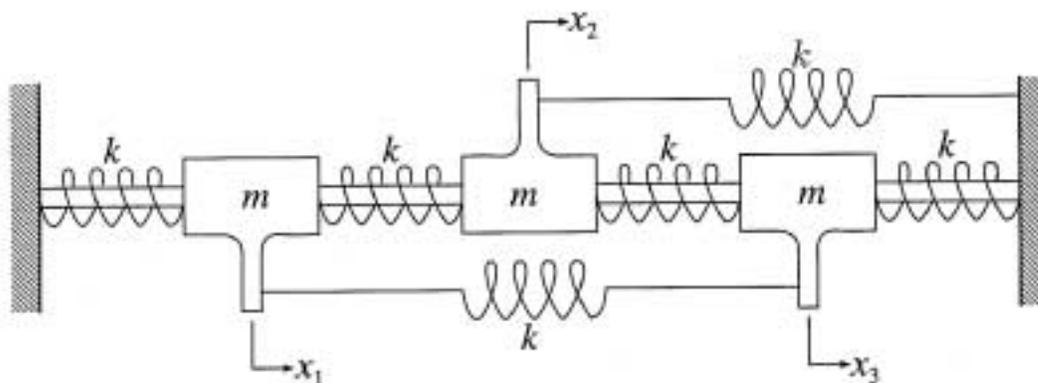


Problem Set No. 8

Problem 1. Three equal masses m slide without friction on a rigid horizontal rod. Six identical springs with spring constant k are attached to the masses as shown in the sketch below. The free vibrations of the system are described by three natural modes of vibration and their corresponding natural frequencies of vibration.



Your problem is to identify as many natural modes and natural frequencies as possible.

Problem 2. Torsional vibration. Probably the most common mechanical power transmission device is the simple shaft. A driving motor or engine applies a torque τ to one end of the shaft and the other end applies the torque τ to the driven element: a propeller, a machine tool, a pump, an electric generator, *etc.* The shaft is twisted by the torque, but if the driver can maintain a rotation rate ω , then the power transmitted along the rotating, twisted shaft is $\pi = \tau\omega$. In the ideal case both the torque τ and the speed ω are constant and there is a steady flow. In actuality the characteristics of both drivers and driven elements are such that the power flow fluctuates throughout each revolution. One of the worst offenders for fluctuating torque is the internal combustion engine. A sequence of explosions produces a fluctuating torque, which is just about the opposite of a steady constant torque. A consequence of fluctuating torque is fluctuating twist in the shaft: torsional vibration.

Historically, torsional vibration was a major technical challenge during the first quarter of the twentieth century. Otto and Diesel engines were being widely adopted in cars and

ships, and there was an epidemic of shaft failures due to torsional vibration. This was a particularly insidious type of vibration because the twisting vibration was superposed on the steady average rate of rotation ω and was unobservable by the crude instrumentation available at that time. Professor Edward Miller, one time head of the Mechanical Engineering Department at MIT, used to tell the story of how he once detected a severe torsional vibration before the shaft failed in fatigue, when he happened to notice, during the initial test of a pump driven by a Diesel Engine, that the shaft connecting the engine to the pump was glowing “cherry red”. Engineering has advanced sufficiently, since then, that torsional vibration is no longer a mystery. In most cases, standard design procedures are available to ensure that the levels of torsional vibration remain within acceptable limits.

In this problem, you are asked to study a particular system consisting of an electric motor driving a pump through a shaft whose *torsional stiffness* (remember this from 2.001?) is

$$K = \frac{\tau}{\Delta\theta} = \frac{GI_Z}{L}$$

where G is the shear modulus, I_Z is the polar moment of inertia of the shaft cross-sectional area, and L is the length of the shaft. The mass moment of inertia of the motor rotor is I_m and the mass moment of inertia of the pump impellor is I_p . A schematic of the system is shown in Fig.1

