Today's goals

- **So far**
  - Modeling the dynamics of electro-mechanical systems
  - Controling the dynamics of electro-mechanical systems
    - *s*-domain (root locus)
    - State space
    - Frequency domain (Bode plots)
  - Types of compensators
    - Proportional
    - Proportional-Derivative
    - Proportional-Integral (aka Ideal Integral)
    - Proportional-Integral-Derivative
- **Today**
  - Passive compensators
    - Lag
    - Lead
    - Lead-Lag
  - Time delays

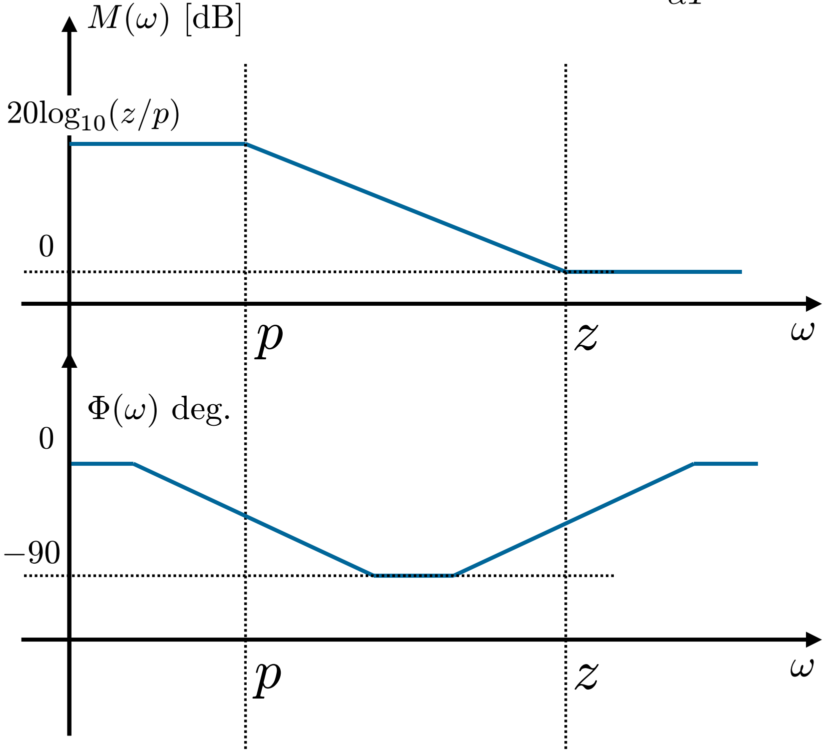
Lag and lead compensators

Compensator transfer function

$$G_c(s) = \frac{s+z}{s+p}.$$

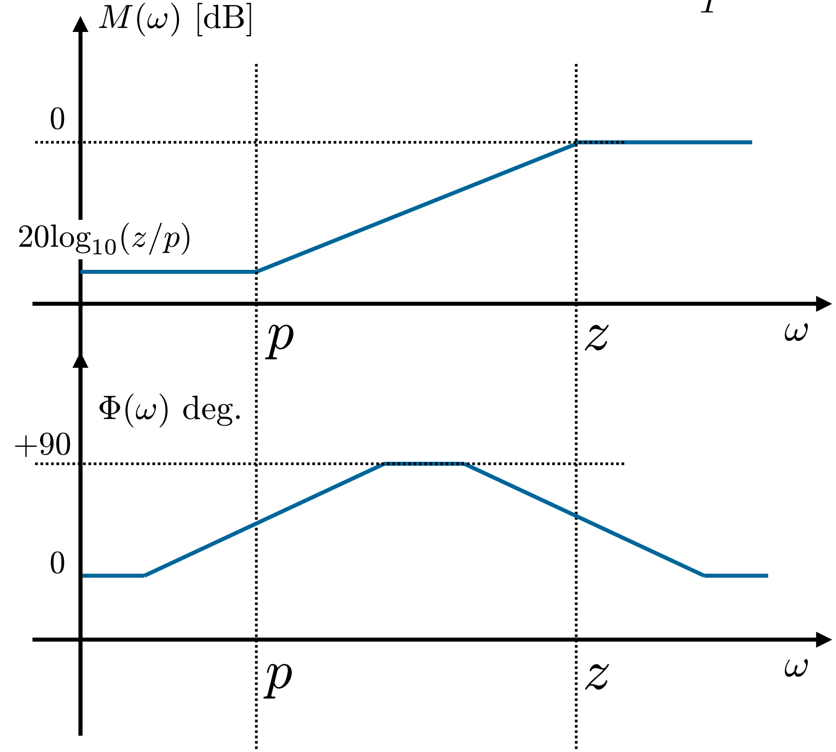
$z > p$: Lag

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \alpha > 1.$$



$z < p$: Lead

$$G_c(s) = \frac{s + \frac{1}{\alpha T}}{s + \frac{1}{T}}, \alpha > 1.$$



Lag compensation

$$G_c(s) = \frac{s+z}{s+p}, \qquad z > p$$

- Improve steady-state error
- Increase phase margin

Steady–state error without compensation:

$$e(\infty) = \frac{1}{1+K_p}, K_p = G_p(0).$$

Steady–state error with lag compensation:

$$e(\infty) = \frac{1}{1+zK_p/p}.$$

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Please see: Fig. 11.4 and 9.10 in Nise, Norman S. *Control Systems Engineering.* 4th ed. Hoboken, NJ: John Wiley, 2004.

Lead compensation

$$G_c(s) = \frac{s+z}{s+p}, \qquad z < p$$

- Increase bandwidth (faster response)
- Increase phase margin

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Please see: Fig. 11.7, 9.24, and 9.25 in Nise, Norman S. *Control Systems Engineering.* 4th ed. Hoboken, NJ: John Wiley, 2004.

Lead-lag compensation

$$G_c(s) = \frac{s + \dfrac{1}{T_1}}{s + \dfrac{\alpha}{T_1}} \times \frac{s + \dfrac{1}{T_2}}{s + \dfrac{1}{\alpha T_2}}, \qquad \alpha > 1.$$

- Essentially equivalent to PID compensation
  - Lead component fixes transient
  - Lag component fixes steady-state error
- Three degrees of freedom: T_1, T_2, α

