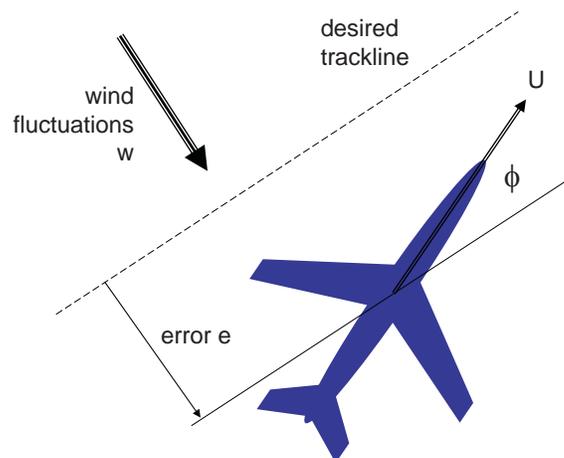


22 Aircraft in Winds

This problem builds on the previous one: you will use the same wind gust spectrum, and study the response of an air vehicle that is being buffeted by it. Here is the spectrum of the wind again, along with a picture of the aircraft:

Frequency, cycles/hr	$S_w^+(\omega)$ m^2/s
0	0.00
10	0.50
14	0.65
20	1.00
32	2.80
50	3.10
72	2.80
100	2.00
141	1.60
200	1.20
316	0.80
500	0.60
717	0.50
1000	0.40
1410	0.20
2000	0.00



The aircraft has a forward speed of $U = 60\text{m/s}$, or about 120 knots. It employs GPS navigation and a simple feedback loop to stay on a given straight-line path. In particular, the feedback law is $\phi = 0.003 \times e$, where ϕ is the controlled heading of the vehicle relative to the desired line of travel, and e is the cross-track error, i.e., the position of the aircraft in the direction normal to the desired line of travel. Thus, a cross-track error of ten meters leads to a heading command of 0.03 radians, which points the vehicle back toward the track. We assume that wind gusts do not affect the heading of the craft directly, but that there is a very good heading controller, that will make the vehicle follow the desired ϕ very closely.

1. Making the linearizing assumption that ϕ is small, and including the feedback law, what is the differential equation relating wind gust velocity $w(t)$ to cross-track error $e(t)$? What is the transfer function $E(j\omega)/W(j\omega)$? Note that the aircraft has negligible mass. *Hint: it is a low-pass filter.*

Solution: The linearization is $\dot{e} = -U\phi + w$ or $\dot{e} = -60 \times 0.003 \times e + w$. Note we have not allowed for any sideslip in the the vehicle - that is, it travels exactly in the direction it is pointing. The transfer function taking the disturbance to the error is $F(j\omega) = E(j\omega)/W(j\omega) = 1/(j\omega + 0.18)$.

2. Make a plot of one thousand seconds of $w(t)$ just as in Homework 4, and show the corresponding system output $e(t)$, in meters. For this, you need to solve the differential equation numerically.

See attached code and the first two plots.

3. Using the Wiener-Khinchine relation, make a plot of $S_e(\omega)$. Then calculate the significant error $\bar{e}_{1/3}$, and plot the plus and minus values of this on your time domain plot above.

The W-K relation says that

$$S_e(\omega) = |F(j\omega)|^2 S_w(\omega),$$

so all you need is a pointwise multiplication in frequency space of the input spectrum $S_w(\omega)$ by the squared magnitude of the transfer function, $|F(j\omega)|^2$. See the code and spectrum plot. The significant value of the error amplitude $\bar{e}_{1/3}$ is 7.44 meters.

4. What happens to $\bar{e}_{1/3}$ as you increase or decrease the feedback gain from 0.003, say to zero and to 0.006?

Setting the heading gain to zero means there is no corrective action, and the cross-track error just follows the integral of the disturbance (in the manner of a random walk). Increasing the gain to 0.006 makes the control action stronger; the error is reduced and the frequency content of the error seems higher. In fact, the higher gain is reducing more of the low-frequency part of the error. See the last two sets of plots.