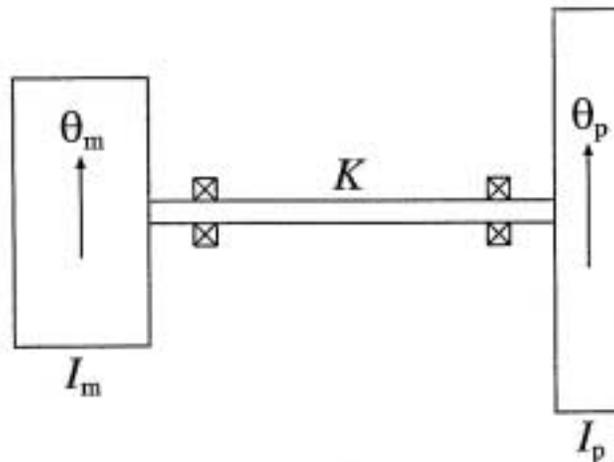


Figure 1: Motor drives pump through elastic shaft.

It is desired to find the natural modes and natural frequencies of the undamped system and the forced response when the motor is known, for the case where

$$I_m = 20 \text{ lb-in-sec}^2$$

$$I_p = 40 \text{ lb-in-sec}^2$$

$$K = 1,000,000 \text{ in-lb/rad}$$

The rotational angles of the motor and pump are designated as θ_m and θ_p , respectively.

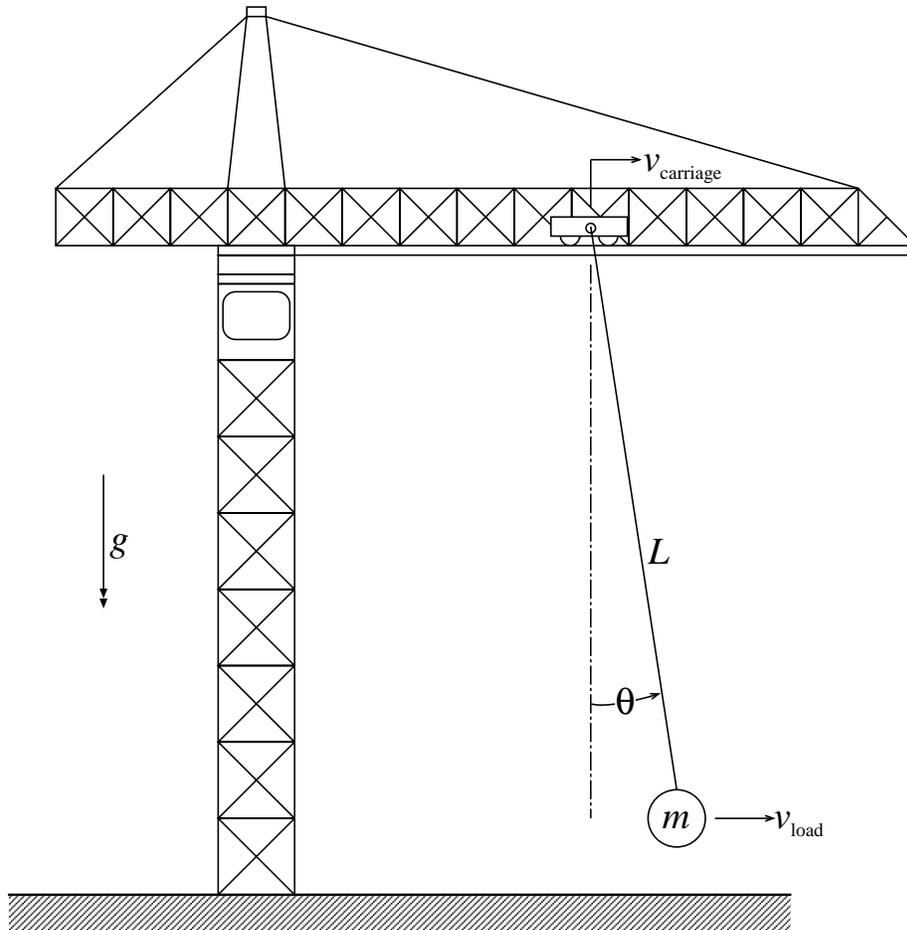
- (a) Obtain equations of motion for the generalized coordinates θ_m and θ_p during free, undamped motion.
- (b) Construct the eigenvalue problem for the natural modes and natural frequencies.
- (c) Solve the eigenvalue problem and obtain the modal matrix and the diagonal matrix of the squares of the natural frequencies.
- (d) Consider the steady-state response of the system when the motor torque has the form

$$\tau_m = \tau_0 + \tau_{alt} \sin \Omega t$$

where τ_0 is the constant average torque produced by the motor, and Ω is the constant average rotational speed of the motor. The quantity τ_{alt} is the amplitude of a small alternating torque due to the variation in magnetic field around the circumference of the gap between the rotor and stator. Find the steady-state responses of the angles θ_m and θ_p .

