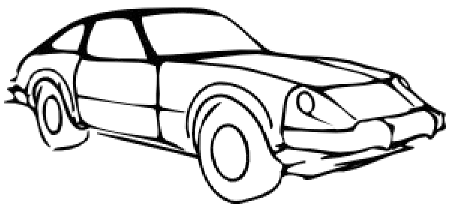
2.004 Dynamics and Control II

Introductions

- Faculty
  - Prof. George Barbastathis (lectures)
  - Prof. Franz Hover (labs)
  - Prof. David Gossard (labs)
- Grader
  - Sebastian Castro
  - TBA
- Administrative Assistant
  - Ms. Kate Anderson

This class is about …

- System modeling



Hardware

Image from the Open Clip Art Library, http://openclipart.org

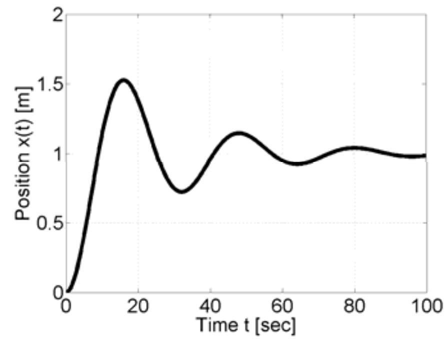
$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t)$$