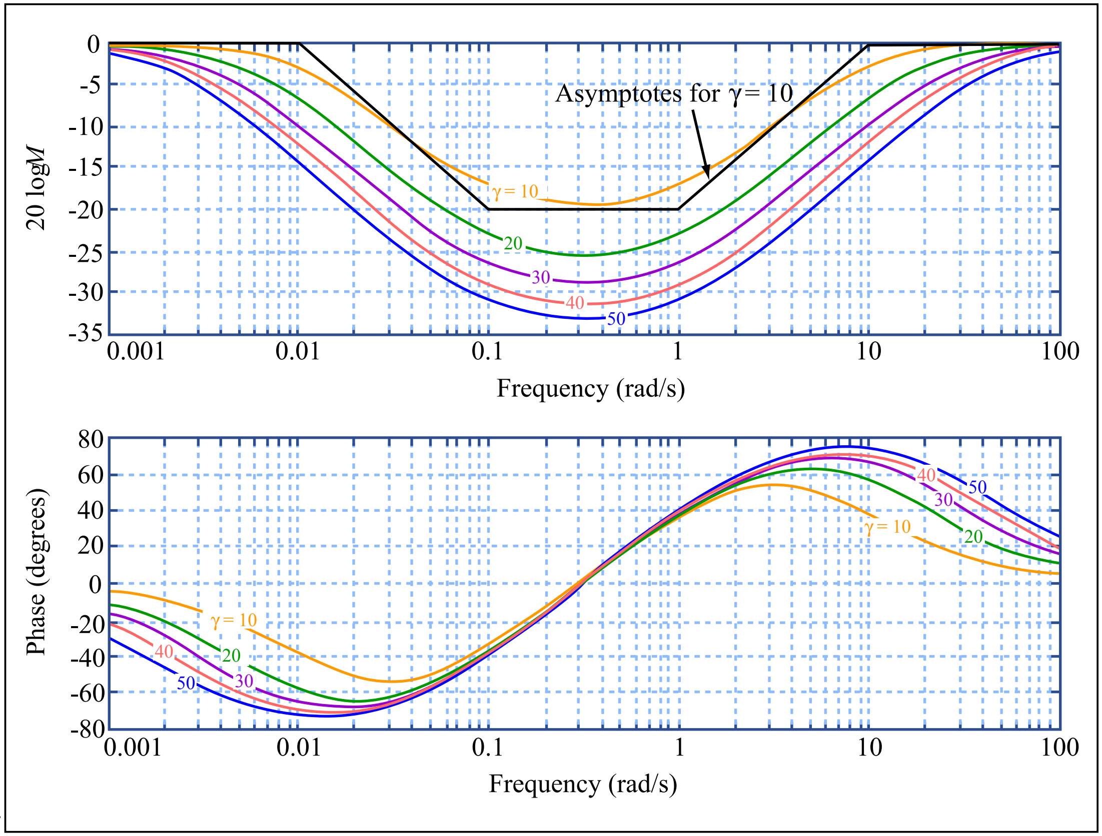
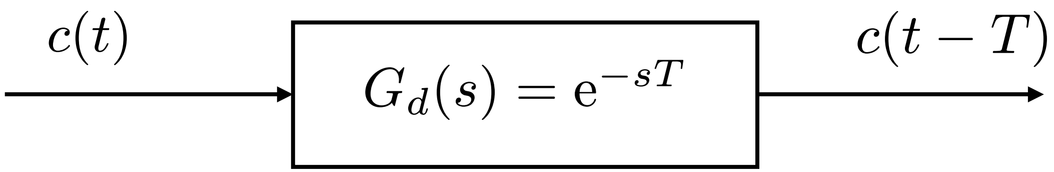


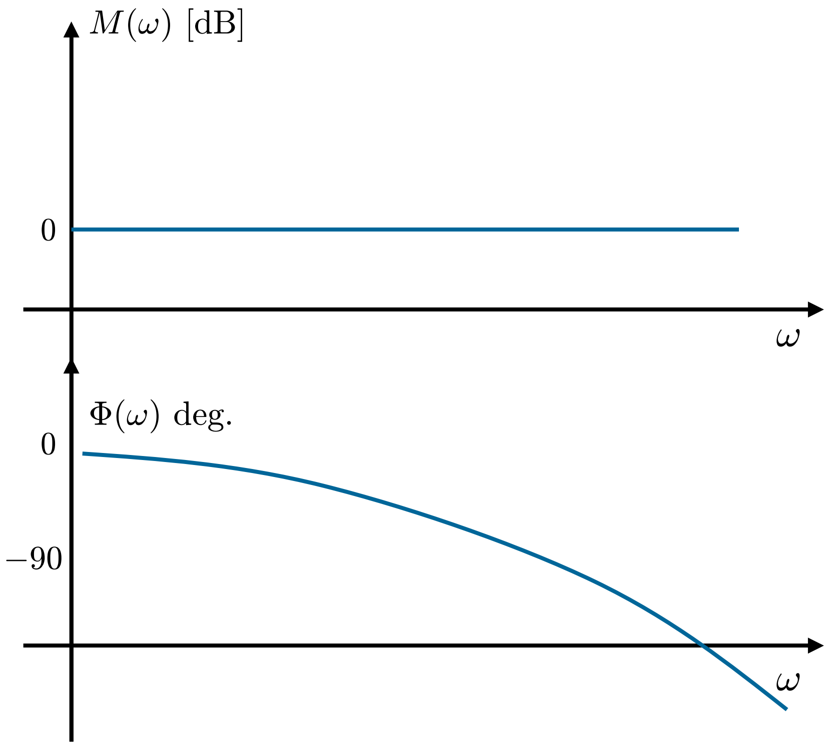
Figure by MIT OpenCourseWare.

