Goals for today

- Block diagrams revisited
  - Block diagram components
  - Block diagram cascade
  - Summing and pick-off junctions
  - Feedback topology
  - Negative *vs* positive feedback
- Example of a system with feedback
  - Derivation of the closed-loop transfer function
  - Specification of the transient response by selecting the feedback gain
- The op-amp in feedback configuration

Block diagram components

Transfer Function

$$\frac{C(s)}{R(s)} = G(s).$$

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Please see: Fig. 5.2 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Cascading subsystems

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Please see: Fig. 5.3 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Note: to cascade two subsystems, we must ensure that the second subsystem **does not <u>load</u>** the first subsystem

Loading and cascade

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Please see: Fig. 5.4 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Cascading with an op-amp

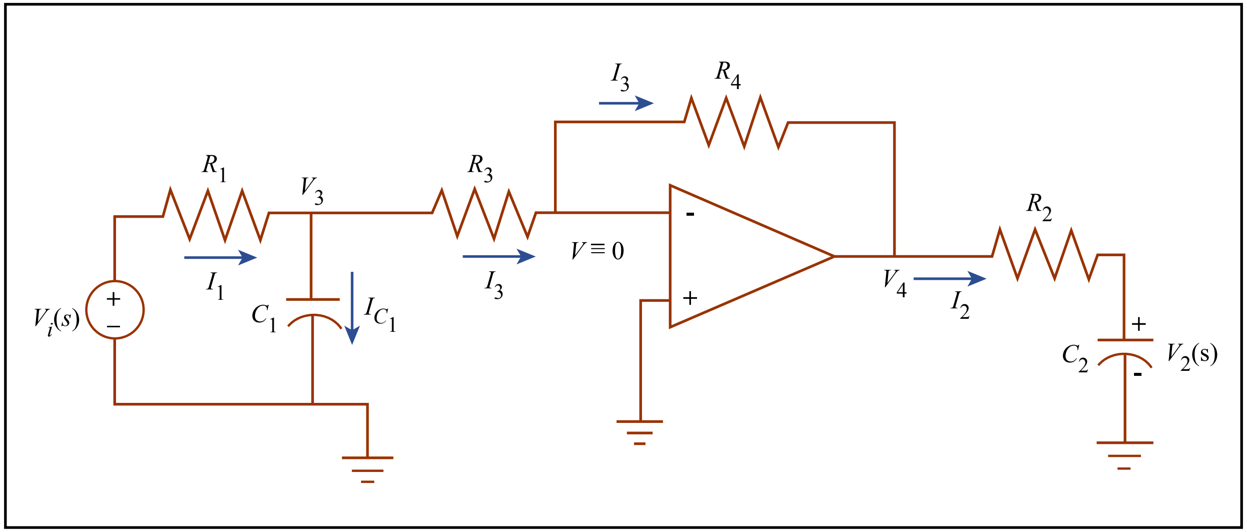

Figure by MIT OpenCourseWare.

We can show that

$$\frac{V_2}{V_i} = \frac{\dfrac{1}{R_1C_1}}{s + \dfrac{1}{R_1C_1} + \dfrac{1}{R_3C_1}} \times \left(-\frac{R_4}{R_3}\right) \times \frac{\dfrac{1}{R_2C_2}}{s + \dfrac{1}{R_2C_2}}.$$

$$\approx \frac{\dfrac{1}{R_1C_1}}{s + \dfrac{1}{R_1C_1}} \times \left(-\frac{R_4}{R_3}\right) \times \frac{\dfrac{1}{R_2C_2}}{s + \dfrac{1}{R_2C_2}} \qquad \text{if} \quad R_3 \gg R_1.$$

Parallel subsystems

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Please see: Fig. 5.5 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Negative feedback