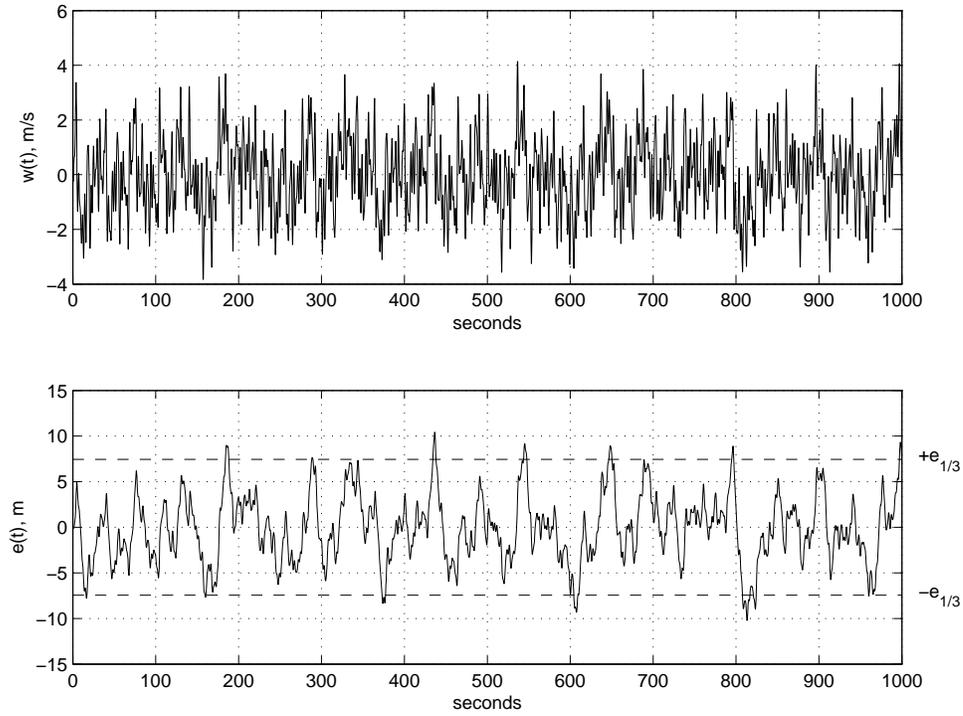


Figure 5: Time series of wind input and error output, for gain of 0.003.

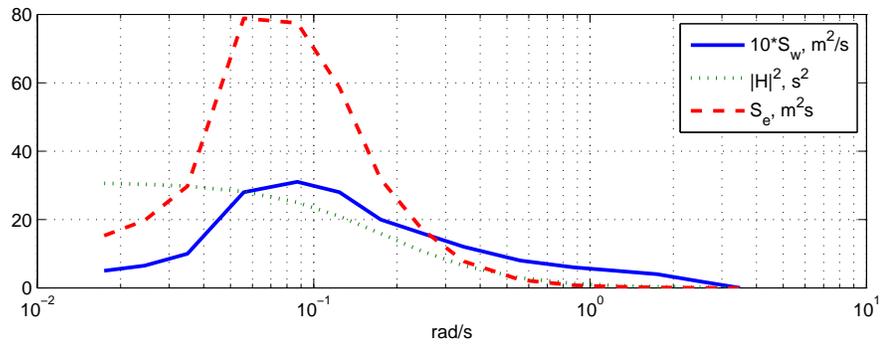


Figure 6: Input and output spectra, with gain 0.003.