Systems with time delay



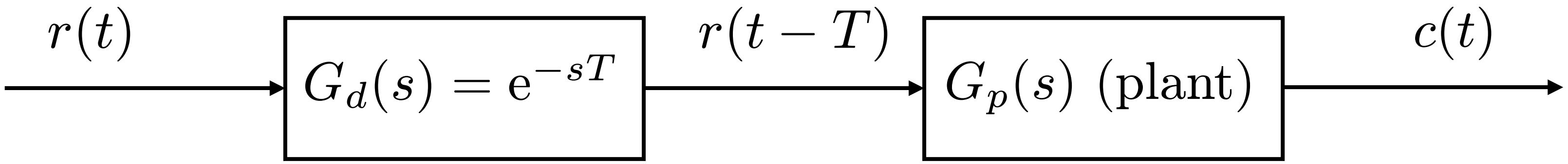
$$
\begin{aligned}
G(j\omega) &= e^{-\omega T} \\
|G(j\omega)| &= 1 \\
\angle G(j\omega) &= -\omega T
\end{aligned}
$$

Delay element (or “delay line”)



Time delay reduces the phase margin; therefore time delay results in reduced stability

Cascading phase delay to a plant



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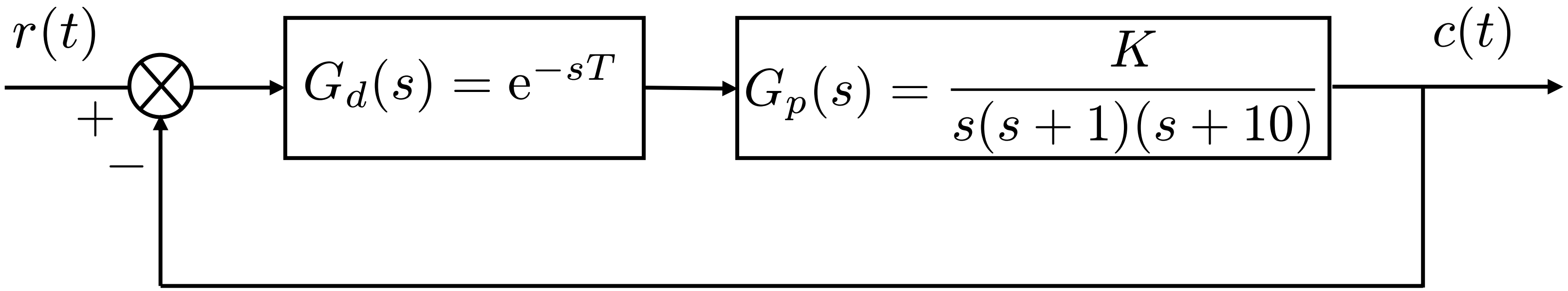
Please see: Fig. 10.54 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Example: time delay in feedback configuration



Time delay *T*=1sec

$(K = 1)$

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Please see: Fig. 10.55 in Nise, Norman S. *Control Systems Engineering.* 4th ed. Hoboken, NJ: John Wiley, 2004.

1. Range of gain for stability

Without phase delay, the phase angle is -180° at frequency 3.16rad/sec so the gain margin is 40.84dB. The closed–loop system is stable for $0 < K \leq 10^{40.84/20} = 110.2$.

With phase delay, the phase angle is -180° at frequency 0.81rad/sec so the gain margin is 20.39dB. The closed–loop system is stable for $0 < K \leq 10^{20.39/20} = 10.46$.