Model: ordinary differential equation (ODE) or other mathematical representation

- System dynamics

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t)$$

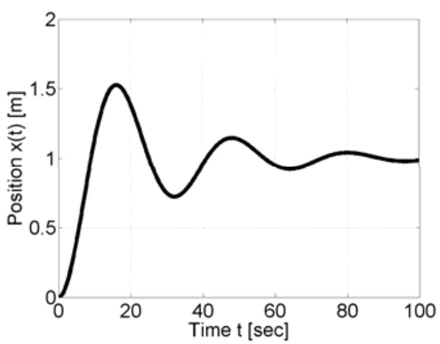
Model

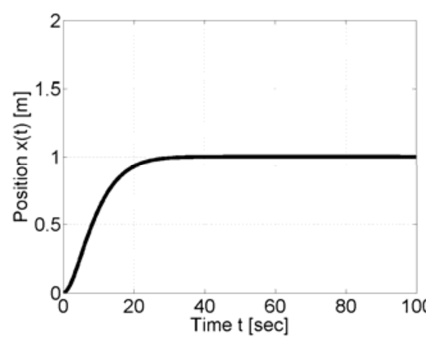


Response

- System control

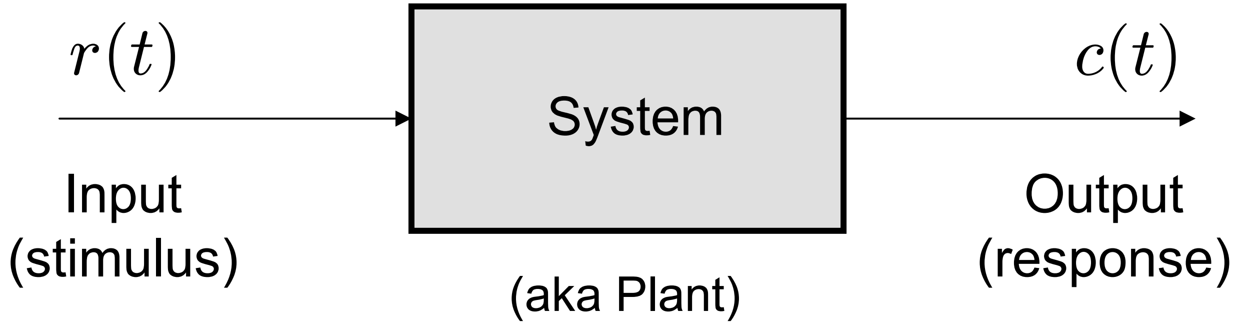
Response



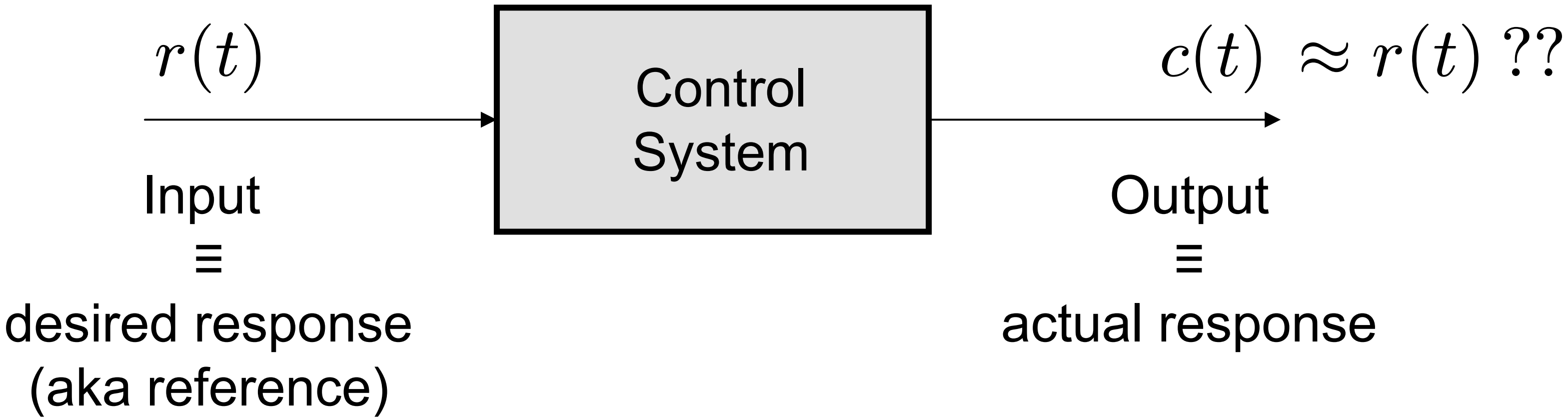


Desired response

Systems



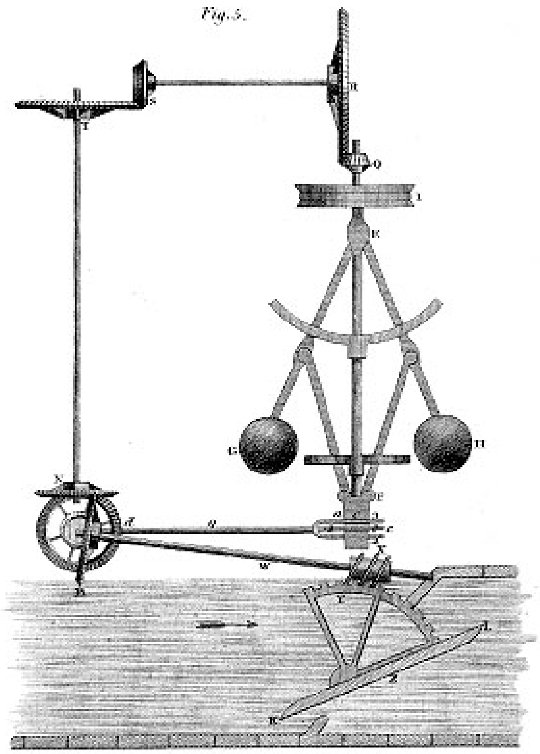
Control Systems



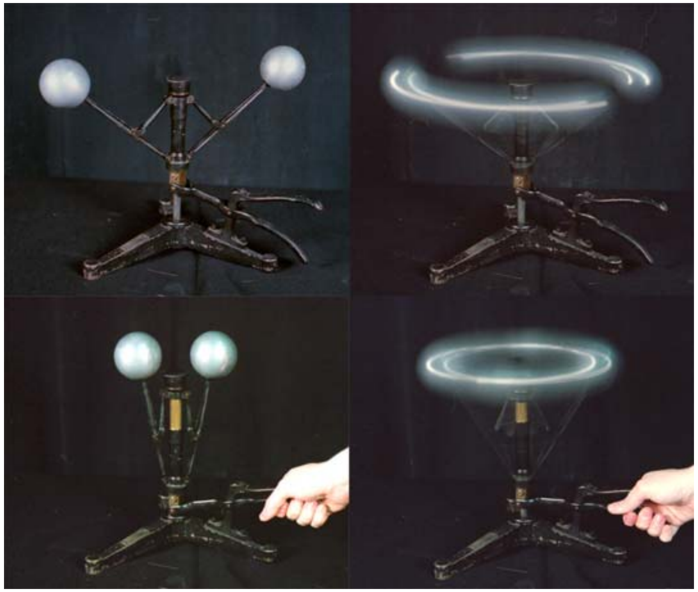
"block diagrams"

Examples of control systems

Flyball Governor
(Watt steam engine)



"As the turbine speeds up, the weights are moved outward by centrifugal force, causing linkage to open a pilot valve that admits and releases oil on either side of a piston or on one side of a spring-loaded piston. The movement of the piston controls the steam valves."



http://www.fas.harvard.edu/~scidemos/NewtonianMechanics/FlyballGovernor/FlyballGovernor.html

Examples of control systems

Elevators

Images and text removed due to copyright restrictions.

Please see: Fig. 1.2 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Examples of control systems

Segway

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http://www.segway.com/img/content/media/product_images/Girli2_high.jpg

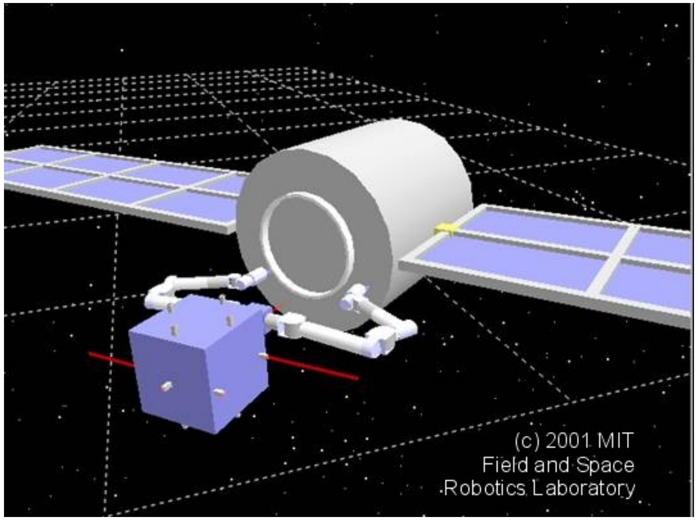
http://www.segway.com/img/content/media/product_images/malex2_high.jpg

http://www.segway.com/img/content/media/product_images/airporti2_high.jpg

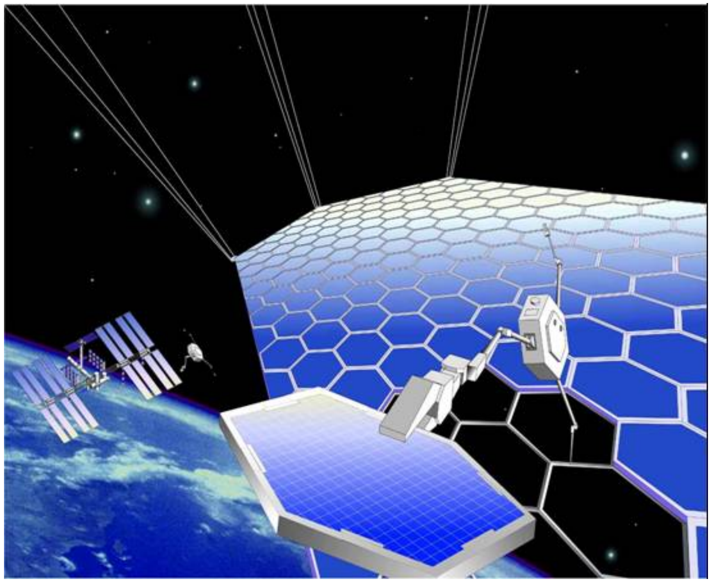
http://www.segway.com/img/content/models/focus-i2-comm-cargo-man-aisle-lg.jpg

Examples of control systems

Rovers for rough terrains and space exploration



A free-flying robot capturing a satellite
in preparation for servicing (Chris Lee).