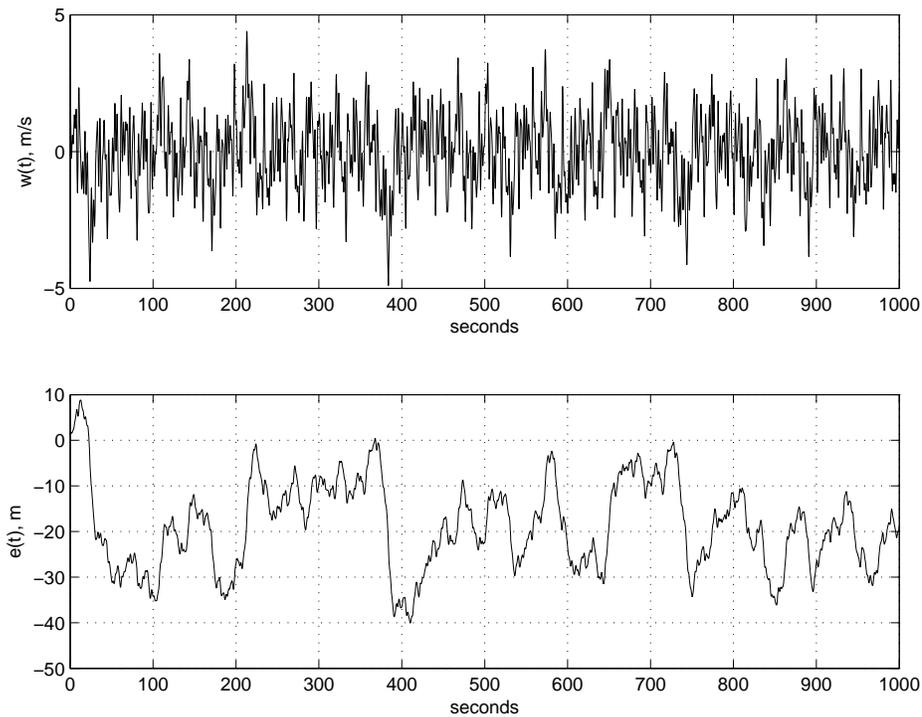


Figure 7: Time series of wind input and error output, for gain of zero - the wind is simply integrated. There is no significant amplitude because $|F|$ is unbounded at low frequencies, and hence the output spectrum is also unbounded here.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Aircraft response under simple trackline control
```

```
function aircraft
```

```
clear all;
```

```
U = 60 ; % aircraft speed
```

```
k = 0.003 ; % heading gain, rad/m
```

```
cph = [0 10 14 20 32 50 71 100 141 200 ...
       320 500 710 1000 1400 2000] ; % freq., in cycles per hour
```

```
Sw = [0 .5 .65 1, 2.8 3.1, 2.8 2 ...
      1.6 1.2 .8 .6 .5 .4 .2 0] ; % spectrum to go with cph frequencies
```

```
omega = cph*2*pi/3600 ; % freq., radians/second
```

```
widths = ([0 diff(omega)] + [diff(omega) 0])/2 ; % make the strip widths
```

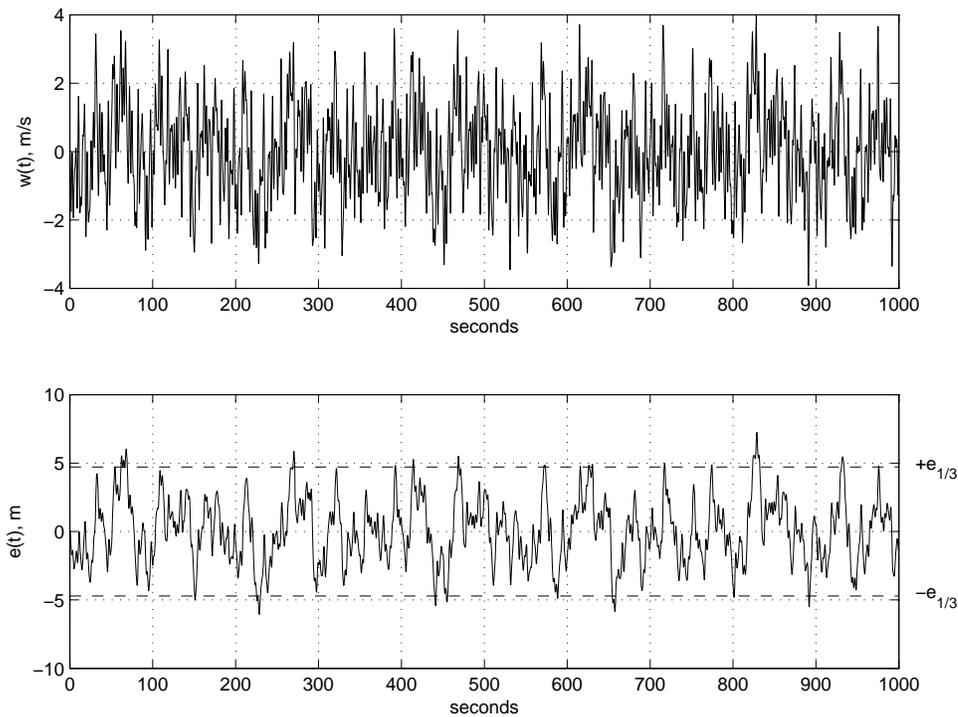


Figure 8: Time series of wind input and error output, for gain of 0.006.

```

% compute the amplitudes that go with each frequency, and pick
% some random phase angles, uniformly distributed in [0,2*pi]
for i = 1:length(widths),
    a(i) = sqrt(2*Sw(i)*widths(i)) ;
    ph(i) = rand*2*pi ;
end;

dt = .1 ; % time step

j = 1:10001 ;
w = zeros(size(j));
t = dt*(j-1) ;
for i = 1:length(widths),
    w = w + a(i)*cos(omega(i)*t + ph(i)) ;
end;

figure(1);clf;hold off;
subplot(211);
plot(t,w) ;
grid on; xlabel('seconds'); ylabel('w(t), m/s');

```


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2.017J Design of Electromechanical Robotic Systems
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