Massachusetts Institute of Technology
2.008: Design and Manufacturing II

Problem Set 3 Solutions

Problem 1-1. Thermoforming

Consider the thermoformed part drawn in Figure 1 which was used to package last semester's hamburger yoyo. This part was unapologetically difficult to manufacture, and this problem will investigate some of its pitfalls.

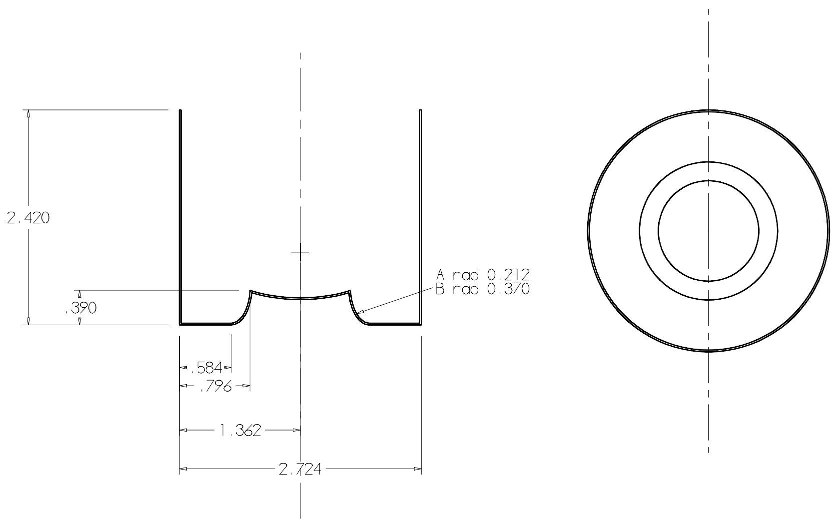

Figure 1: Thermoformed part

(a) Compute the part's draw ratio. Is it reasonable?

The draw ratio is the depth over the width $= \frac{2.420}{2.724} = 0.888$ *which is perfectly reasonable.*

(b) The part was formed from a sheet of clear polystyrene with thickness 0.030 inches and dimensions 4 inches by 4 inches square. The part was vacuum-drawn with a clamp that enclosed an area of approximately 3.5" by 3.5". Assuming that the drawn part has uniform thickness, what is the wall thickness.

First we need to approximate the surface area of the inside of the part. Kudos go to Brian Ruddy who generated a solidworks model that does all the hard work. His model reports an area of $A = 27.57$ *inches*^{2}. *The material that is drawn depends on the geometry of the mold. If you are optimistic and assume all of the material from the* 3.5 *inches*^{2} *clamped area gets drawn, then the part thickness would be*

$$(0.030)\frac{3.5^2}{27.57} = 0.013in$$

In reality, as soon as the lip of the warm plastic touches the sides of the mold no more of that external area gets drawn into the mold. Therefore, a more realistic calculation is to assume that drawing area is approximately 120 *percent of the diameters of the part.*

$$(0.030)\frac{\pi(2.724*1.20)^2}{4*27.57} = 0.009\ in$$

(c) The part certainly did not have uniform thickness. Which areas of the part would you expect to be the thickest and why?

As empirically proven, the bottom center cup and the very top of the part were the thickest. In both cases, this represents were the plastic first touched the mold.

(d) Recall that the formula for Euler buckling is

$$P_{cr} = \pi^2 EI/(kL^2)$$

where E is the modulus of elasticity, I is the moment of inertia L is the length, and k is a factor that takes the support conditions into consideration. Make reasonable assumptions and compute the buckling load that the part can take. Recall that $I = \frac{1}{4}\pi(r_o^4 - r_i^4)$ for a tube with outside and inside diameters r_o and r_i respectively. Also, the modulus for standard polystyrene is $3350 N/\text{mm}^2$. Comment on the appropriateness of the value you computed.

First we compute the moment of Inertia:

$$I = \frac{1}{4}\pi((2.724/2)^4 - (2.715/2)^4) = 0.724$$

Substituting this value, we have

$$P_{cr} = \pi^2(3350/25.4)(0.724)/(2.420)^2 = 160.8\ N$$

Which sounds fantastic, but unfortunately, the walls are not uniform.

(e) What other problems does the part have?