2. Percent overshoot for $K = 5$

Since $K = 5$, the magnitude curve is raised by $20\log_{10}5 = 13.98$dB. The zero dB crossing occurs at frequency 0.47rad/sec.

Without phase delay, the phase at the zero dB crossing is -118°, so the phase margin is $-118^\circ - (-180^\circ) = 62^\circ$. From the phase margin *vs* damping curve (2nd–order approx.) we find $\zeta = 0.64 \Rightarrow \%OS = 7.3\%$.

With phase delay, the phase at the zero dB crossing where the phase is -145°, so the phase margin is 35°. From the phase margin *vs* damping curve (2nd–order approx.) we find $\zeta = 0.33 \Rightarrow \%OS = 33\%$.

Example: time delay in feedback configuration

$r(t)$ → (+/−) → $G_d(s) = e^{-sT}$ → $G_p(s) = \dfrac{K}{s(s+1)(s+10)}$ → $c(t)$

Without phase delay — With phase delay T=1sec

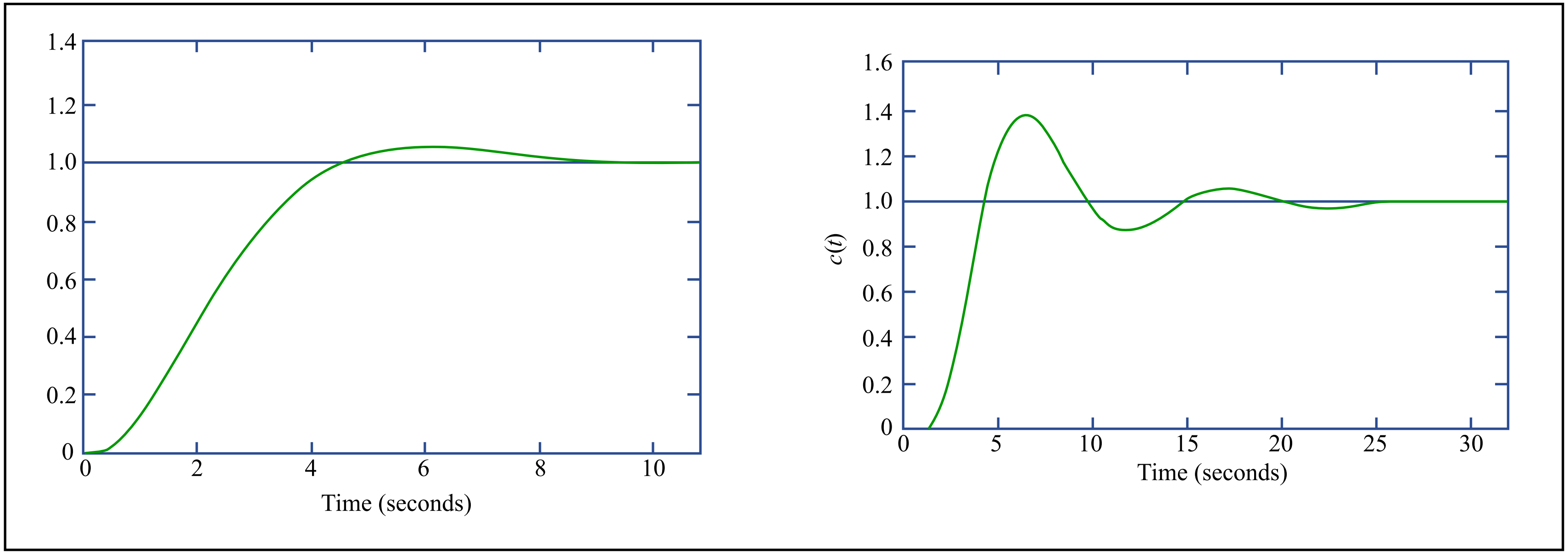

Figure 10.56

Figure by MIT OpenCourseWare.