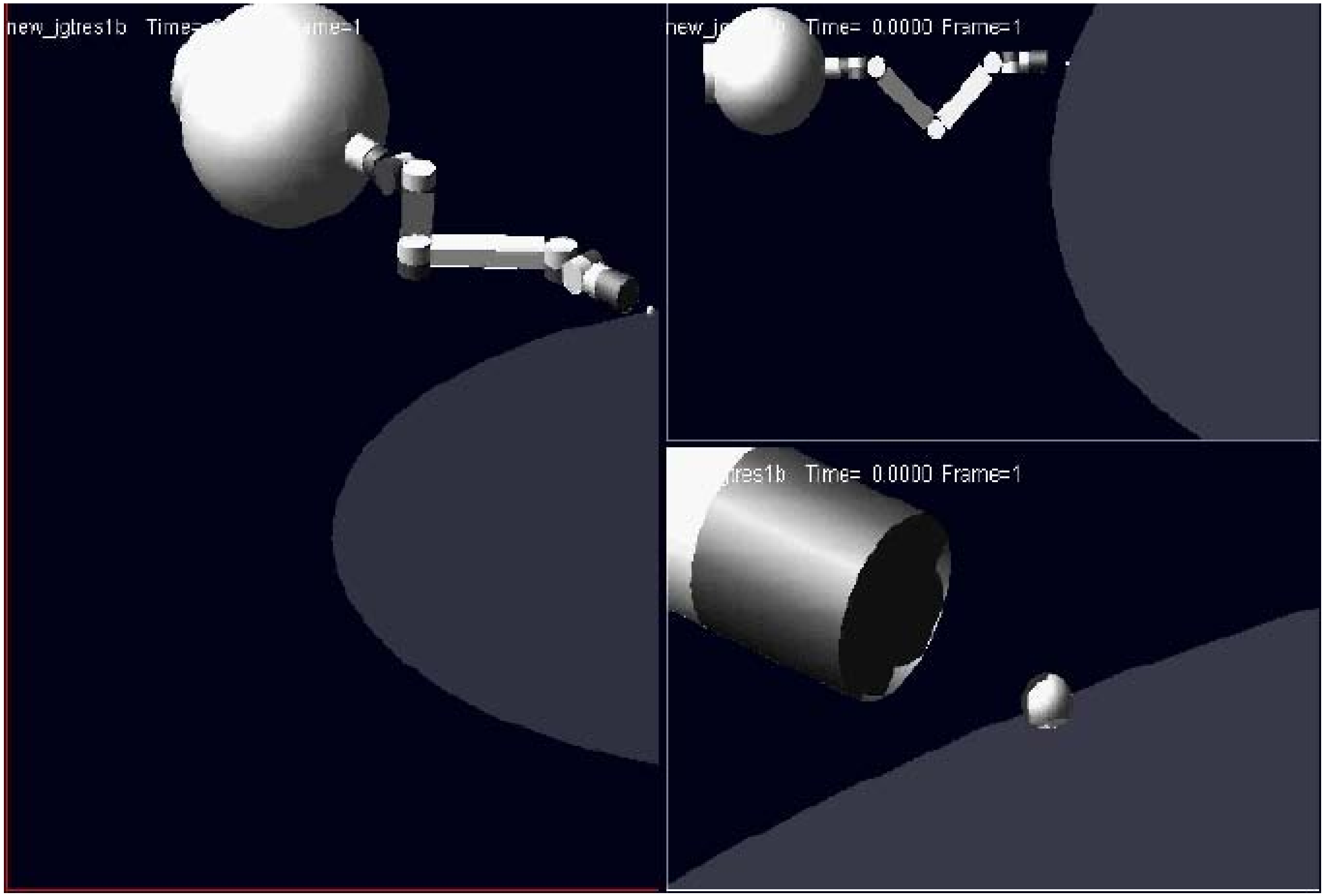


A free-flying robot assembling a large
solar power grid from sub-modules (JAXA).

MIT Field and Space Robotics Laboratory,
http://robots.mit.edu

Examples of control systems

Rovers for rough terrains and space exploration



Courtesy Steven Dubowsky, MIT Space Lab. Used with permission.

MIT Field and Space Robotics Laboratory,
http://robots.mit.edu

Examples of control systems

Hard disk drives

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Fig. 1 in Tam, Karman, et al. 1992. Disk drive power control circuit and method.
US Patent 5, 412, 809, filed Nov. 12, 1992, and issued May 2, 1995.

Fig. 3.3 in Workman, Michael L. "Adaptive Proximate Time-Optimal Servomechanisms." PhD thesis, Stanford, 1987.

Fig. 3 in Al Mamun, Abdullah, and Ge, Shuszi Sam. "Precision Control of Hard Disk Drives."
IEEE Control Systems Magazine 25 (August 2005): 14-19.

Types of control

- Regulator (e.g. flyball governor)
  - maintains constant output despite disturbances
- Compensator (e.g. elevator)
  - drive system from an initial to a final state according to specifications on the transient response
- Tracking (e.g. space robot)
  - match output to a non-stationary input despite disturbances
- Optimal control (e.g. hard disk drive)
  - drive system from an initial to a final state while optimizing a merit function (e.g. minimum time to target or minimum energy consumption)
- Combinations of the above
  (e.g. Segway might regulate a constant trajectory or drive a transient to turn trajectory while minimizing energy consumption)

Transient specifications: e.g., elevator response

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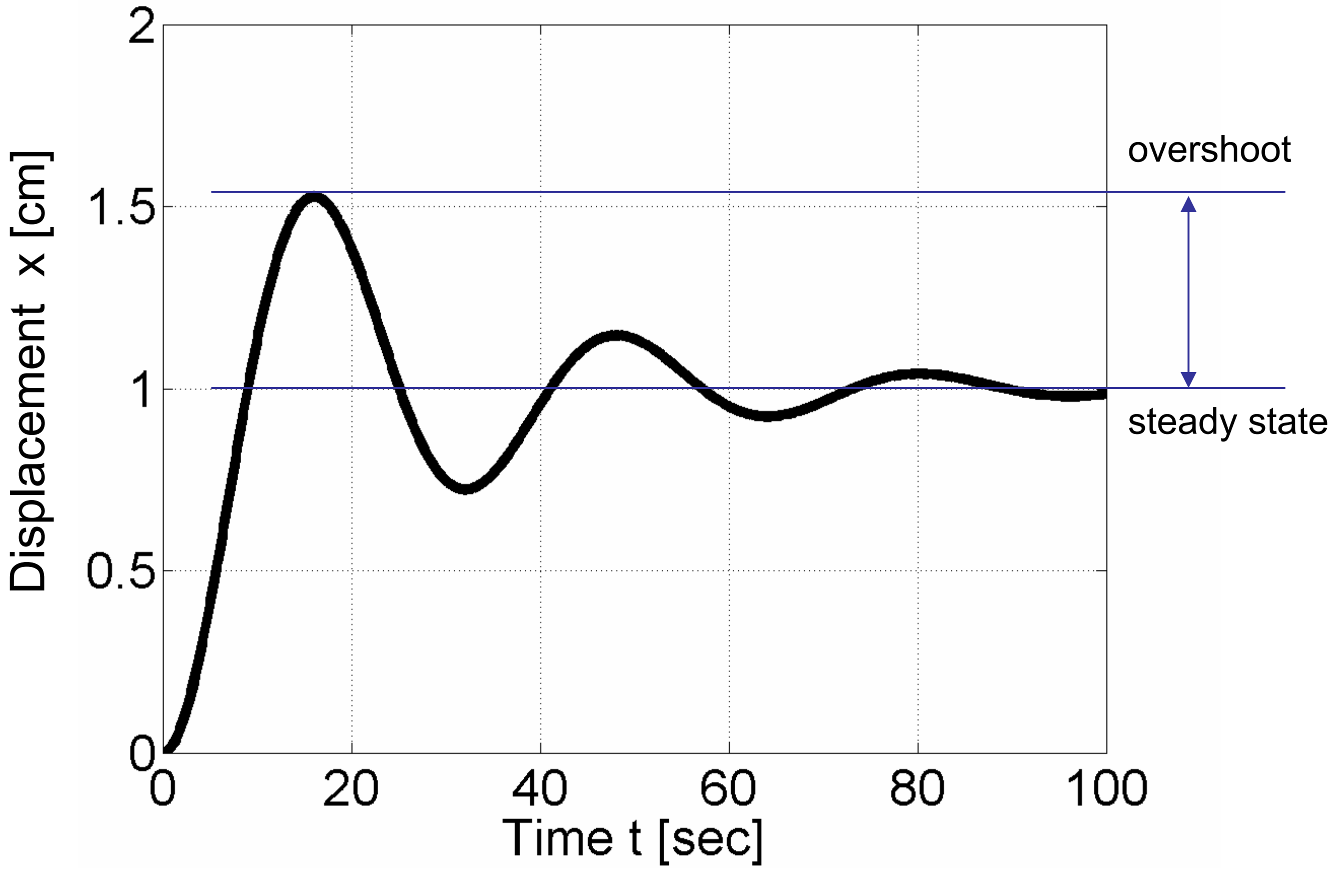
Please see: Fig. 1.5 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

