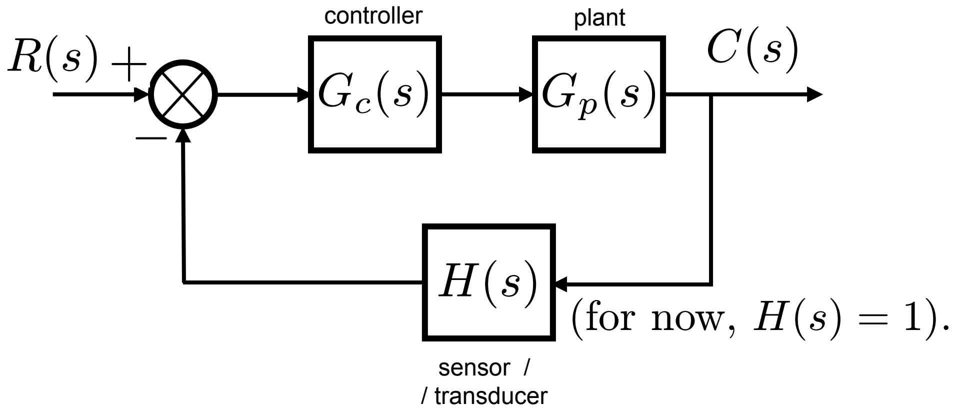
Review



- **Adjusting gain in uncompensated feedback system (G_c=K, proportional control)**
  - Allows us to move the poles only along the given Root Locus specified by the plant's open-loop poles/zeros; desirable pole locations away from the given Root Locus are inaccessible:
  - e.g., we saw that we can adjust to high gain to reduce steady-state error, but at the expense of increasing the overshoot.
- **Cascade compensation (G_c="judiciously chosen transfer function")**
  - Allows us to reshape the Root Locus; therefore, desirable pole locations that were not allowed in the uncompensated system now become accessible
  - e.g. we saw that we can completely eliminate steady-state error by cascading an integrator (pole@origin) and compensate the resulting slow-down (due to the integrator) by cascading a zero: "PI controller"

Compensator rules of thumb

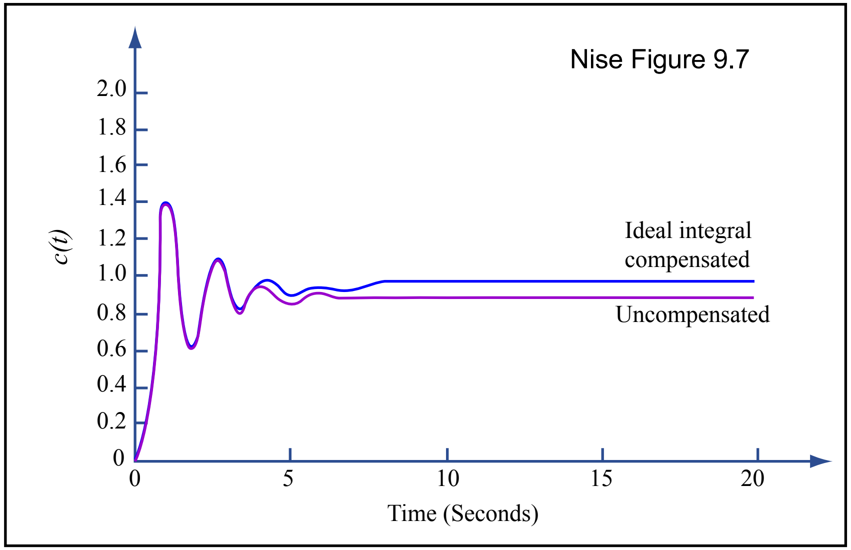
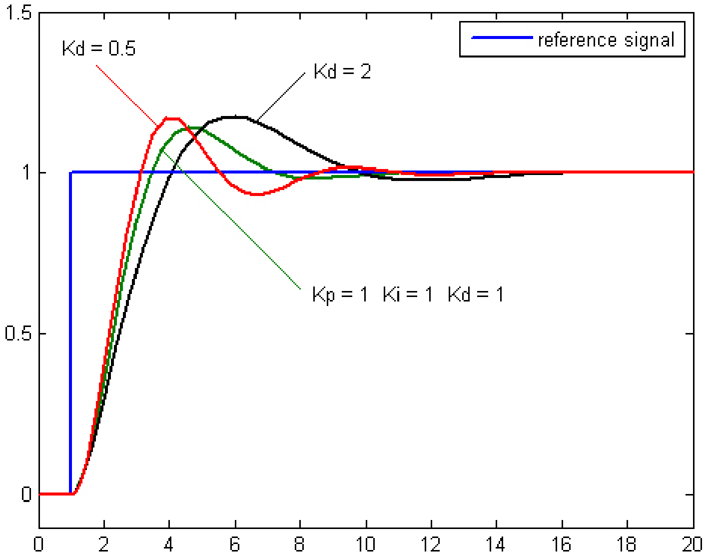


Figure by MIT OpenCourseWare.



