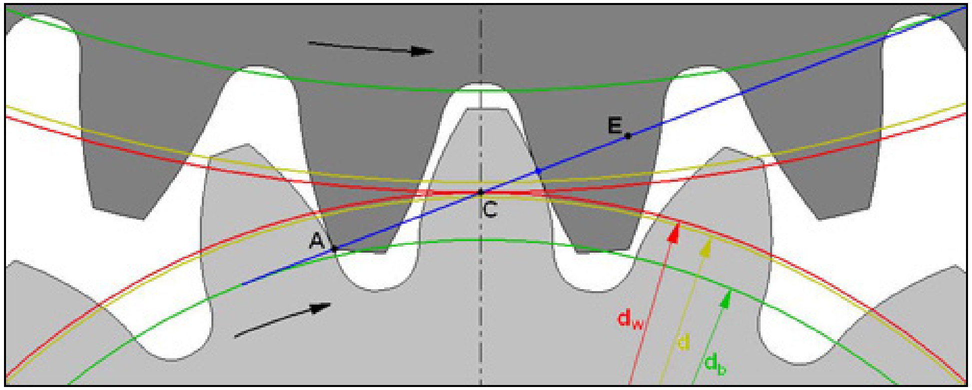
Kinematic compatibility for friction cylinders: $r_1\omega_1 = r_2\omega_2$

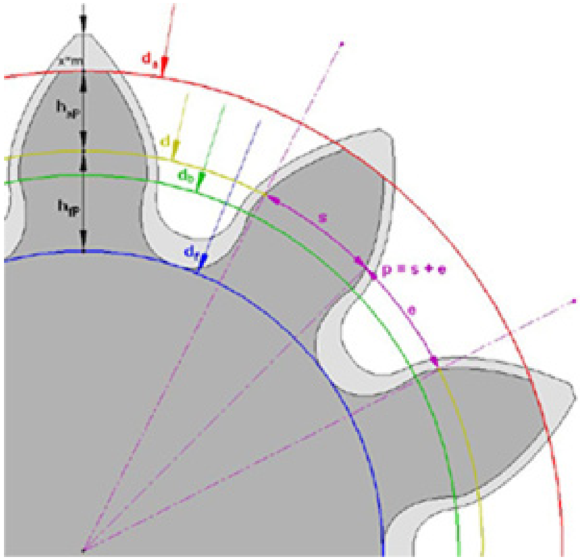
Fundamental Law of Gears:
If the velocity of the driving gear is constant, so is the velocity of the driven gear
Fundamental Law dictates certain tooth shapes!

Example of Involute gear teeth →
Cycloidal teeth also satisfy Fund. Law

Rolling contact when interface is between gear centers, otherwise sliding contact

Load is always applied along AB – so actual loading is the power transfer load, amplified by $1/\cos\phi$





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Adapted from M. Spotts, 1985

Epicyclic/Planetary Gearing!

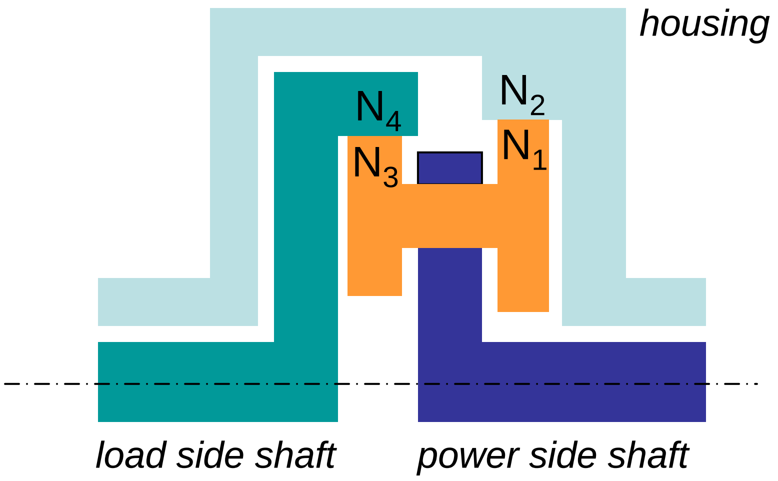
Angle α on the power side (crank):

leads to
rotation of the planet by $-\alpha N_2/N_1$
and
rotation of the crank arm by α

The planet rotation alone (fix the crank angle to zero) drives the output shaft through an angle
$(N_3/N_4)x(-\alpha N_2/N_1) = -\alpha N_3 N_2/N_4 N_1$
while
the crank rotation alone (fix the planet angle to zero) rotates the output shaft by α

The net gear ratio is

$$\omega_{load}/\omega_{power} = 1 - N_3N_2/N_1N_4$$



Super-compact form

Because slight variations between N_2 and N_4, and N_1 and N_3, are easy to achieve, very high reductions are possible in a single stage, e.g., 100:1

Image sources

two spur gears
http://www.globalspec.com/NpaPics/23/3125_083020069743_ExhibitPic.JPG

epicyclic gears
http://www.swbturbines.com/products/images/img18.jpg

radial ball bearings (3)
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