Solution Homework #3: Metal Cutting

2.008 Design and Manufacturing II
Spring 2004

Problem 1:

(a) Consider the Merchant's Cutting Force diagram in Figure 1 and the chip-workpiece interface during orthogonal cutting in Fig. 2.

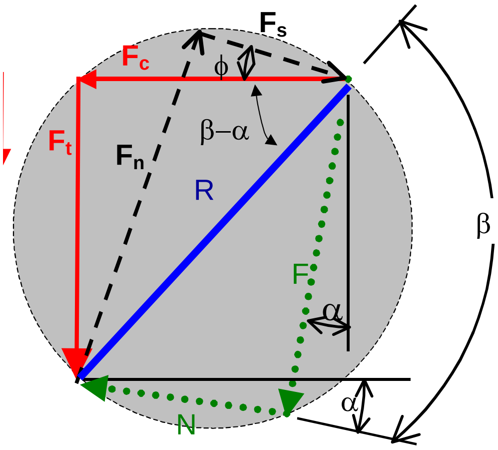


Fig 1. Merchant's diagram

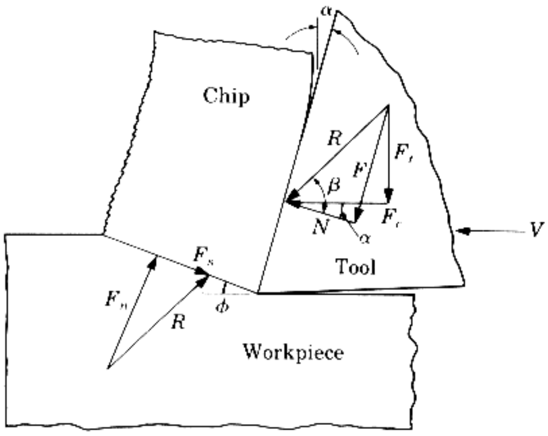


Fig 2. Cutting FBD

Merchant's hypothesis is that the shear plane is located to minimize the cutting force, or where the shear stress is maximum. Derive the Merchant's relationship between shear angle, rake angle, and friction angle as below from the diagram above.

$$\varphi = 45^{o} + \frac{\alpha}{2} - \frac{\beta}{2}$$

(b) Consider the directions of the cutting force and the thrust force. Fc, cutting force, is always positive, since the material is removed. Is the thrust force, Ft, also positive at all times? If not, explain why. Also explain how you can make Ft = 0 for a given friction coefficient between the tool and the work piece.

Solution
(a)
From the Force diagram (we discussed it in class so you know how each of the elements came in), we know that

Fs = Fc*cosφ - Ft*sinφ (1)
Fn = Fc*sinφ + Ft*cosφ (2)
Where
Fc = net horizontal cutting force
Ft = net vertical thrust force

Fs = shear force along shear plane
Fn = shear force normal to shear plane

There are two ways to derive Merchant´s Relationship: find the minimum of the cutting force or the maximum of the shear stress. We will have a look at both.

i) Find the minimum of the cutting force Fc

$$F_s = \sigma_s \bullet A_s = \sigma_s \frac{A}{\sin\varphi} = \sigma_s \frac{t_0 \bullet w}{\sin\varphi} \qquad (3)$$

$$\frac{F_t}{F_c} = \tan(\beta - \alpha) \qquad (4)$$

With σ_s as the shear stress and A_s the shear area; t_0 is the thickness of the uncut chip, called the depth of cut, w is the width of the work.

(3) and (4) in (2):

$$\sigma_s \frac{t_0 \bullet w}{\sin\varphi} = F_c \cos\varphi - F_c \sin\varphi \tan(\beta - \alpha)$$

$$\Leftrightarrow F_c = \frac{t_0 \bullet w \bullet \sigma_s}{\sin\varphi} \bullet \frac{1}{\cos\varphi - \sin\varphi \tan(\beta - \alpha)} \qquad (5)$$

In order for the cutting force to be minimum at angle φ, the first derivative has to be zero:

$$\frac{\partial F_c}{\partial \varphi} = 0$$

Substituting equation (5) and solving:

$$\frac{\partial F_c}{\partial \varphi} = t_0 \bullet w \bullet \sigma_s \left[-\frac{\cos\varphi}{\sin^2\varphi} \bullet \frac{1}{\cos\varphi - \sin\varphi \tan(\beta - \alpha)} - \frac{1}{\sin\varphi} \bullet \frac{-\sin\varphi - \cos\varphi \tan(\beta - \alpha)}{(\cos\varphi - \sin\varphi \tan(\beta - \alpha))^2} \right] \overset{!}{=} 0$$

$$\Leftrightarrow \cos\varphi \bullet [\cos\varphi - \sin\varphi \tan(\beta - \alpha)] = \sin\varphi \bullet [\sin\varphi + \cos\varphi \tan(\beta - \alpha)]$$

$$\Leftrightarrow \cos^2\varphi - \sin^2\varphi = 2\cos\varphi \sin\varphi \tan(\beta - \alpha)$$