Nise Figure 9.1

<u>Integral action</u>

- eliminates steady-state error; but,
- by itself, the integrator slows down the response;
    - therefore, a zero (derivative action) speeds the response back up to match the response speed of the uncompensated system

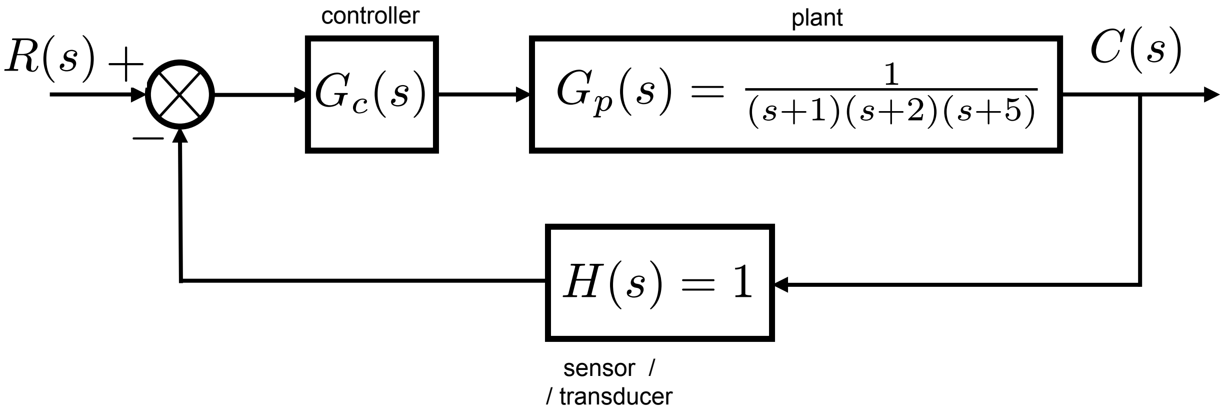
$$\text{PI controller: } G_c(s) = K\frac{s+z}{s}.$$

<u>Derivative action</u>

- speeds up the transient response;
- it *may* also improve the steady-state error; but
- differentiation is a ***noisy*** process
    - (we will deal with this later in two ways: the lead compensator and the PID controller)

$$\text{PD controller: } G_c(s) = K\,(s+z)\,.$$

Example



We wish to speed up the system response while maintaining $\zeta = 0.4 \Leftrightarrow \%\mathrm{OS} \approx 25.4\%$.

Evaluating different PD controllers

Nise Figure 9.15

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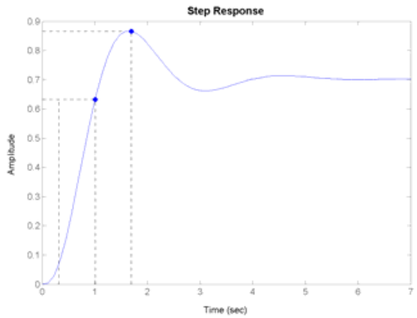
Please see Fig. 9.15 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.



K=23.7
%OS=23.2
T_r=0.688 sec

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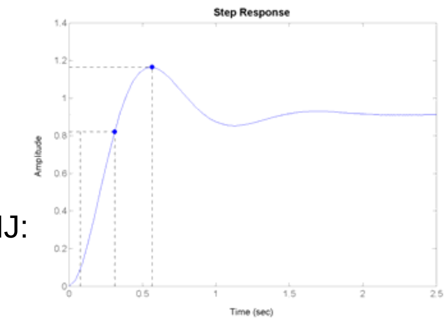
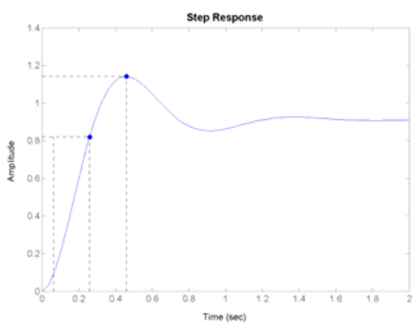
Please see Fig. 9.15 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.