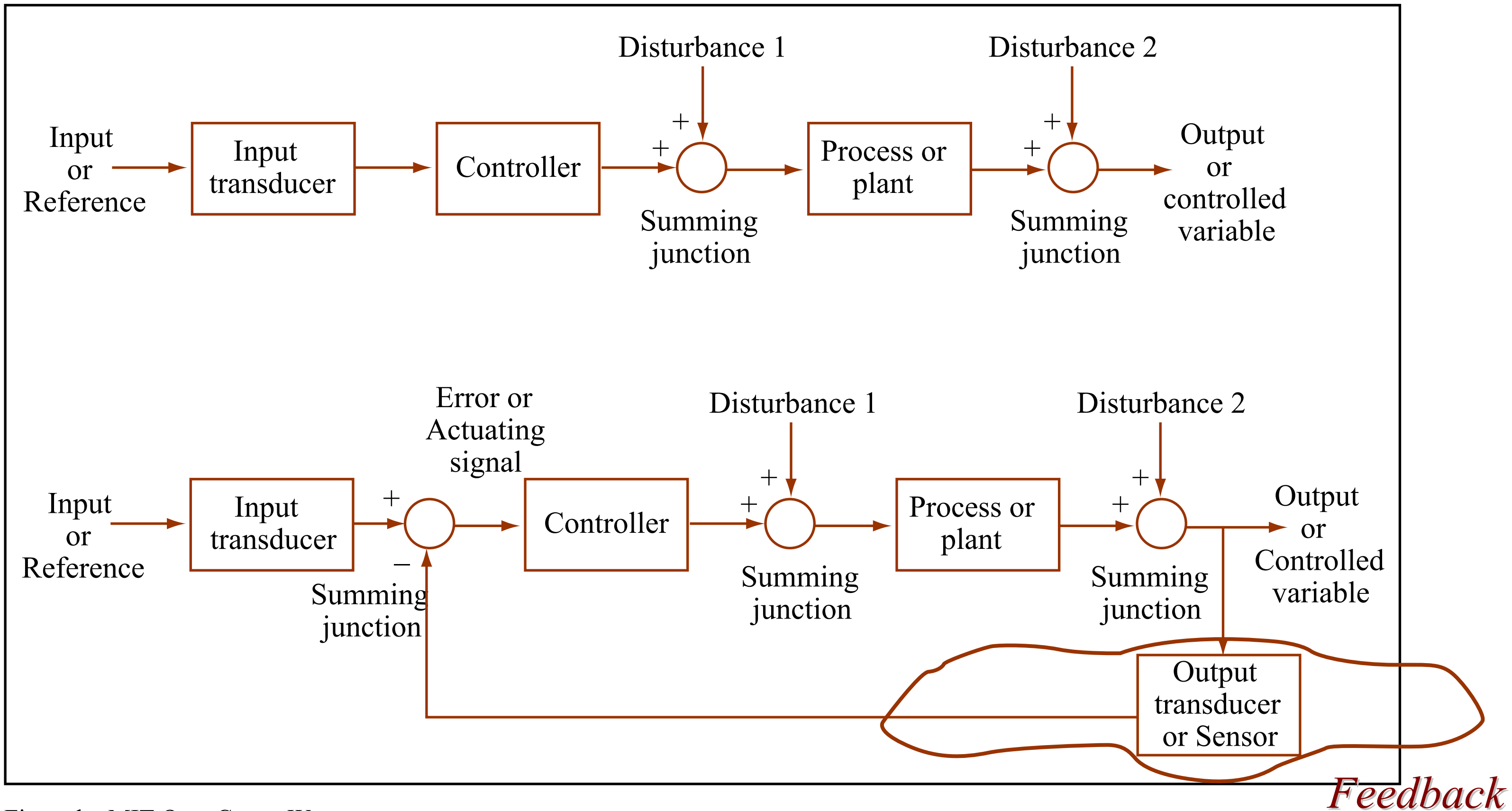
Transient specifications: e.g., car suspension



Control: open *vs* closed loop

OPEN LOOP

CLOSED LOOP



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Using feedback: the importance of gain

Image removed due to copyright restrictions.

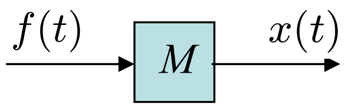
Please see Fig. 1.10 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

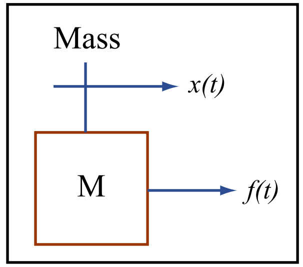
Today's goals

- Introduction and motivation √
- Modeling mechanical elements by ordinary differential equations (ODEs)
  - Translation
    - mass, damper, spring
  - Rotation
    - inertia, rotary damper, rotary spring, gear
- Definition of linear systems
- **Next lecture (Friday):** solving the ODE model for a simple system of mechanical translation

Mechanical system components: translation

- Mass

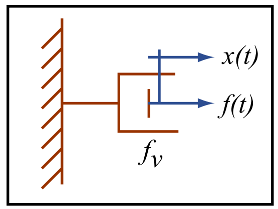




Figures by MIT OpenCourseWare.

- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE (Newton's law): $M\dfrac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} \equiv M\ddot{x}(t) = f(t)$

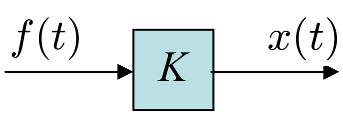
- Damper (friction)
  - viscous



- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE: $f_v\dot{x}(t) = f(t)$

  - Coulomb → Component ODE: $f_c\mathrm{sgn}[\dot{x}(t)] = f(t)$
  - drag → Component ODE: $f_d\,|\dot{x}(t)|\,\dot{x}(t) = f(t)$

- Spring (compliance)



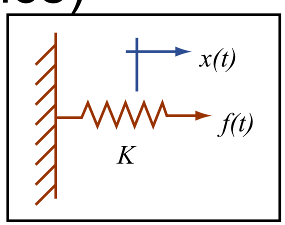
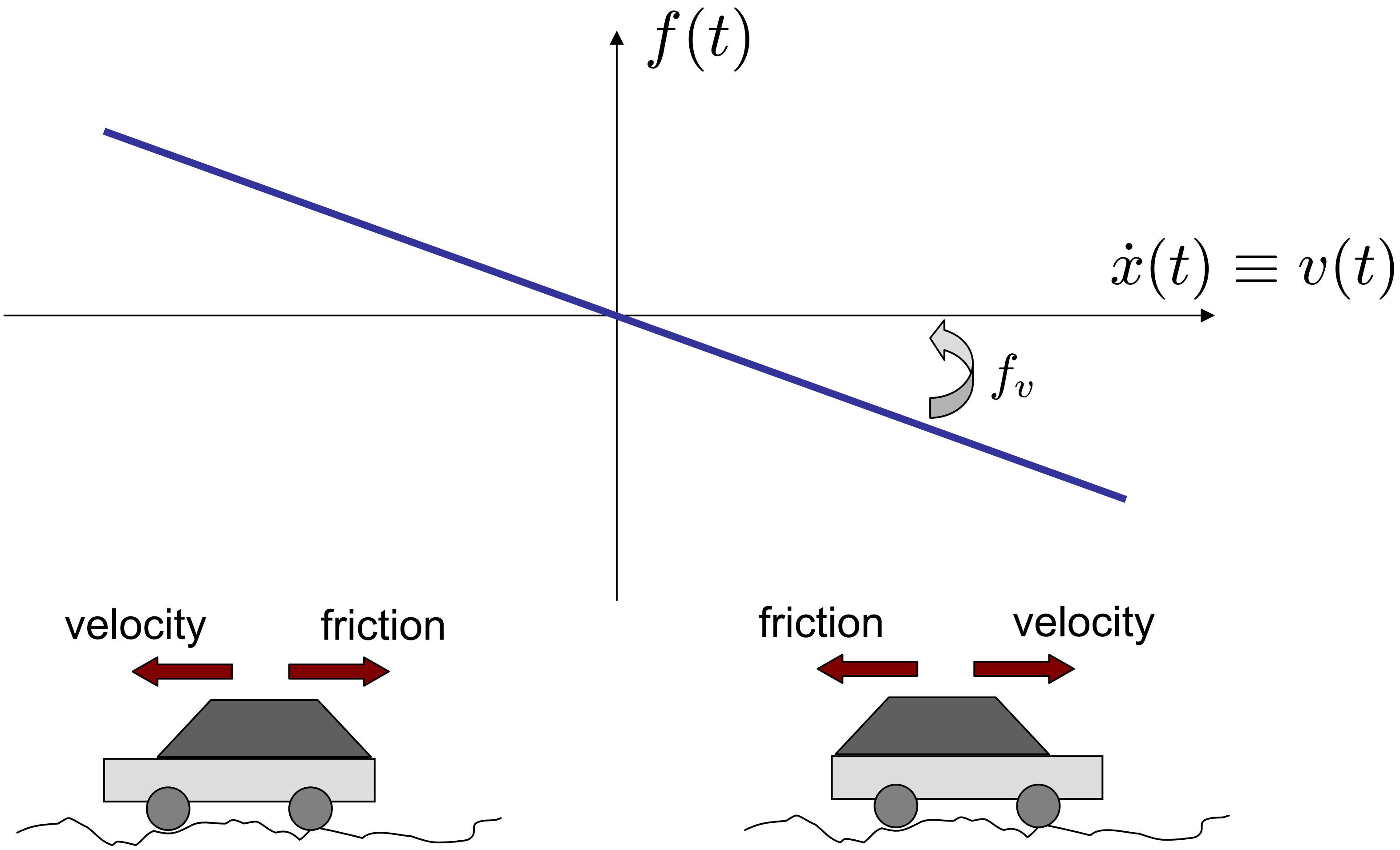


Figure by MIT OpenCourseWare.

- Component input: **force** $f(t)$
- Component output: **position** $x(t)$
- Component ODE (Hooke's law): $Kx(t) = f(t)$

Nise Table 2.4

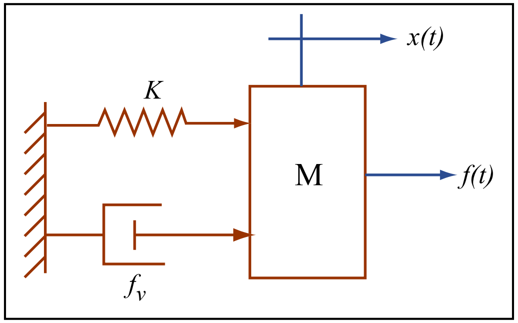
Viscous friction



Viscous friction is in opposite direction to the velocity;
the magnitude of the friction force is proportional to the magnitude of the velocity

Mass – spring – viscous damper system

e.g., car suspension



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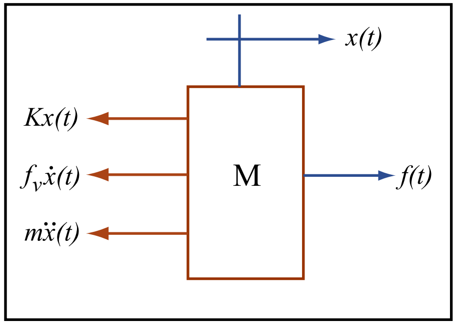

