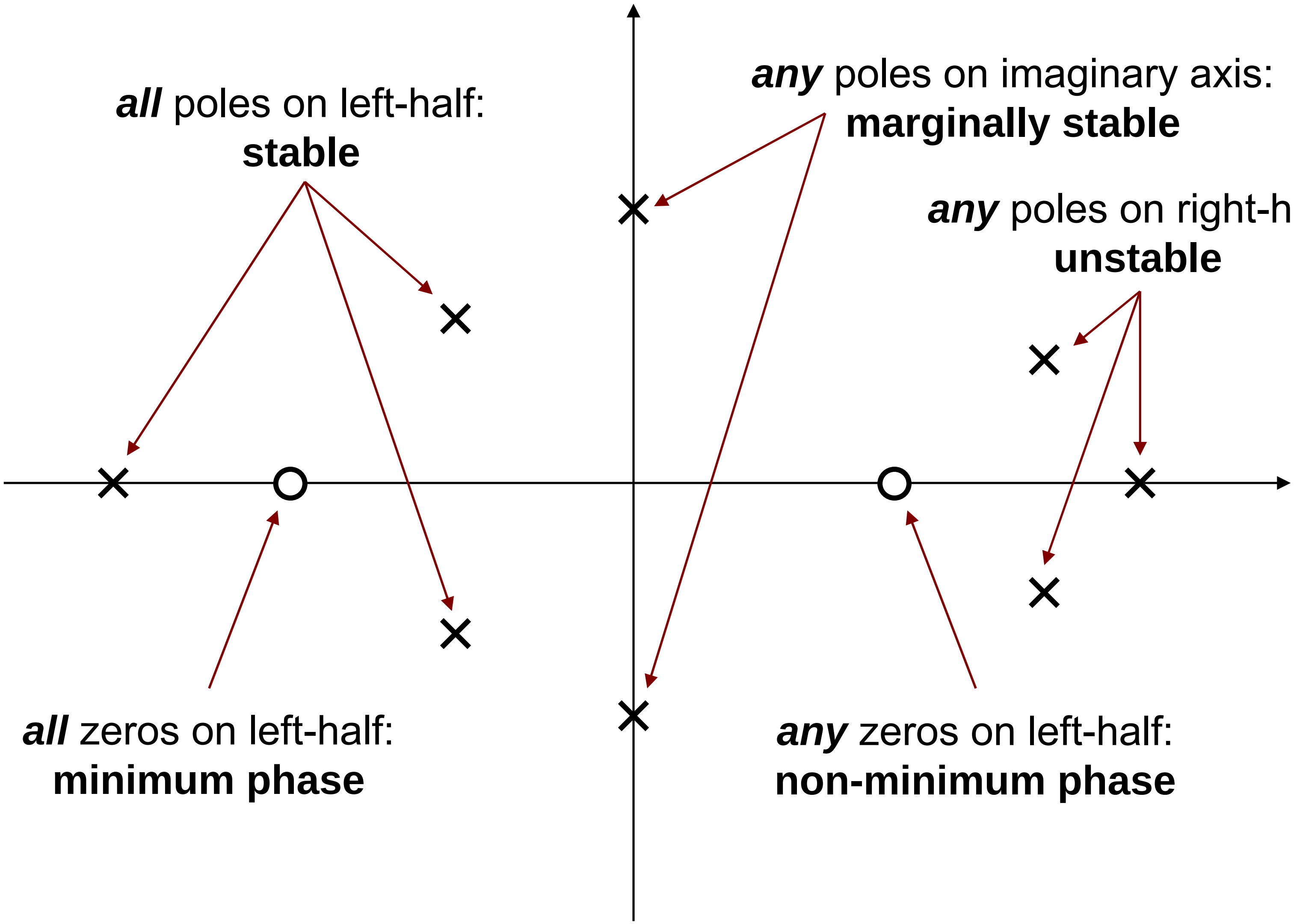
Definitions of stability

$$\text{Reminder: } c(t) = c_{\text{natural}}(t) + c_{\text{forced}}(t).$$

- A system is **stable** if
  - the natural response decays exponentially to zero as $t \to \infty$
  - for every bounded input the output is also bounded as $t \to \infty$
- A system is **unstable** if
  - the natural response increases exponentially as $t \to \infty$
  - there is at least one bounded input for which the output is undbounded (increases without bound) as $t \to \infty$
- A system is **marginally stable** if
  - the natural response oscillates as $t \to \infty$ (i.e. neither decays exponentially to zero nor increases exponentially)
  - there is at least one bounded input for which the output oscillates as $t \to \infty$ (i.e. neither decays exponentially to zero nor increases exponentially)

Stability on the *s*-plane