K=35.3
%OS=27.5
T_r=0.236 sec



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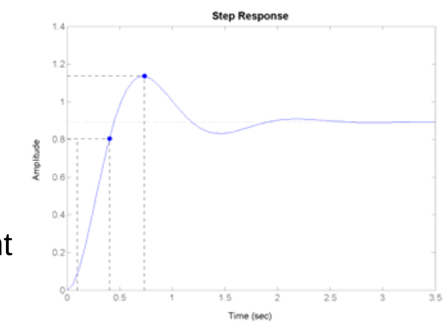
Please see Fig. 9.15 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

K=51.4
%OS=25.4
T_r=0.197 sec



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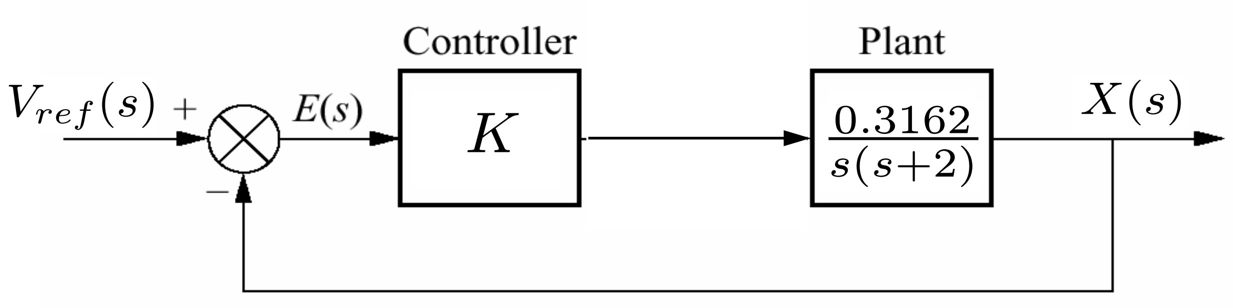
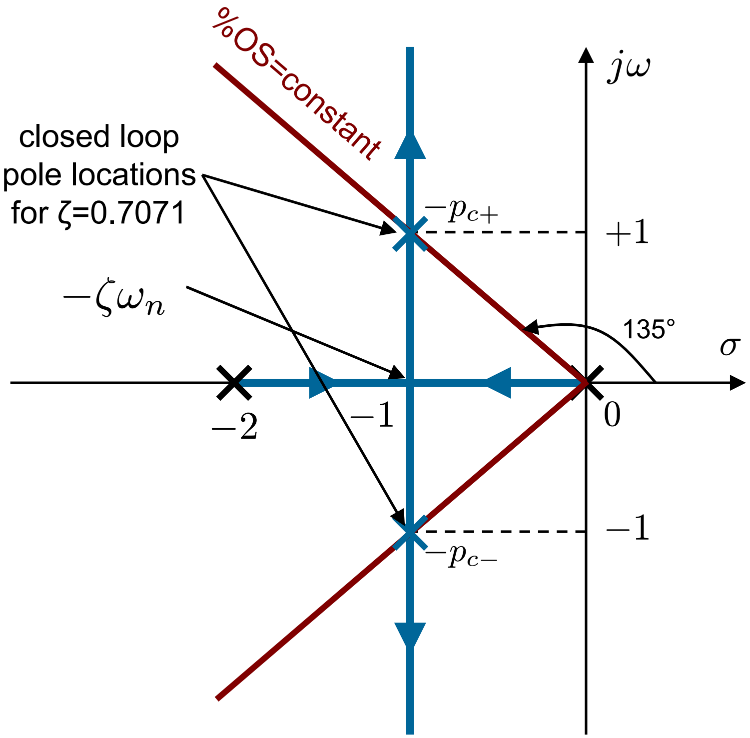
Please see Fig. 9.15 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

K=20.86
%OS=27.2
T_r=0.305 sec

Example: speeding up the pinion-rack response





In lecture 20, we designed this system with proportional control for

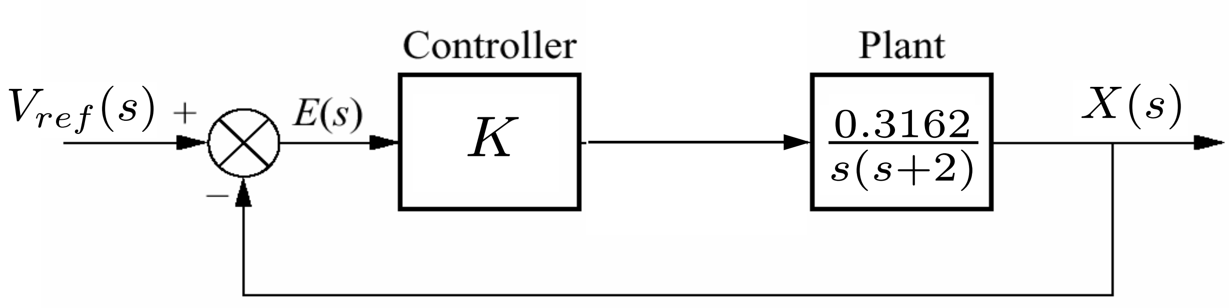
$$\zeta = 1/\sqrt{2} = 0.7071 \Leftrightarrow \%\text{OS} = 4.32\%.$$