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Model

Force balance

System ODE

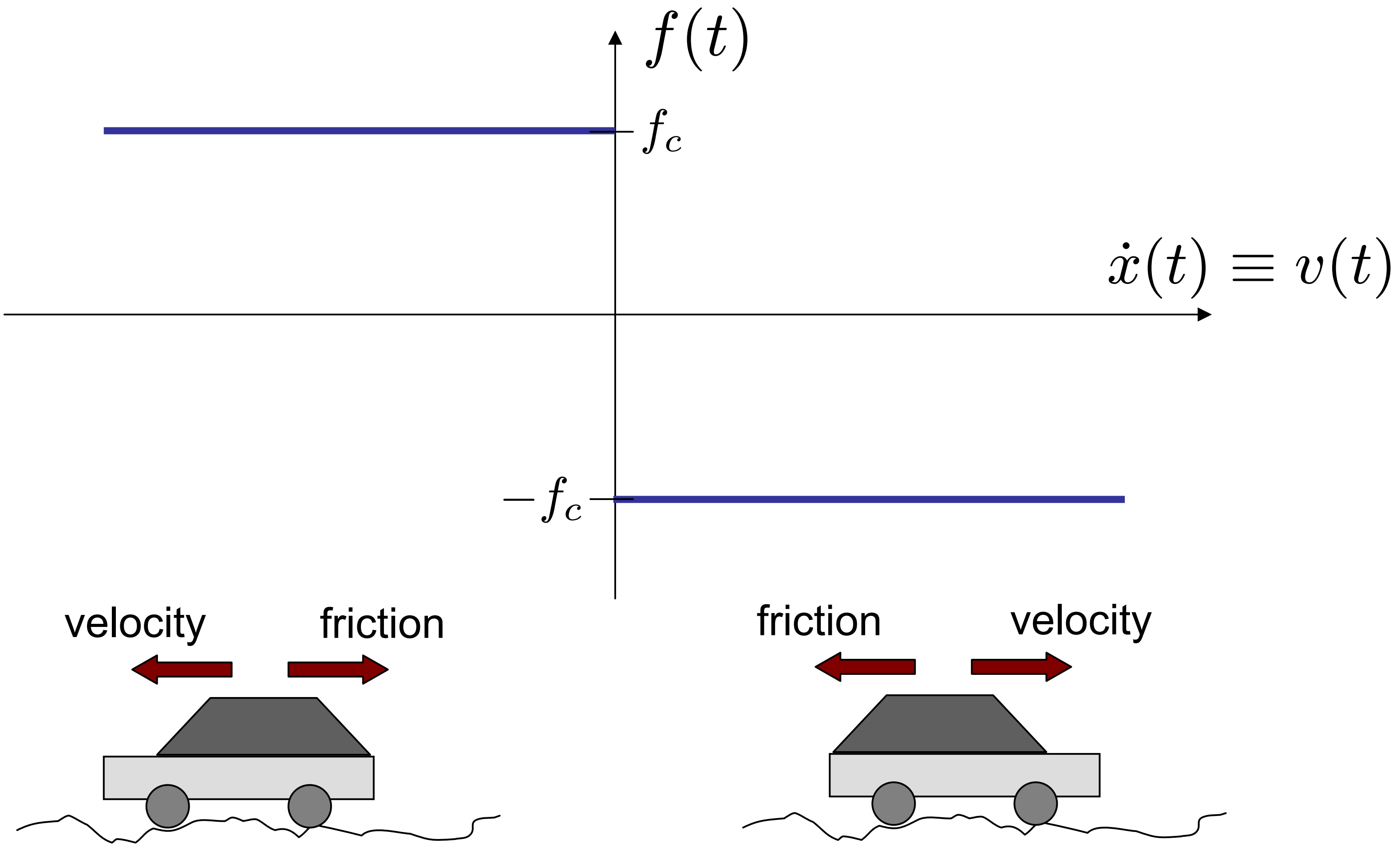
(2nd order ordinary <u>linear</u> differential equation)

$$M\ddot{x}(t) + f_v\dot{x}(t) + Kx(t) = f(t)$$

Equation of motion

Nise Figure 2.15a/2.16a

Coulomb friction



Coulomb friction is in opposite direction to the velocity;
the magnitude of the friction force is <u>independent</u> of the magnitude of the velocity

Mass – spring – Coulomb damper system

e.g., car suspension

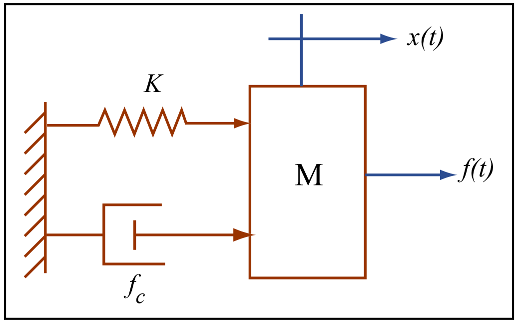


Figure by MIT OpenCourseWare.

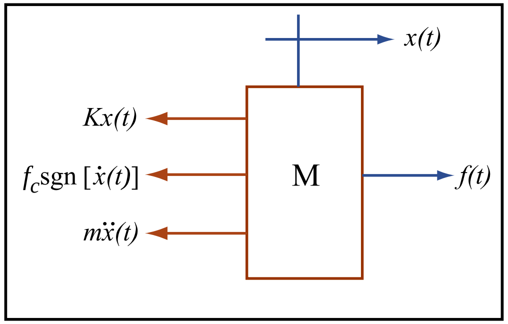


Figure by MIT OpenCourseWare.

Model

Force balance

System ODE

(2nd order ordinary <u>nonlinear</u> differential equation)

$$M\ddot{x}(t) + f_c \operatorname{sgn}\left[\dot{x}(t)\right] + Kx(t) = f(t)$$

Equation of motion

Nise Figure 2.15a/2.16a (modified)

Linear systems: mathematical definition

Consider the mass–spring–*viscous* damper system

$$M\ddot{x}(t) + f_v\dot{x}(t) + Kx(t) = f(t). \qquad (1)$$

Suppose that the response to input $f_1(t)$ is output $x_1(t)$, *i.e.*

$$M\ddot{x}_1(t) + f_v\dot{x}_1(t) + Kx_1(t) = f_1(t); \qquad (2)$$

and the response to input $f_2(t)$ is output $x_2(t)$, *i.e.*

$$M\ddot{x}_2(t) + f_v\dot{x}_2(t) + Kx_2(t) = f_2(t). \qquad (3)$$

If instead the input is replaced by the scaled sum $f_s(t) = a_1 f_1(t) + a_2 f_2(t)$, where a_1 and a_2 are complex constants; then the output is the identically scaled sum $x_s(t) = a_1 x_1(t) + a_2 x_2(t)$. This can be verified directly by adding equations (2) and (3). A system with this property is called **linear**.

You should verify for yourselves that mass–spring–damper systems with Coulomb or drag friction are **nonlinear**.

Linear systems: definition by block diagram

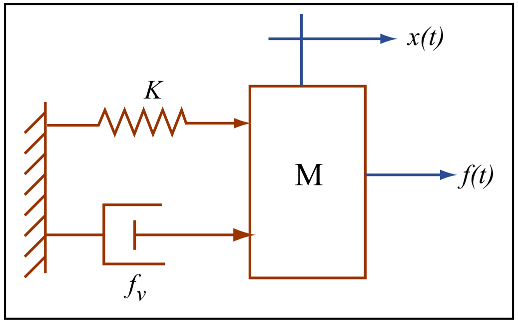
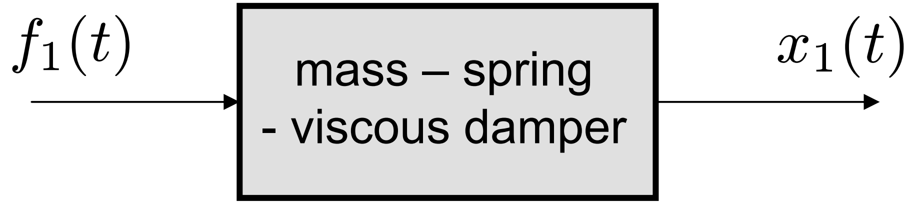
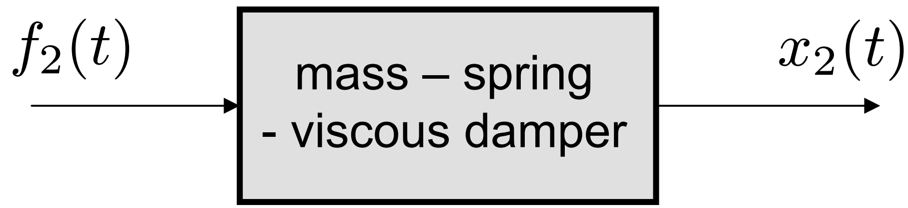


