Goals for today

- The operational amplifier
  - input-output relationships
  - feedback configuration
- More about zeros
  - zero on the right-half plane: non-minimum phase response
  - Zero-pole cancellation
- Next week
  - Block diagram operations
  - Analysis of a simple feedback system

The operational amplifier (op-amp)

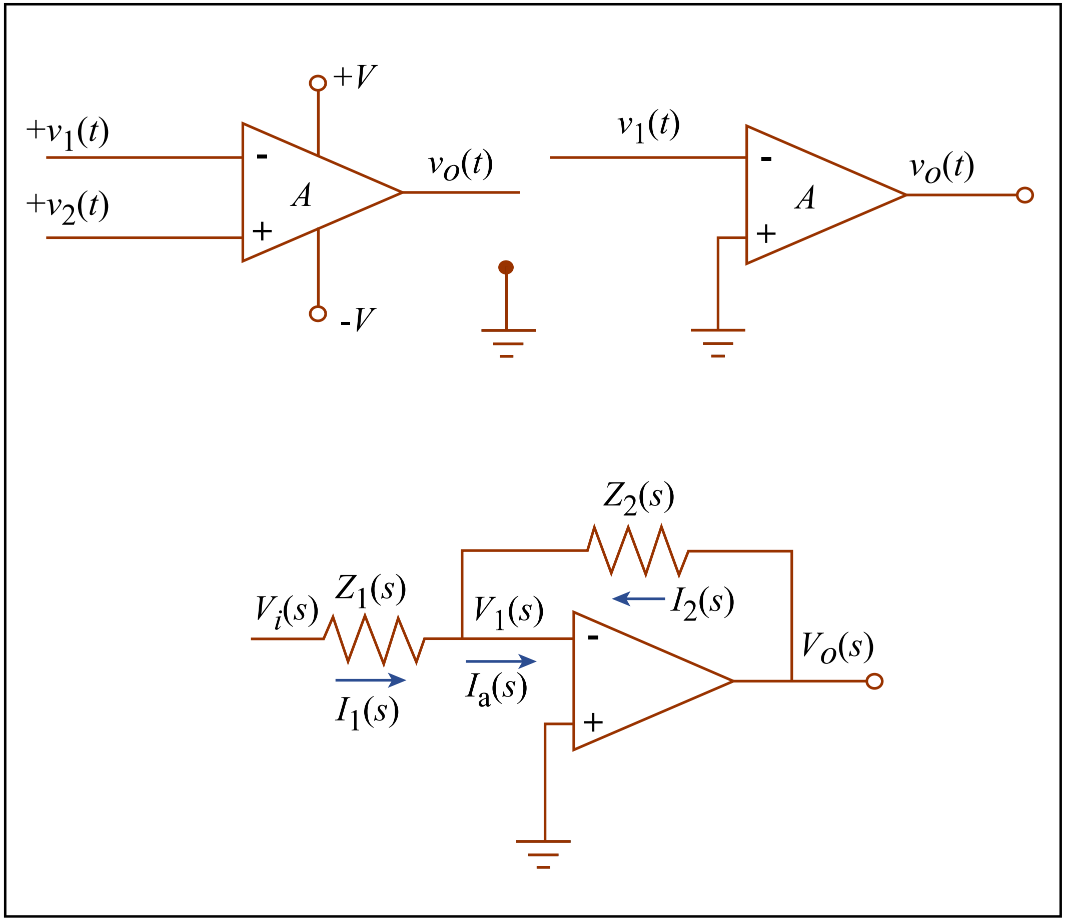

Figure by MIT OpenCourseWare.

Figure 2.10
(see also Lecture 04 – page 16)

(a) Generally, $v_o = A(v_2 - v_1)$, where A is the amplifier **gain.**

(b) When v_2 is grounded, as is often the case in practice, then $v_o = -Av_1$. (Inverting amplifier.)

(c) Often, A is large enough that we can approximate $A \to \infty$. Rather than connecting the input directly, the op–amp should then instead be used in the **feedback** configuration of Fig. (c). We have:

$$V_1 = 0; \qquad I_a = 0$$

(because V_o must remain finite) therefore

$$I_1 + I_2 = 0;$$

$$V_i - V_1 = V_i = I_1 Z_1;$$

$$V_o - V_1 = V_o = I_2 Z_2.$$

Combining, we obtain

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}.$$

Example: *PID controller*

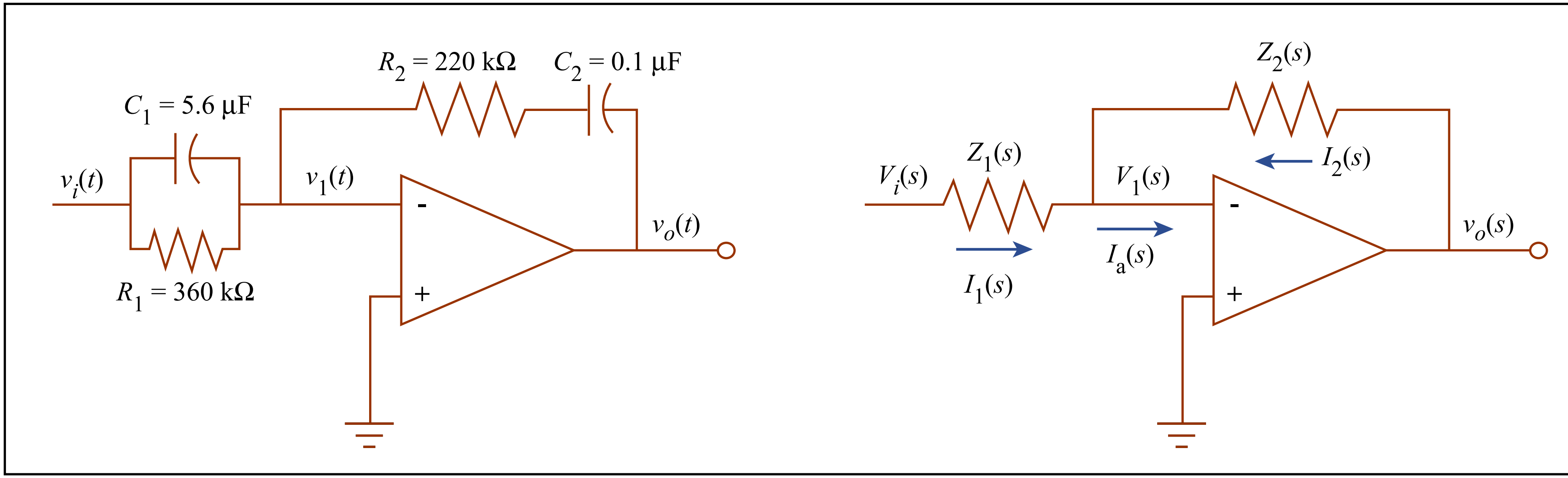

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Figure 2.11

Figure 2.10

Equivalent impedances:

(R_1, C_1 connected in parallel)

$$\frac{1}{Z_1(s)} = \frac{1}{R_1} + C_1 s \Rightarrow Z_1(s) = \frac{R_1}{1 + R_1 C_1 s} = \frac{360 \times 10^3}{1 + 2.016 s};$$

(R_2, C_2 connected in series)

$$Z_2(s) = R_2 + \frac{1}{C_2 s} = 220 \times 10^3 + \frac{1}{10^{-7} s}.$$

$\Rightarrow$ Transfer Function:

$$\frac{V_o(s)}{V_i(s)} = -1.232 \frac{s^2 + 45.95 s + 22.55}{s}.$$

Example: *all-pass filter*

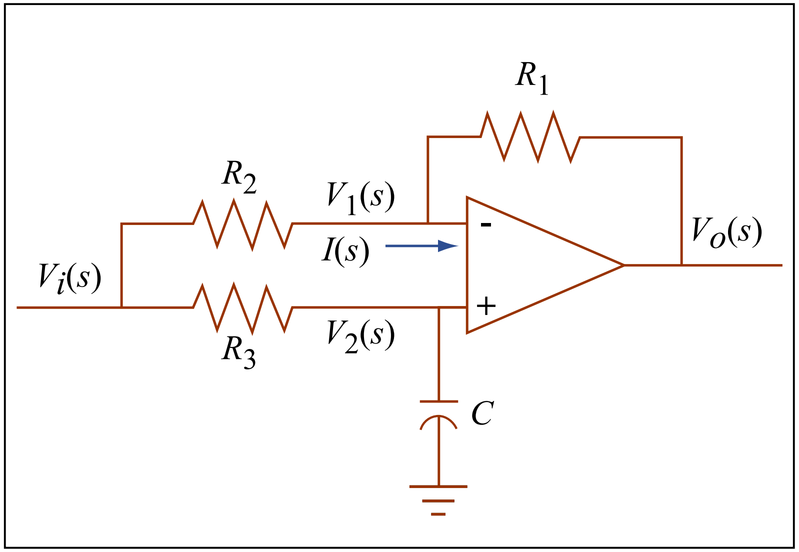


Figure by MIT OpenCourseWare.