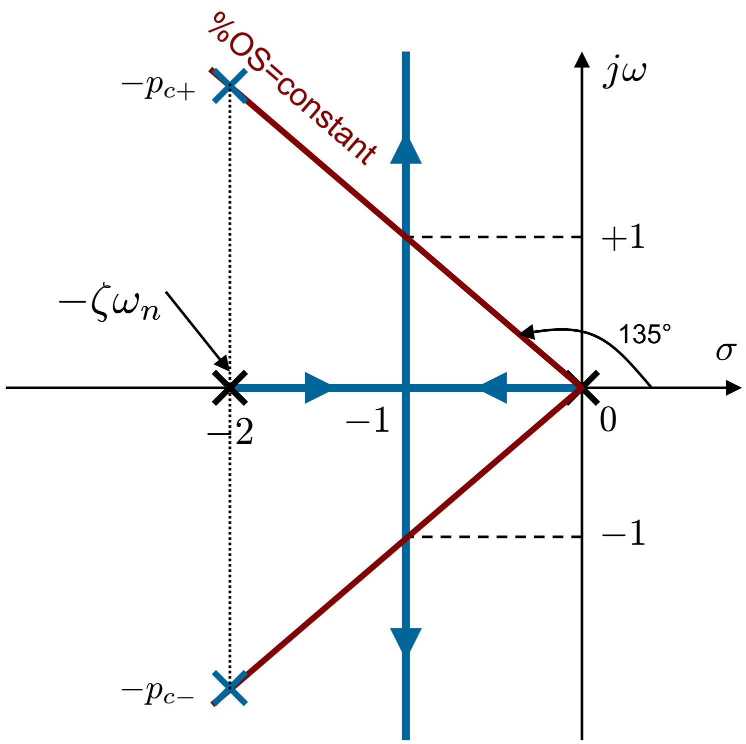
We found that the overshoot target is achieved with proportional gain $K = 6.325$. From the root locus we can see that for this value of gain, the settling time is

$$T_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{1} = 4 \text{ sec.}$$

How can we "speed up" the system to $T_s = 2$ sec while maintaining the same %OS value?