With the trigonometric expressions

$$\cos^2\varphi - \sin^2\varphi = \cos 2\varphi$$

$$2\cos\varphi \sin\varphi = \sin 2\varphi$$

we get

$$\cos 2\varphi = \sin 2\varphi \tan(\beta - \alpha)$$
$$\Leftrightarrow \tan 2\varphi = \cot(\beta - \alpha)$$

And another trigonometric expression: $\tan 2\varphi = \cot(\pi / 2 - 2\varphi)$

Which leads us to

$$\frac{\pi}{2} - 2\varphi = \beta - \alpha$$
$$\Rightarrow$$
$$\varphi = 45^{o} + \frac{\alpha}{2} - \frac{\beta}{2}$$

ii) Find the maximum of the shear stress

The shear stress is given by

σ_s **= Fs/Area** (6)

If the width of the work is given by **w**, and the uncut chip thickness is t_o, the area along which Fs acts is

Area = t_o***w/sin**φ (7)

Equation (6) then becomes with (1) and (7)

σ_s **= (Fc*cos**φ **- Ft*sin**φ**) / (**t_o***w/sin**φ**)** (8)

From the force diagram we also know that

Ft/Fc = tan(β**-**α**)** (9)

Substituting (9) in (8), we have

σ_s **= [Fc*sin**φ**(cos**φ **- tan(**β**-**α**)*sin**φ**)] / (**t_o***w)** (10)

Now, in order for the shear stress to be maximum at angle φ

$\partial\sigma_s / \partial\varphi$ **= 0**

Substituting equation (10) and solving

$\partial\sigma_s / \partial\varphi$ **= 1/t_o*w[Fc($\cos^2\varphi$ - $\sin^2\varphi$) – Fc(tan(β-α)*2*sinφ*cosφ)]**

From trigonometry we know
$$\cos^2\varphi - \sin^2\varphi = \cos 2\varphi$$
$$2\cos\varphi\sin\varphi = \sin 2\varphi$$

Substituting and equating to zero, we have

Fc*cos2φ = Fc*sin2φ*tan(β-α) → **tan2φ = cot(β-α)** (11)

Again, from trigonometry: $\tan 2\varphi = \cot(\pi/2 - 2\varphi)$
The principal solution is

$\pi/2 - 2\varphi = \beta - \alpha \rightarrow 2\varphi = \pi/2 + \alpha - \beta \rightarrow$ $\boldsymbol{\varphi = \pi/4 + \alpha/2 - \beta/2}$

(b)
We know that

Ft = R sin(β - α) and **Ft = Fc tan(β - α)**

The magnitude of Fc is always positive (since the direction of the feed is always towards the material); the sign of Ft can be either positive or negative, depending on the relative magnitudes of β and α. When β > α, the sign of Ft is positive (downward) and when β < α, it is negative (upward). Therefore, in order to make the thrust force = 0, we make β = α. This can be achieved in two ways:
1) Change the rake angle of the tool, α.
2) By adding lubricant, effectively changing the friction angle β.

It is the second step that we most commonly employ in machining when something is "wrong" in the cut.

Problem 2:

Your lab group decided to make fancy transparent-look Yo-Yos and designed the body of the Yo-Yo to be injection molded as follows. For a solid look, two identical parts are supposed to be assembled with an insert-molded bolt and nut.

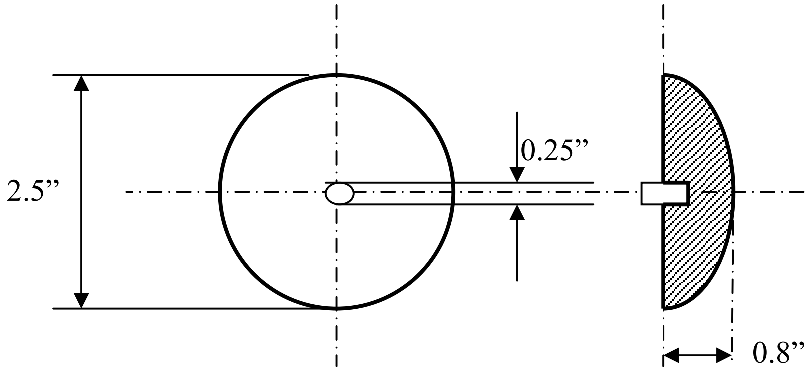
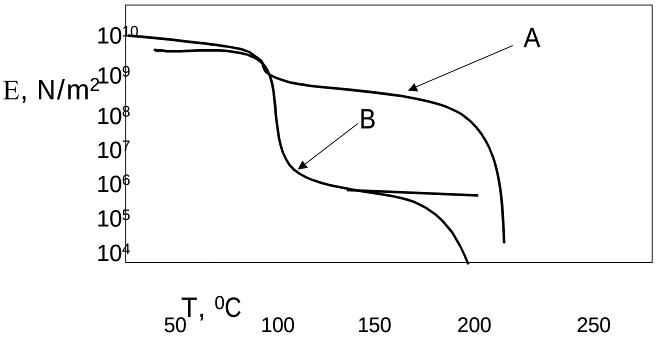