Step response without zero:

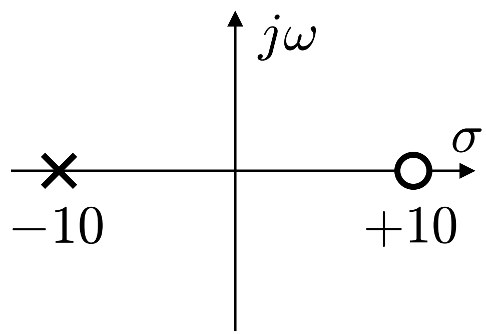
$$C_o(s) = -\frac{1}{s\,(s+10)} = -\frac{1/10}{s} + \frac{1/10}{s+10}$$

$$c_0(t) = -\frac{1}{10}\left(1 - \mathrm{e}^{-10t}\right) u(t).$$

Images removed due to copyright restrictions.
Please see Fig. 4.28 in Nise, Norman S. *Control Systems Engineering*.
4th ed. Hoboken, NJ: John Wiley, 2004.

Transfer function:

$$\frac{V_o(s)}{V_i(s)} = -\frac{R_1}{R_2}\,\frac{s - \dfrac{R_2}{R_1 R_3 C}}{s + \dfrac{1}{R_3 C}}.$$



Step response:

$$\begin{aligned} C(s) &= -\frac{s-10}{s\,(s+10)} \\ &= \frac{1}{s} - \frac{2}{s+10} \Rightarrow \\ c(t) &= \left(1 - 2\mathrm{e}^{-10t}\right) u(t). \end{aligned}$$

Substituting $R_1 = R_2$, $R_3 = 100\mathrm{k}\Omega$, $C = 1\mu\mathrm{F}$,

$$\frac{V_o(s)}{V_i(s)} = -\frac{s-10}{s+10}$$ zero in the r.h.p.

Non–minimum phase system

Nonminimum-phase response

Consider a system without a zero, whose step response is $C_o(s)$ and recall that the effect of the zero is $C(s) = (s+a)C_o(s) = sC_o(s) + aC_o(s)$. In the time domain, $c(t) = \dot{c}_o(t) + ac_o(t)$. Therefore, the system response with the zero is the sum of the derivative of the original response plus the original response amplified by a gain equal to a ("proportional term.")

If $a < 0$ and the derivative term $\dot{c}_o(t=0)$ is larger than the proportional term $ac_o(t=0)$, then the response will initially follow the derivative term in the *opposite direction* of the proportional term.

Please see: Fig. 4.26 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Zero-pole cancellation

Compare the step responses

$$C_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)}$$

$$C_2(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)}.$$

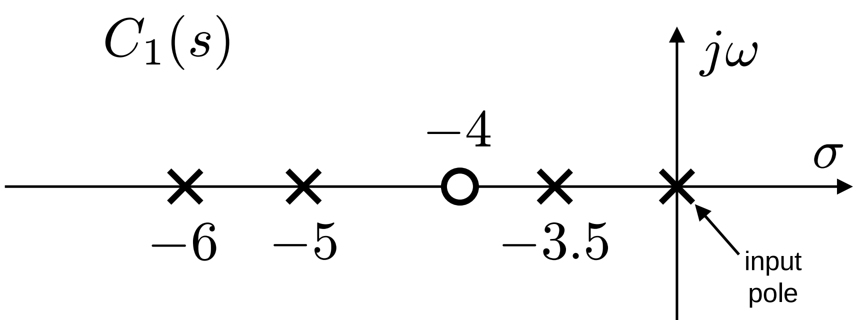
Partial fraction expansion yields

non-negligible

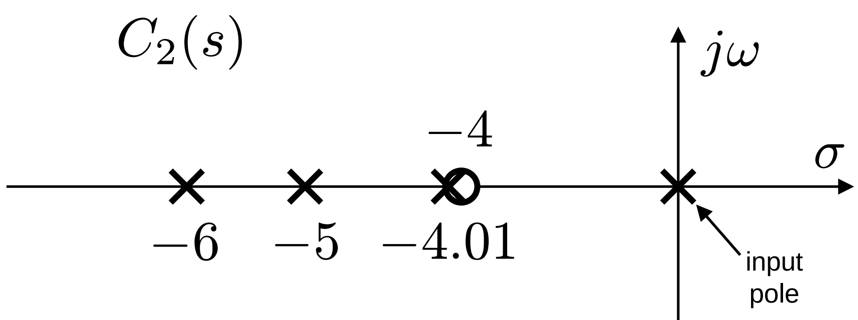
negligible

$$C_1(s) = \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5}$$

$$C_2(s) = \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} - \frac{0.033}{s+4.01}.$$



zero-pole cancellation does not occur



zero-pole cancellation occurs