Example: speeding up the pinion-rack response



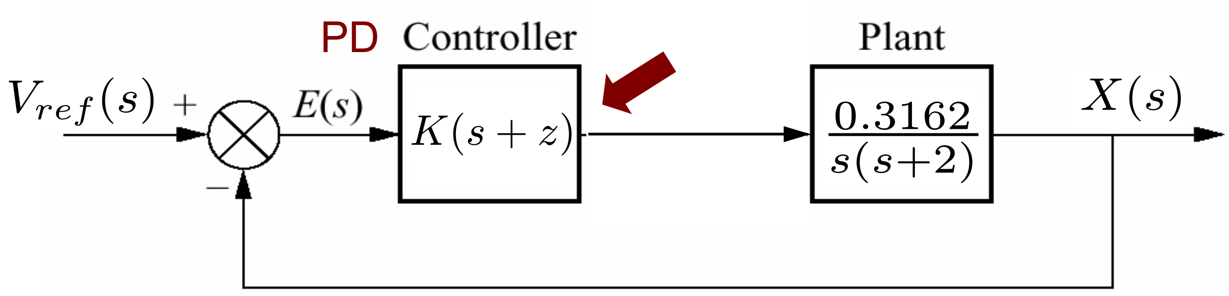


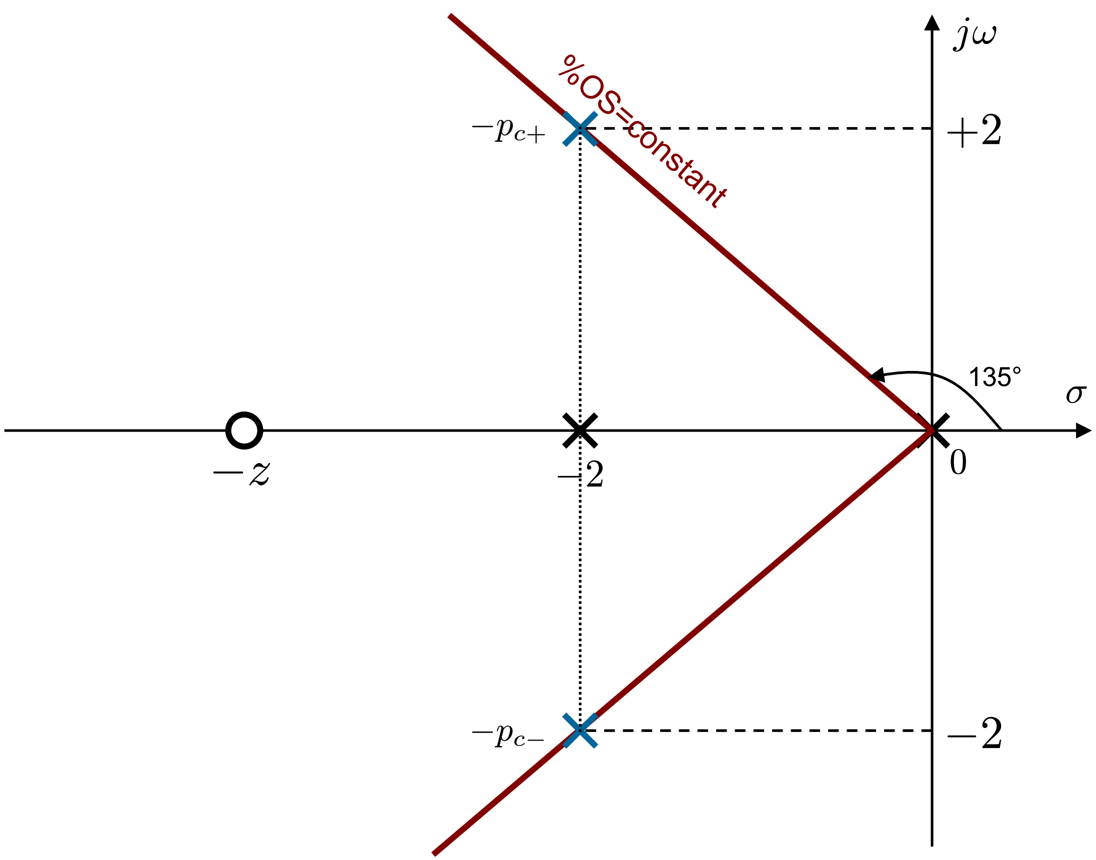
From the shorter settling time requirement, we have

$$T_s \approx \frac{4}{\zeta\omega_n} = 2 \Rightarrow \zeta\omega_n = 2.$$

Moreover, to maintain the same %OS, the poles must be located on the $\zeta = 0.707$ line. The new desired pole locations are shown on the left. Unfortunately, **they do not belong** to the uncompensated root locus. To achieve the desired poles, we propose to use a proportional–derivative (PD) compensator.