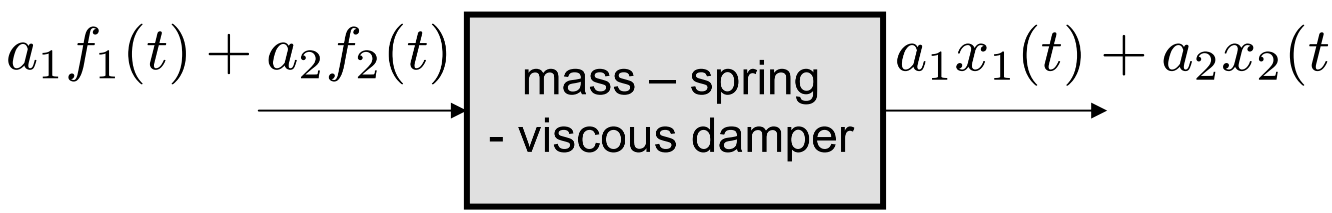
Figure by MIT OpenCourseWare.

if





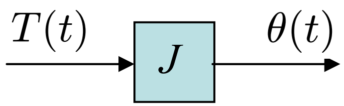
then

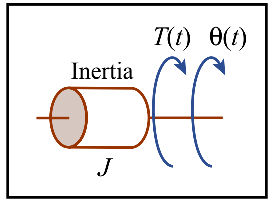


Nise Figure 2.15a

Mechanical system components: rotation

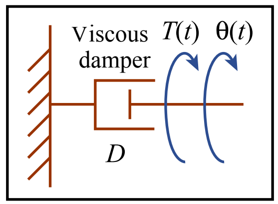
- Inertia





- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE (Newton's law): $m\dfrac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} \equiv m\ddot{\theta}(t) = T(t)$

- Damper (rot.)
  - viscous

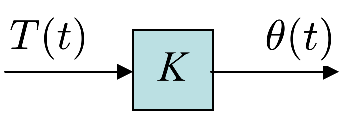


Figures by MIT OpenCourseWare.

- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE: $D\dot{\theta}(t) = T(t)$

  - Coulomb → Component ODE: $D_c \mathrm{sgn}[\dot{\theta}(t)] = T(t)$
  - drag → Component ODE: $D_d \left|\dot{\theta}(t)\right| \dot{\theta}(t) = T(t)$

- Spring (rot.)



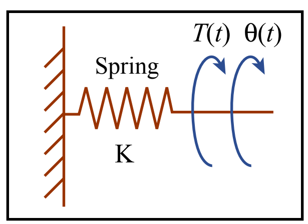


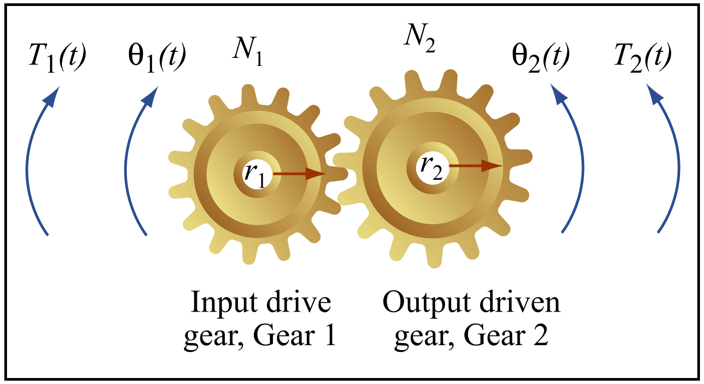
Figure by MIT OpenCourseWare.

- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE (Hooke's law): $K\theta(t) = T(t)$

- Gear (next page)

Nise Table 2.5

Mechanical system components: rotation: gears



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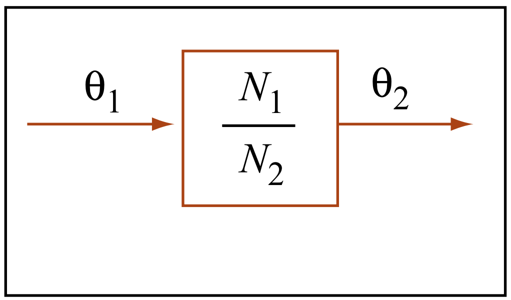

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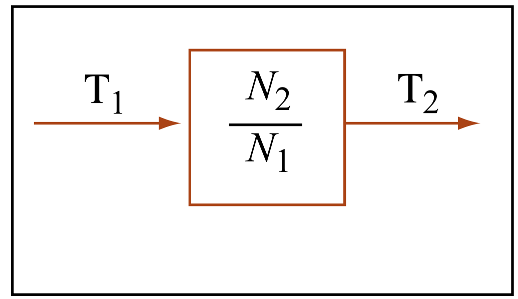


Figure by MIT OpenCourseWare.

- Component input: **angle** $\theta_1(t)$
- Component output: **angle** $\theta_2(t)$
- Component ODE: $$\theta_2 = \frac{N_1}{N_2}\theta_1$$

- Component input: **torque** $T_1(t)$
- Component output: **torque** $T_2(t)$
- Component ODE: $$T_2 = \frac{N_2}{N_1}T_1$$