- (e) What is the average power delivered to the pump by the motor?
- (f) The shaft connecting the motor and the pump was designed very conservatively to transmit the power predicted in (e), never-the-less it was found that, when the system was installed, there was a very early fatigue failure of the shaft. If you were called in to consult on this failure, could you suggest a reason for the failure?
- (g) Could you suggest a remedy for this problem which would not require major changes in the hardware?

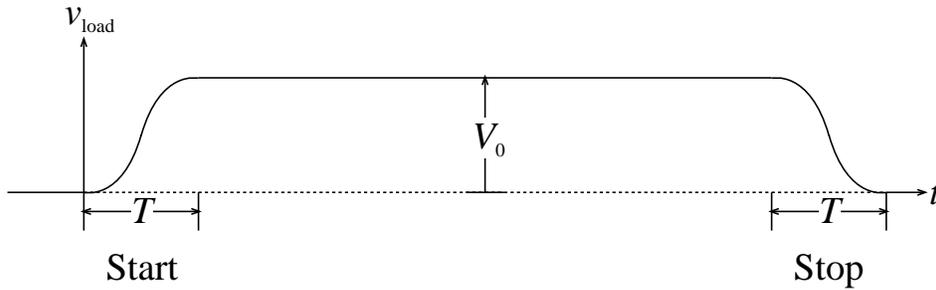
Problem 3. Crane Dynamics. The crane shown is used to move loads from one place on a construction site to another.



Consider horizontal transport (L is constant). The crane operator controls the carriage velocity $v_{carriage}$ but the load velocity is not necessarily the same as the carriage velocity because of the pendulum action. A major problem for crane operators is to control the carriage velocity in such a way that the load is moved smoothly and rapidly to its destination without being left swinging back and forth through such a large angle that it takes several minutes to decay down to a level that can be safely handled.

- Using the angle θ as a generalized coordinate, derive a linearized equation of motion for the response $\theta(t)$ due to the input $v_{carriage}(t)$.
- Obtain a differential equation relating the load velocity $v_{load}(t)$ to the carriage velocity $v_{carriage}(t)$.
- Find the transfer function from the carriage velocity $v_{carriage}(s)$ to the angle $\theta(s)$.
- Find the transfer function from the carriage velocity $v_{carriage}(s)$ to the load velocity $v_{load}(s)$.

A desirable time history for the load velocity is sketched below. There is a short *start* interval of duration T in which the load velocity smoothly accelerates upto speed V_0 . At the end of this period both the carriage and the load are travelling at velocity V_0 with the angle $\theta = 0$. Then there is a short *stop* interval in which the load velocity smoothly decelerates to zero speed and both the carriage and the load come to rest with the angle $\theta = 0$.



The analytical description of the load velocity during the *start* interval is

$$v_{load} = \frac{V_0}{2} \left(1 - \cos \pi \frac{t}{T} \right)$$

if the origin of t is placed at the beginning of the *start* interval. The analytical description of the load velocity during the *stop* interval is

$$v_{load} = \frac{V_0}{2} \left(1 + \cos \pi \frac{t}{T} \right)$$

if the origin of t is placed at the beginning of the *stop* interval. In order to obtain quantitative results in the following questions, take the interval T to be

$$T = 2\pi \sqrt{\frac{L}{g}}$$

- (e) Determine the time history of the carriage velocity required to produce the above load velocity in the *start* interval. Draw a sketch comparing the carriage velocity to the load velocity during the *start* interval.
- (f) Determine the time history of the carriage velocity required to produce the above load velocity in the *stop* interval. Draw a sketch comparing the carriage velocity to the load velocity during the *stop* interval.
- (g) Determine the time history of θ during the *start* and *stop* intervals. Draw a sketch of the time history of θ during the *start* and *stop* intervals.