Example: speeding up the pinion-rack response



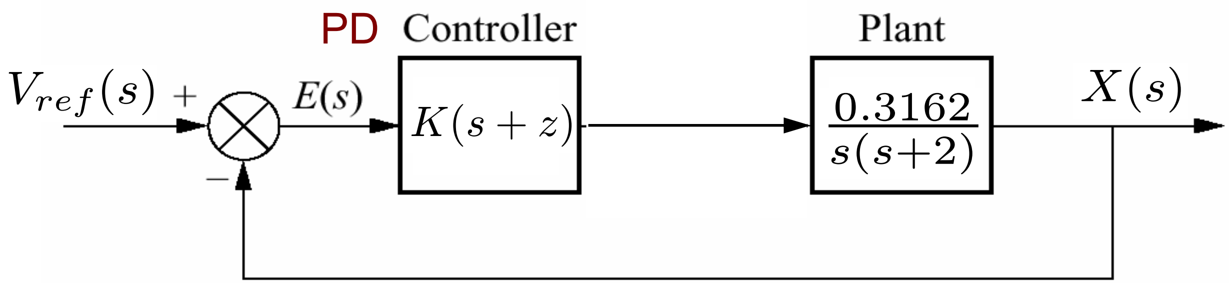


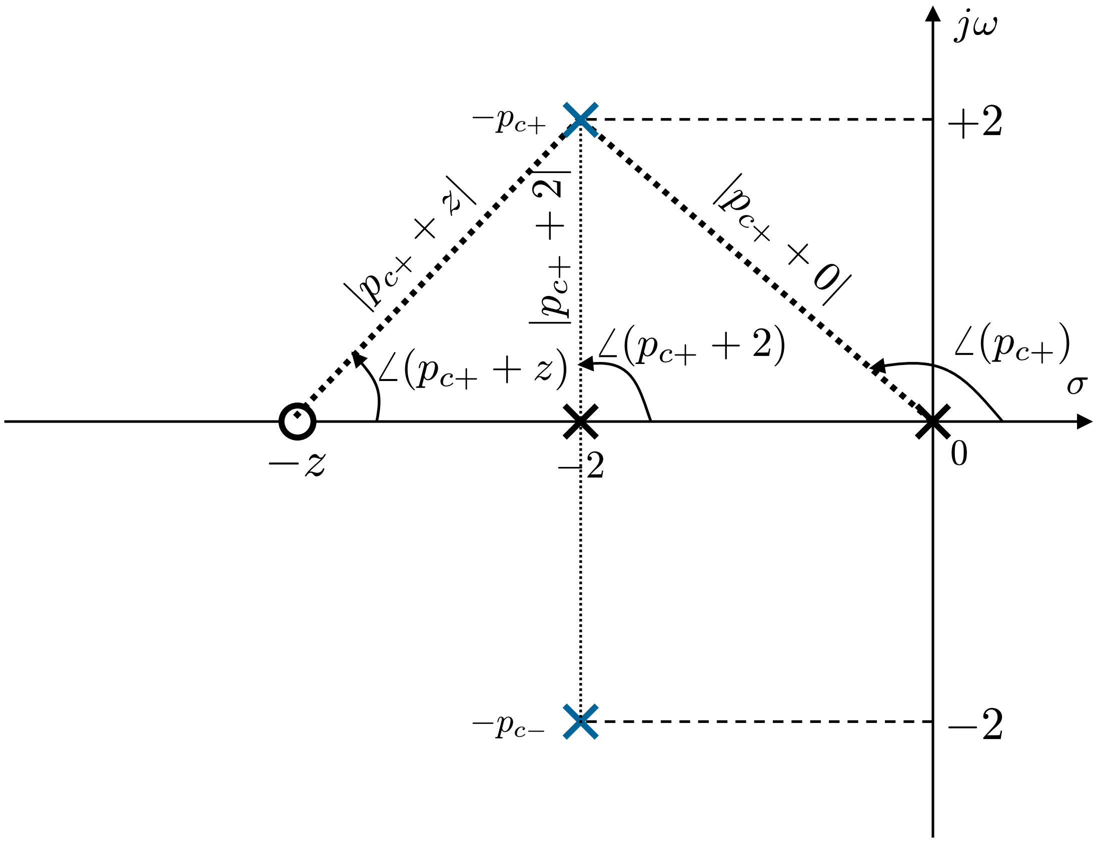
The new compensated system is represented on the left on the s–plane. The desired pole locations are

$$-p_{c\pm} = -2 \pm j2.$$

The PD compensator (see the block diagram above) contributes a zero at $-z$. We now must answer the question: where should the zero be such that the desired poles belong on the root locus? The process of placing z is the **"design of the PD compensator."**

Example: speeding up the pinion-rack response





If the new $p_{c\pm}$ are to belong to the root locus, the phase contributions from the open–loop poles must add up to $(2m+1)\pi$, where m is an integer.

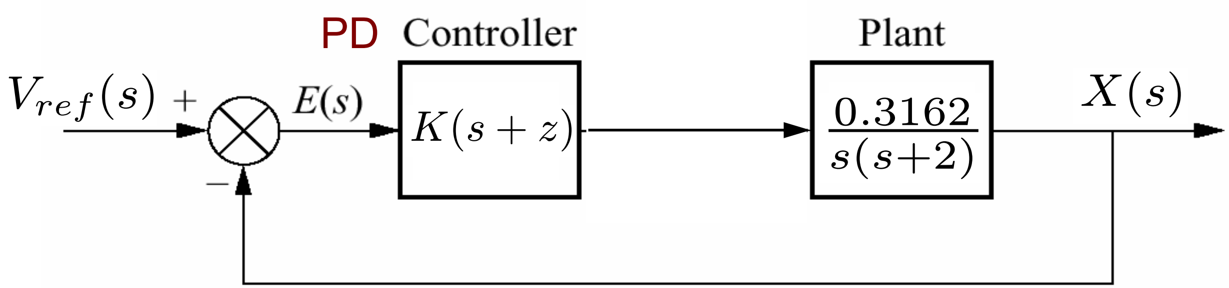
$$\angle(p_{c+}+z) - \angle(p_{c+}+2) - -\angle(p_{c+}) = (2m+1)\pi.$$