Figure 1. Injection molded YoYo body

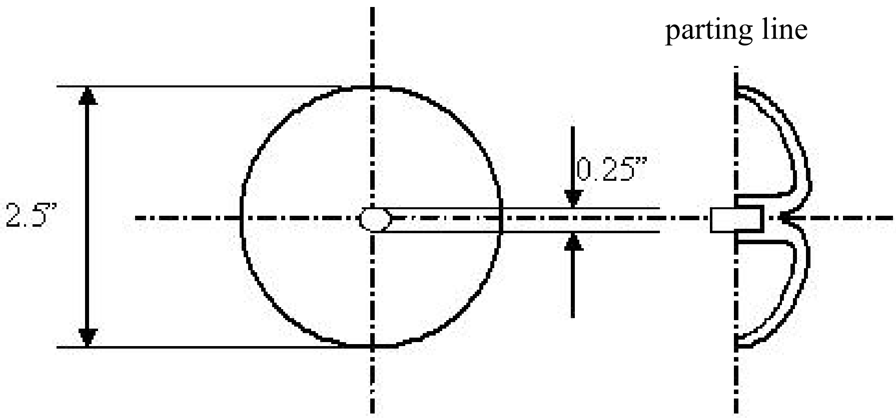
a) After learning about polymer materials and processes, you now know this design is not good for injection molding. Make necessary design changes on the figure below and explain why you chose to make them.
b) The material provided by Dave and Pat is Polypropylene, which is translucent. Explain in one sentence why Polypropylene (PP) is not transparent. After searching for crystal clear and high impact resistant materials, you find out that polycarbonate (PC) is the right one for your nude Yo-Yo. Why is PC transparent? (1 sentence) Are those materials thermosetts or thermoplastic? How you can prove that?
c) From the manufcaturer of the Polycarbonate resin, you received the modulus-temperature curves shown below. Which is the right M-T curve of Polycarbonate? (A or B?) What is the approximate Tg of PC?
d) What would be the clamping force required for this part? Choose a proper parting line and assume the injection pressure is 1000 psi with a packing process which requires a packing pressure about twice the injection pressure.
f) The cycle time of the part (after making necessary design changes in (a)) is still measured as 2 minute. It is composed of mold filling (10%), part cooling (75%), ejecting and mold closing, etc. This cycle time cannot justify the selling price at the market. Therefore you redesigned it with 40% reduction of the thickness of the part. Assume that the thickness is changed uniformly over the whole part. Please estimate the new cycle time.

Modulus-temperature of PC



Solution

a)



Reasons for the design changes:

- Use uniform wall thicknesses throughout the part. This will minimize sinking, warping, residual stresses, and improve mold fill and cycle times.
- Use generous radius at all corners. The inside corner radius should be a minimum of one base wall thickness??
- Use the least thickness compliant with the process, material, or product design requirements. Using the least wall thickness for the process ensures rapid cooling, short cycle times, and minimum shot weight. All these result in the least possible part cost.

b)
PP is translucent because it is a crystalline material. PC is transparent because it is amorphous.
PP and PC are both thermoplastics. A method to find out if this is true would be to reheat the finished product to processing temperature. If it melts it is a thermoplastic.

c)
Curve B is clearly a temperature-modulus curve for an amorphous material and therefore applies to PC. Tg is approximately 90 - 100° C.

d)
A packing pressure of twice the injection pressure means that after the first injection with 1000 Psi an additional stroke with 2000 Psi is applied in order to pack more material to compensate the shrinkage. The required clamping force is area*pressure. With the parting line as indicated in the figure in a) the clamping force becomes:
2000 Psi * $3.14 * 1.25^2$ = 9812.5 lbs =about 5 tons

e)
A reduction of the wall thickness and therefore of the volume of material injected will reduce the injection time as well as the cooling time. The new cycle time can be calculated according to the following equation:

(1) The filling time will be decreased by 40%: the new filling time = 0.6x (0.1x120 sec) = 7.2 sec
(2) The cooling time is be proportional to the square of the thickness decrease: new cooling time = $(0.6)^2$ x (0.75 x 120) = 32.4 sec
(3) Ejecting, mold closing time will be the same = 0.15 x 120 = 18sec

The new cycle time: 57.6 sec (52% reduction !!!)