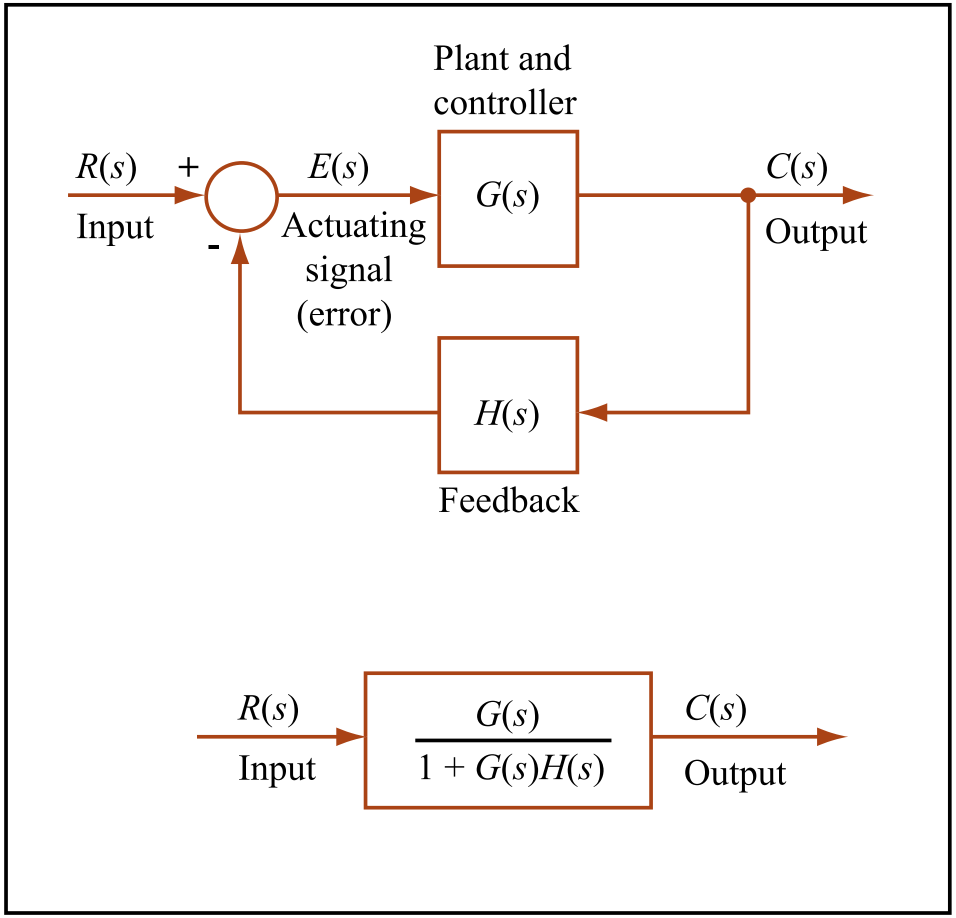

Figure by MIT OpenCourseWare.

$$\begin{aligned} E(s) &= R(s) - H(s)C(s) \\ C(s) &= E(s)G(s) \\ \Rightarrow C(s) &= \Big[R(s) - H(s)C(s)\Big]\; G(s) \\ \Rightarrow \frac{C(s)}{R(s)} &= \frac{G(s)}{1+G(s)H(s)}. \end{aligned}$$