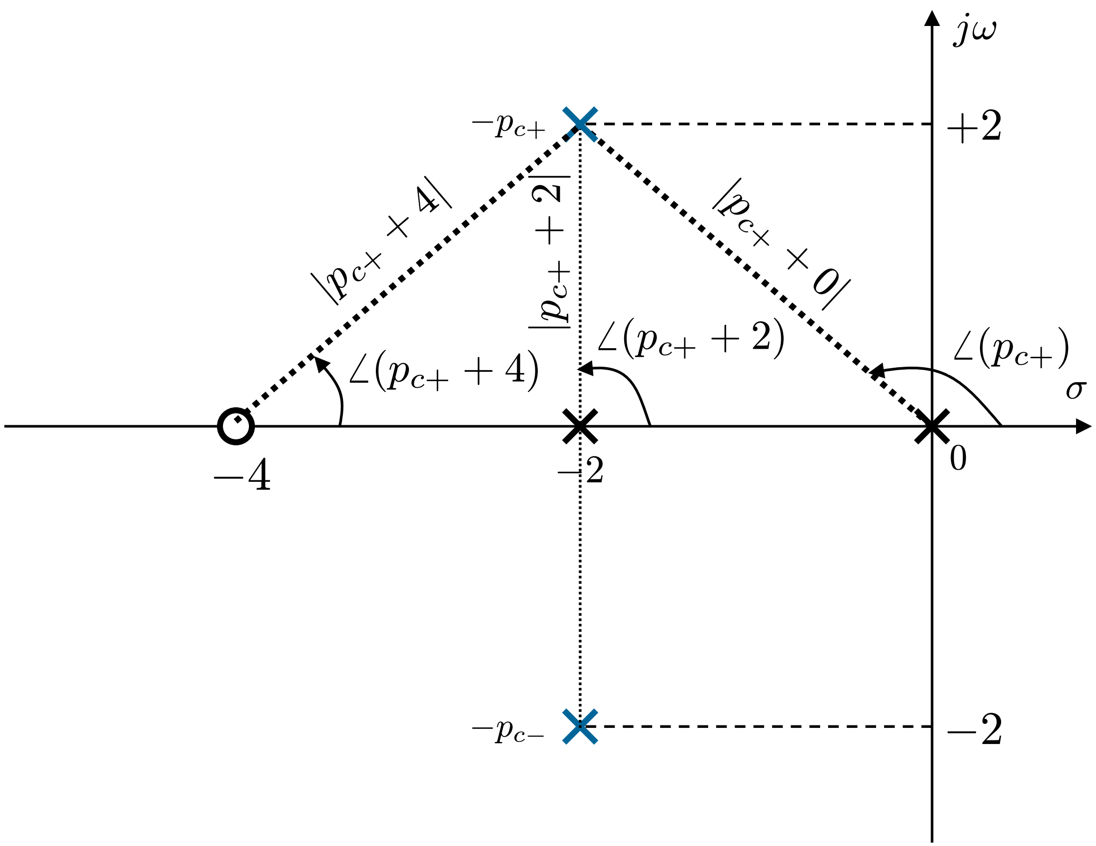
From the geometry,

$$\begin{aligned} \angle(p_{c+}+2) &= \pi/2 \quad (90^\circ) \\ \angle(p_{c+}) &= 3\pi/4 \quad (135^\circ), \\ \Rightarrow \angle(p_{c+}+z) &= \pi/4 \quad (45^\circ), \end{aligned}$$

which places the zero at $-z = -4$.

Example: speeding up the pinion-rack response





Now that we have placed the zero at $-z=-4$, we can also compute the gain required to reach the desired closed–loop poles at $p_{c\pm}=2\pm j2$:

$$|G_c(s)|\,|G_p(s)|=1, \quad \text{where}$$

$$G_c(s) = K(s+4)$$

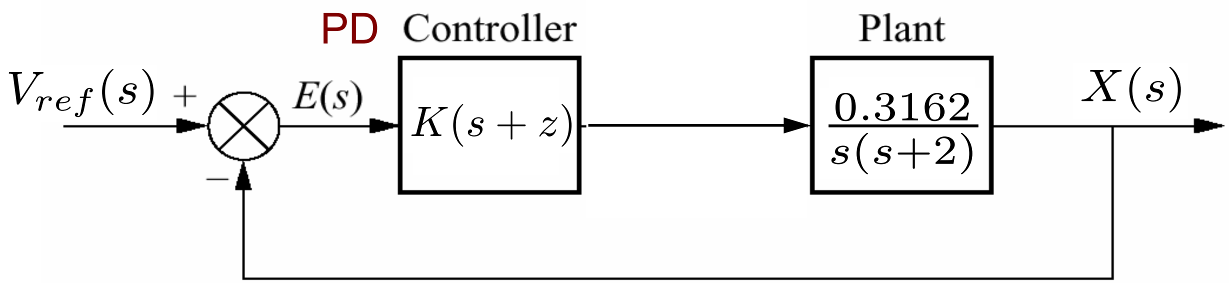
$$G_p(s) = \frac{0.3162}{s(s+2)}.$$

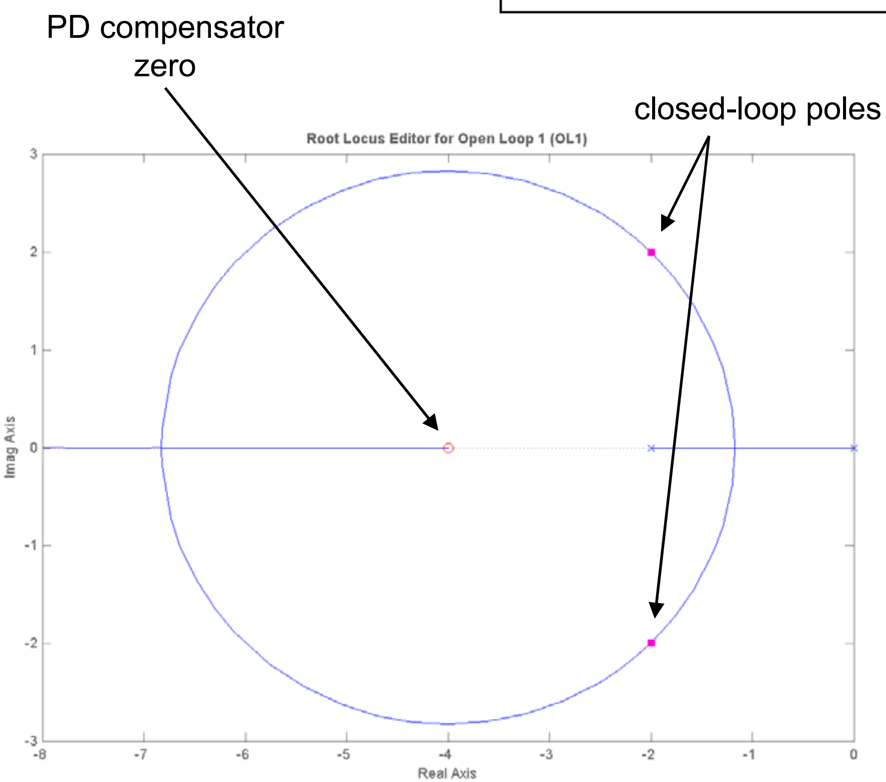
For the pole of interest at $s=-p_{c\pm}$,

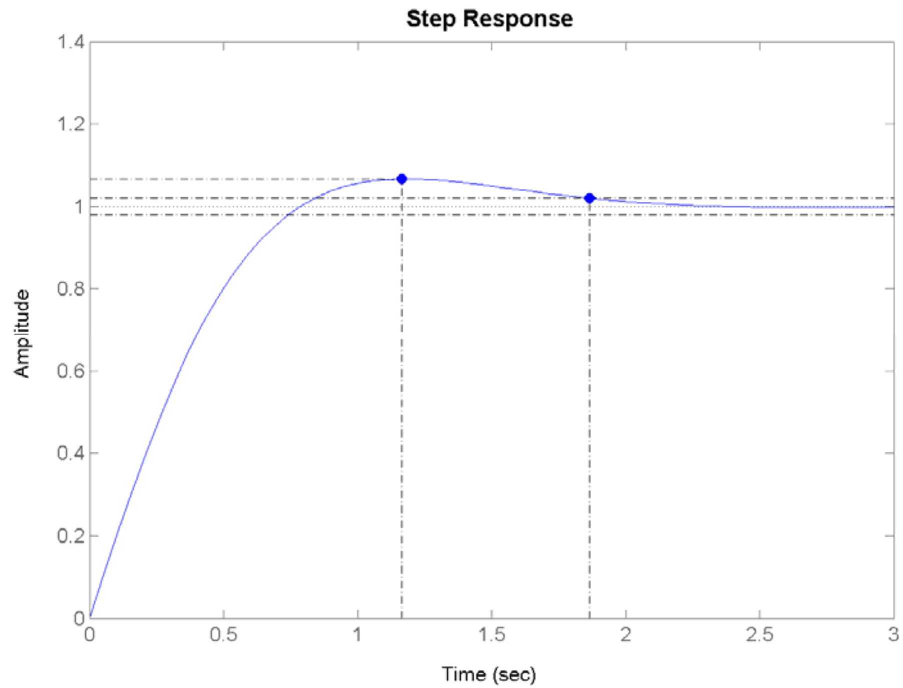
$$\Rightarrow K=\frac{|p_{c+}|\,|p_{c+}+2|}{0.3162\,|p_{c+}+4|}$$

$$=\frac{(2\sqrt{2})\times 2}{0.3162\times(2\sqrt{2})}=6.325.$$

Example: speeding up the pinion-rack response







K=6.325 gives %OS=6.7; T_s=1.86sec