Stability and feedback

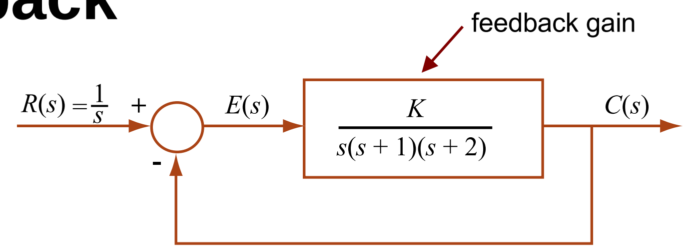
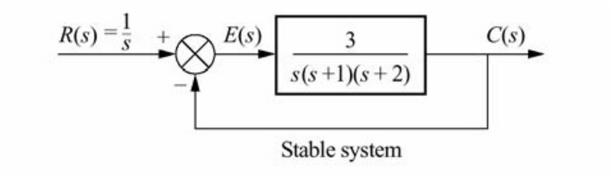
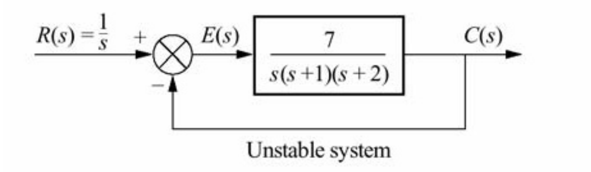


Figure by MIT OpenCourseWare.

Small gain: *stable*



Large gain: *unstable*



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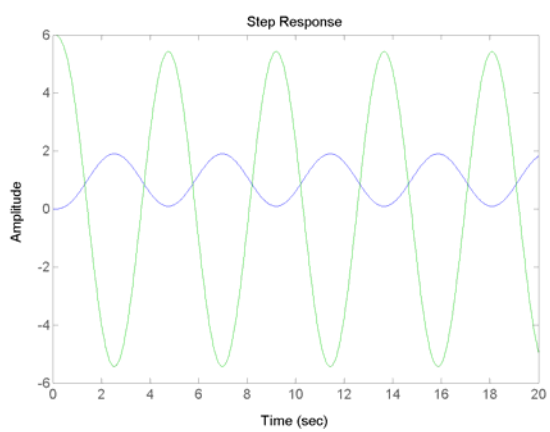
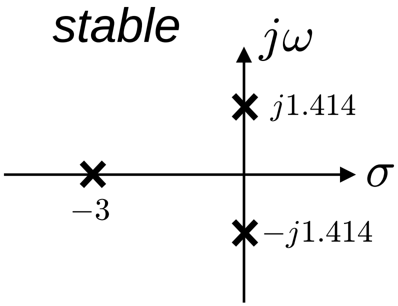
Please see: Fig. 6.1 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.



K=6: *marginally stable*



We can show that for an arbitrary gain K the closed–loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{s^3 + 3s^2 + 2s + K} \quad \Rightarrow$$

$$\begin{aligned} &\text{if} \quad 0 < K < 6 \quad \rightarrow \quad \text{stable} \\ &\text{if} \quad K = 6 \quad \rightarrow \quad \text{marginally stable} \\ &\text{if} \quad K > 6 \quad \rightarrow \quad \text{unstable} \end{aligned}$$