$$\frac{C(s)}{R(s)} \quad : \quad \text{Closed–loop TF}$$

$$G(s)H(s) \quad : \quad \text{Open–loop TF.}$$

equivalent system

Figure 5.6

Positive feedback

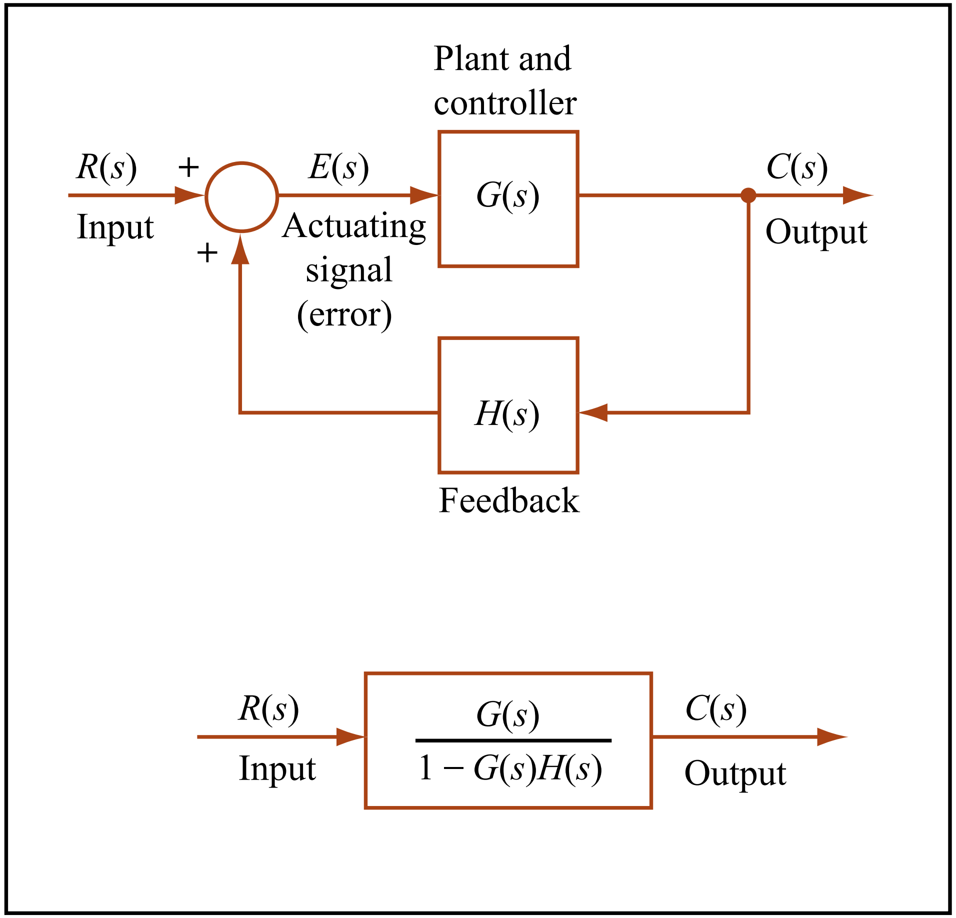

Figure by MIT OpenCourseWare.

Figure 5.6

$$E(s) = R(s) + H(s)C(s)$$
$$C(s) = E(s)G(s)$$
$$\Rightarrow C(s) = \left[R(s) + H(s)C(s)\right] G(s)$$
$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}.}$$
$$\frac{C(s)}{R(s)} : \text{Closed–loop TF}$$
$$G(s)H(s) : \text{Open–loop TF.}$$

equivalent system

Generally, positive feedback is dangerous: it may lead to unstable response (i.e. exponentially increasing) if not used with care

A more general feedback system

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Please see: Fig. 5.6 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Plant: the system we want to control (e.g., elevator plant: input=voltage, output=elevator position)
Controller: apparatus that produces input to plant (i.e. voltage to elevator's motor)
Transducers: converting physical quantities so the system can use them
(e.g., input transducer: floor button pushed→voltage;
output transducer: current elevator position →voltage)
Feedback: apparatus that contributes current system state to error signal (e.g., in elevator system, error=voltage representing desired position – voltage representing current position

Transient response of a feedback system

Figure 5.15

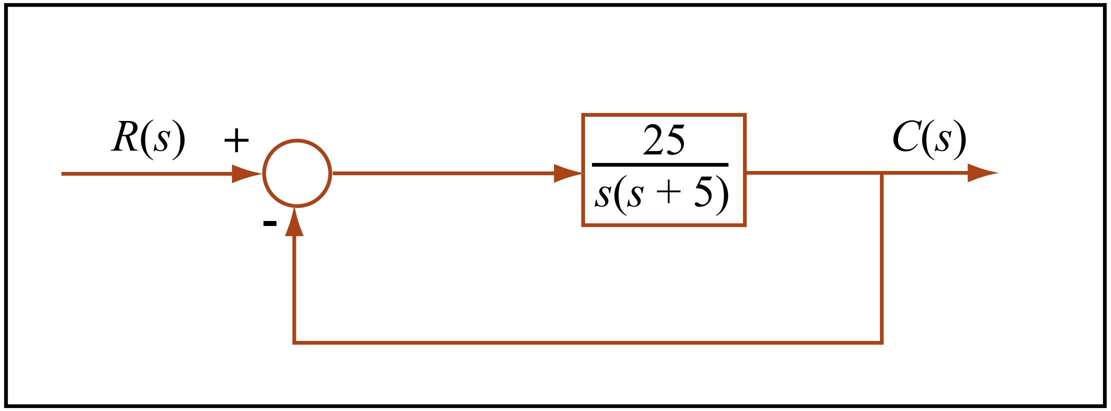

Figure by MIT OpenCourseWare.

$$
\begin{aligned}
\text{Plant \& Controller} \quad & G(s) && = \frac{25}{s(s+5)}, \\
\text{Gain} \quad & K && = 25, \\
\text{Feedback} \quad & H(s) && = 1. \\
\text{Open loop TF} \quad & G(s)H(s) && = \frac{25}{s(s+5)}; \\
\text{Closed loop TF} \quad & \frac{G(s)}{1+G(s)H(s)} && = \frac{25}{s^2+5s+25}.
\end{aligned}
$$