In this original drawing, there is no draft angle, and the outside corners do not have any radius.

Problem 1-2. Cutting model

(a) Using the data from the first experiment in Monday's lecture, calculate the sheer angle ϕ for the experiment. Show your calculations. The data for the experiment will be posted on the website.

We find the shear angle from the measured value of t_0, t_c, α *and the cosine identity* $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$*:*

$$\begin{aligned}
\frac{t_0}{t_c} &= \frac{\sin(\phi)}{\cos(\phi-\alpha)} \\
\frac{t_0}{t_c} &= \frac{\sin(\phi)}{\cos(\phi)\cos(-\alpha) - \sin(\phi)\sin(-\alpha)} \\
t_0(\cos(\phi)\cos(\alpha) + \sin(\phi)\sin(\alpha)) &= t_c\sin(\phi) \\
t_0(\cos(\alpha) + \tan(\phi)\sin(\alpha)) &= t_c\tan(\phi) \\
t_0\cos(\alpha) &= \tan(\phi)(t_c - t_0\sin(\alpha)) \\
\frac{t_0\cos(\alpha)}{t_c - t_0\sin(\alpha)} &= \tan(\phi) \\
\phi &= \tan^{-1}(\frac{t_0\cos(\alpha)}{t_c - t_0\sin(\alpha)})
\end{aligned}$$

Plugging in values for $\alpha = 10, t_0 = 0.015, t_c = 0.0445$ *from the first experiment, we get* $\phi = 19.4°$ *deg. In the last experiment, we get that* $\phi = 40°$ *deg which is quite steep.*

(b) What is the chip velocity (V_c) and sheer velocity (V_s) from this experiment? Why does this make intuitive sense?

$$\frac{V_c}{V} = \frac{t_0}{t_c}$$

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In the first experiment, the diameter was 3 inches and the spindle was set at 96 RPM. Therefore, $V = \pi(3)(96) = 904$ *in/min and therefore the* $V_c = 904(0.015/0.0445) = 304$ *in/min.*

The shear velocity relationship is

$$\frac{V_s}{\cos(\alpha)} = \frac{V_c}{\sin(\phi)}$$

and is therefore $V_s = \frac{304\cos(10)}{\sin(19.4)} = 901.3$*. This makes sense since most of the cutting energy is spent shearing the material.*

(c) Using the variables F_c and F_t (which would normally be measured experimentally using a dynamometer), determine the shear strength (τ_s) of the material that we were cutting.

The formally, I see, is quite readily presented in the lecture notes

$$\tau_s = \frac{F_c\cos(\phi) - F_t\sin(\phi)}{(t_0/\sin(\phi))w}$$

(d) Compute the power of the machine. Make assumptions about the motor efficiency, the energy lost to friction, noise, heat, and vibrations.

This is meant to be a rough estimate. The specific cutting energy of 4140 is $3.35J/mm^3$*. When the machine stalled, the spindle was 540rpm, and* $t_0 = 0.027$*. Therefore, the material removal rate was* $MRR = (540 * \pi * 3) * 0.027 * 0.075 = 10.3008$ in^3/min*. Since the machine stalled, the power of the machine had to be less than* $P = mrr * \mu_c = 10.3 * 3.335 * 25.4^3 \leq 562,900J/min \leq 9400W < 12.6Hp$ *which is reasonable.*

(d) Based on the drawing on page 15 of the Cutting notes (L6), I argue that the chip thickness t_c is the same dimension as the length of the shear plane. This would imply that $t_c = t_0/\sin(\phi)$ which contradicts the equations from page 12. Explain what is wrong.

The chip is not necessarily perpendicular to the shear zone. This is only the case when the friction angle β *is zero.*

Problem 1-3. Process choice

Describe how you would make a decision on whether to use thermoforming or injection molding for a part.

What is the difference between amorphous and crystalline plastics, and which ones are better for thermoforming?

Brian Ruddy provided this crips answer:

> *Crystalline plastics exhibit strong polymer chain alignment; amorphous plastics have no alignment. This difference makes crystalline plastics stronger but more brittle than amorphous ones. For thermoforming, amorphous plastics ar e preferred because at higher temperatures, there strength and stiffness are red uced, therefore making it easier to form the part.*