$$\textbf{Question:}\ \text{Why is } T_1\theta_1 = T_2\theta_2?$$

Nise Figure 2.27, 2.28

Gear transformations

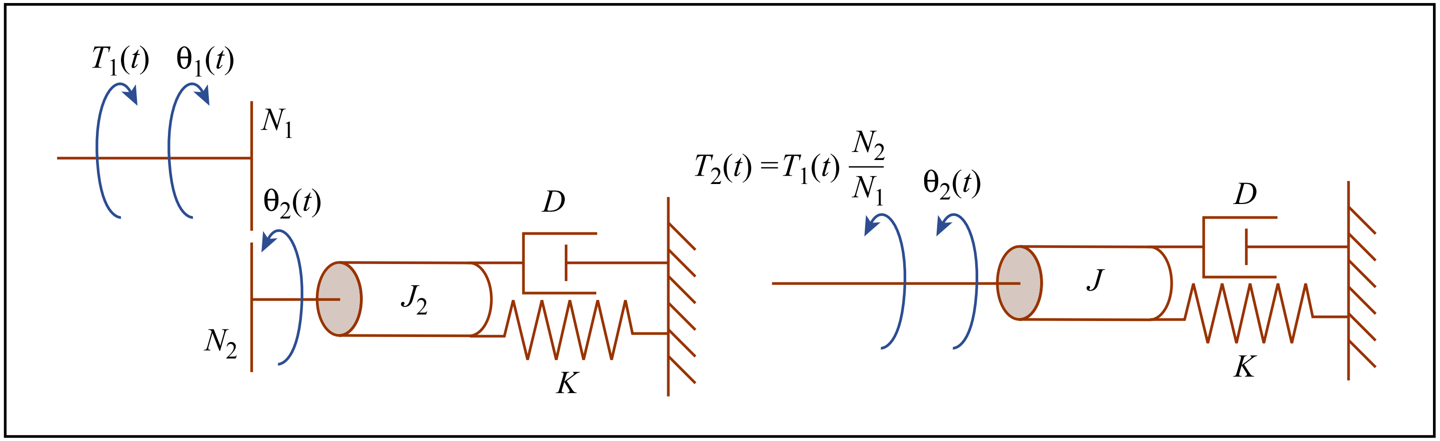


Figure by MIT OpenCourseWare.

Let T_2 denote the torque applied to the left of the inertia J. The equation of motion is

$$J\ddot{\theta}_2 + D\dot{\theta}_2 + K\theta_2 = T_2,$$

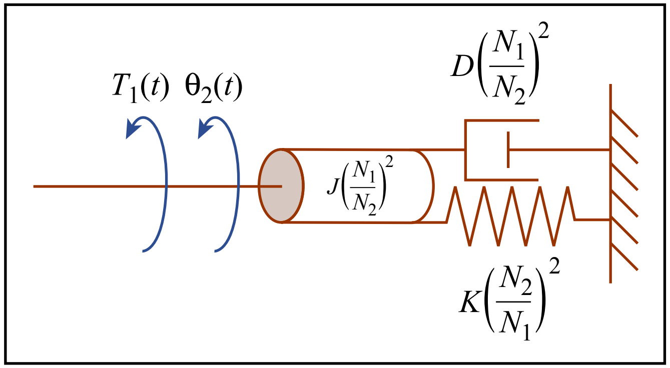
while from the gear equations we have

$$T_2 = T_1\frac{N_2}{N_1} \qquad \text{and} \qquad \theta_2 = \theta_1\frac{N_1}{N_2}.$$

Combining, we obtain

$$\left[\left(\frac{N_1}{N_2}\right)^2 J\right]\ddot{\theta}_1 + \left[\left(\frac{N_1}{N_2}\right)^2 D\right]\dot{\theta}_1 + \left[\left(\frac{N_1}{N_2}\right)^2 K\right]\theta_1 = T_1.$$

This is the equation of motion of the equivalent system shown in (c).



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Nise Figure 2.29

Rotational mechanical system: example

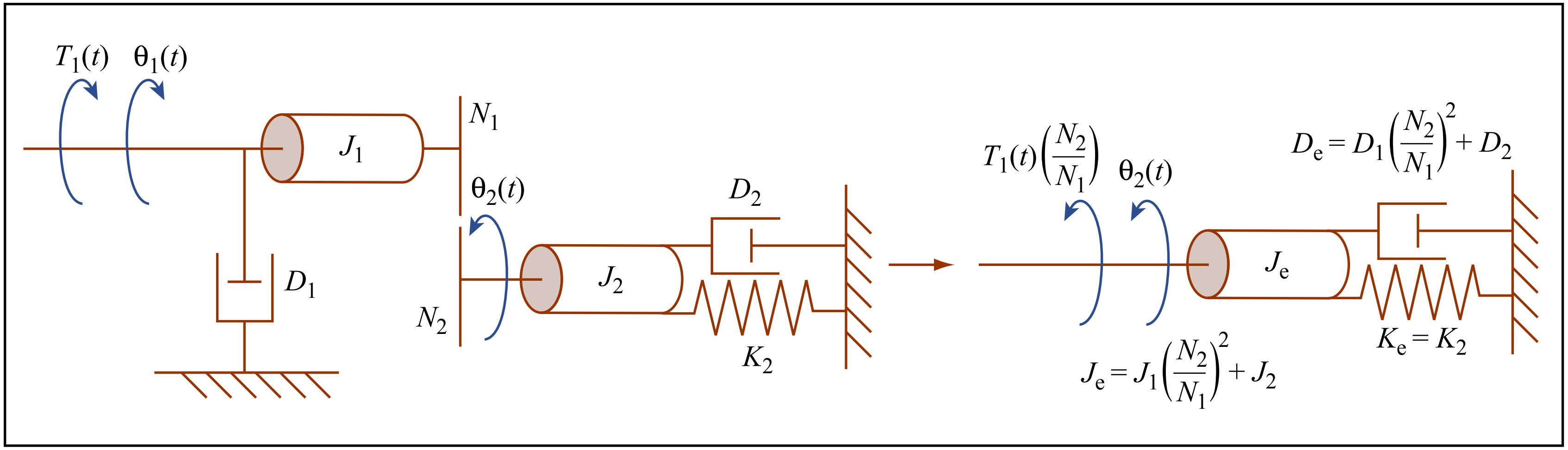


Figure by MIT OpenCourseWare.

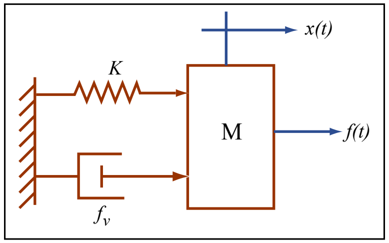
Equation of motion:

$$\left[\left(\frac{N_1}{N_2}\right)^2 J_1 + J_2\right]\ddot{\theta}_2 + \left[\left(\frac{N_1}{N_2}\right)^2 D_1 + D_2\right]\dot{\theta}_2 + K_2\theta_2 = \left(\frac{N_2}{N_1}\right) T_1.$$

Nise Figure 2.30a-b

Summary

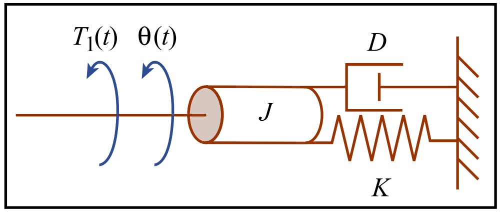
- Control systems
  - regulators, compensators, trackers
  - open & closed loop
- Mechanical system models
  - Translation



$$M\ddot{x}(t) + f_v\dot{x}(t) + Kx(t) = f(t)$$

Figures by MIT OpenCourseWare.

  - Rotation



$$J\ddot{\theta}(t) + D\dot{\theta}(t) + K\theta(t) = T(t)$$

- Viscous damping → <u>linear</u> ODE model

- **Next lecture (Friday):** how to solve linear & nonlinear ODEs