Transient response of a feedback system

Figure 5.15

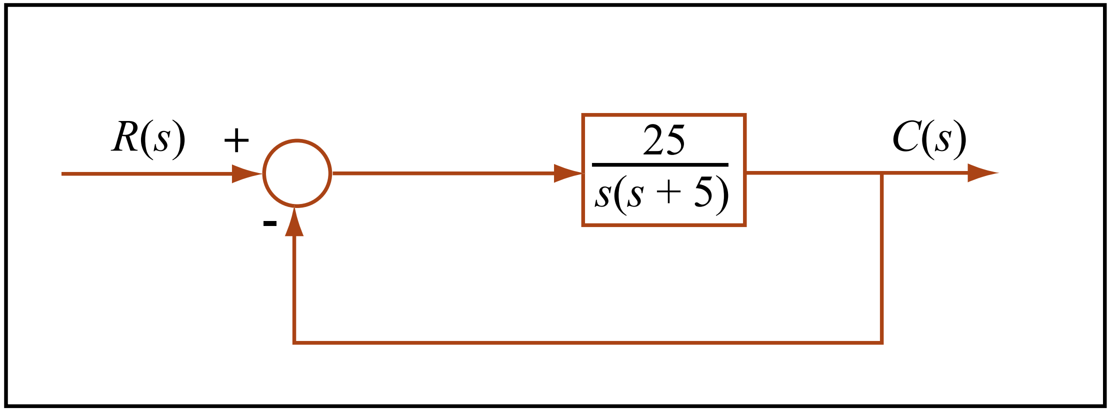

Figure by MIT OpenCourseWare.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{25}{s^2 + 5s + 25}.$$

$$\text{Recall} \quad \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n = \sqrt{25} = 5 \quad \text{rad/sec}; \qquad 2\zeta\omega_n = 5 \Rightarrow \zeta = 0.5;$$

$\Rightarrow$ The feedback system is **underdamped**.

Transient response of a feedback system

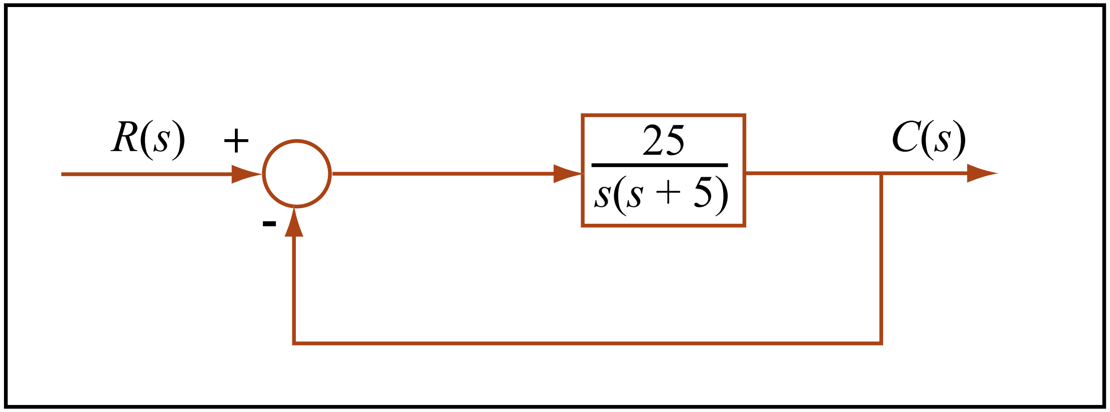

Figure 5.15

Figure by MIT OpenCourseWare.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{25}{s^2 + 5s + 25}.$$

$$\text{Peak time} \quad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.726 \quad \text{sec},$$

$$\%\text{OS} = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100 = 16.3,$$

$$\text{Settling time} \quad T_s = \frac{4}{\zeta\omega_n} = 1.6 \quad \text{sec}.$$

Adjusting the transient by feedback

Figure 5.16

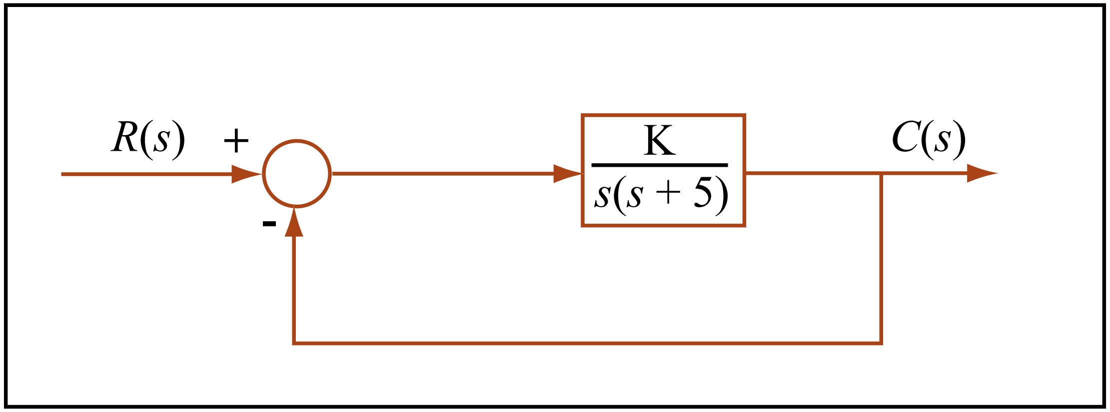

Figure by MIT OpenCourseWare.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{K}{s^2 + 5s + K}.$$

We wish to reduce overshoot to %OS=10% or less.

$$\omega_n = \sqrt{K}; \quad 2\zeta\omega_n = 5 \quad \Rightarrow \zeta = \frac{5}{2\sqrt{K}}.$$

For 10% overshoot or less, we need $\zeta = 0.591$ or more. Therefore,

$$K = 17.9 \quad \text{or less.}$$

The op-amp as a feedback system

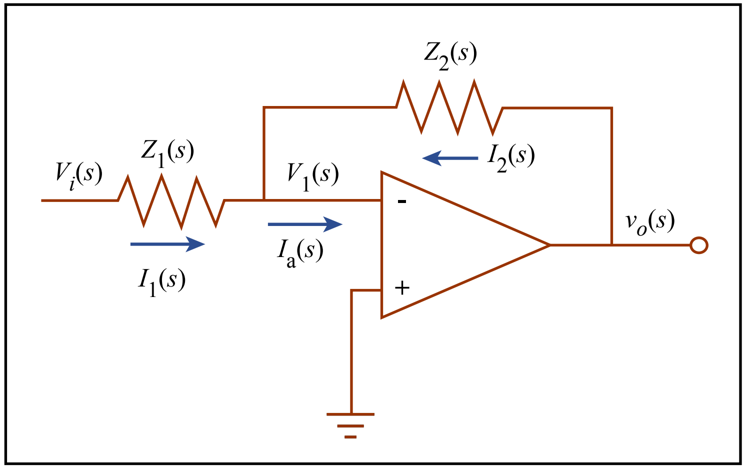

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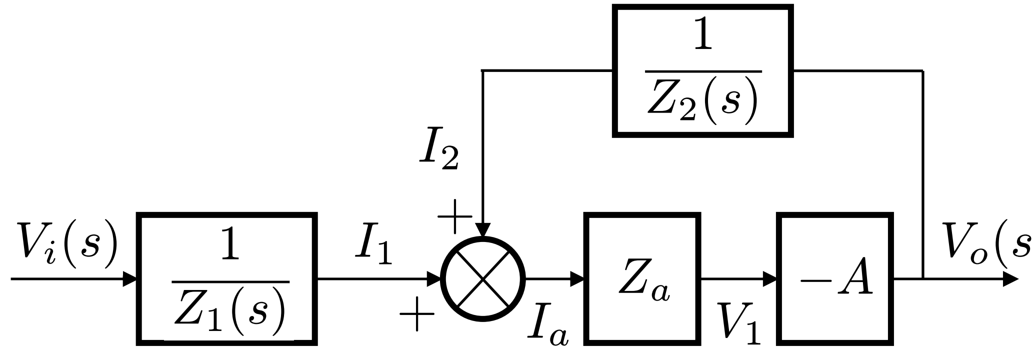
Figure 2.10(c)



$$I_1 = \frac{V_i - V_1}{Z_1} \approx \frac{V_i}{Z_1} \quad \text{if} \quad V_1 \approx 0$$

$$I_2 = \frac{V_o - V_1}{Z_2} \approx \frac{V_o}{Z_2} \quad \text{if} \quad V_1 \approx 0$$

$$V_o = -AV_1 = -AI_aZ_a = (I_1 + I_2)\,Z_a$$

$$\approx -A\left(\frac{V_i}{Z_1} + \frac{V_o}{Z_2}\right) Z_a.$$

$$\Rightarrow V_o\left(1 + \frac{AZ_a}{Z_2}\right) = -AV_i\frac{Z_a}{Z_1}.$$

$$\frac{V_o}{V_i} = \frac{-\dfrac{AZ_a}{Z_1}}{1 - \dfrac{-AZ_a}{Z_2}}$$

$$= -\frac{-\dfrac{1}{Z_1}}{\dfrac{1}{AZ_a} + \dfrac{1}{Z_2}}$$

$$\approx -\frac{Z_2}{Z_1} \quad \text{if} \quad A \to \infty.$$