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2.004 Dynamics and Control II
Spring 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

Dynamics and Control II
Spring Term 2008

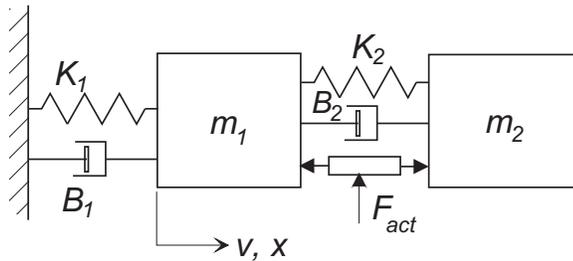
Solution of Problem Set 9

Assigned: April 18, 2008

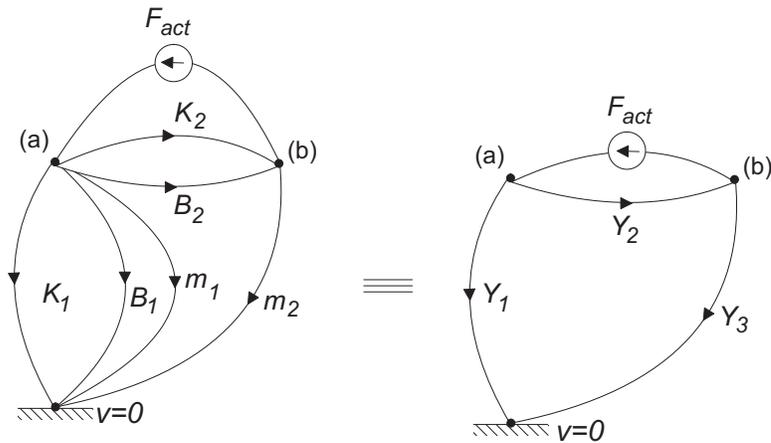
Due: April 25, 2008

Problem 1: (Laboratory Preparation for week 2 of the project.)

(a) (From the lab handout) For the transfer function relating the building motion to the actuator force:



Reduce the system graph to a reduced impedance graph as shown below:



where

$$\begin{aligned}
 Y_1 &= \frac{1}{Z_1} = m_1 s + B_1 + \frac{K_1}{s} \\
 Y_2 &= \frac{1}{Z_2} = B_2 + \frac{K_2}{s} \\
 Y_3 &= \frac{1}{Z_3} = m_2 s
 \end{aligned}$$

Use node equations:

$$\begin{aligned} \text{At node (a)} \quad & F_{Z_1} + F_{Z_2} = F_{act} \\ \text{At node (b)} \quad & F_{Z_2} - F_{Z_3} = F_{act} \end{aligned}$$

Substitute admittances

$$\begin{aligned} v_a Y_1 + (v_a - v_b) Y_2 &= F_{act} \\ (v_a - v_b) Y_2 - v_b Y_3 &= F_{act} \end{aligned}$$

and express in matrix form

$$\begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} F_{act} \\ F_{act} \end{bmatrix}$$

Use Cramer's Rule to solve for v_a :

$$\begin{aligned} v_a &= \frac{\begin{vmatrix} F_{act} & -Y_2 \\ F_{act} & -(Y_2 + Y_3) \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{vmatrix}} \\ &= \frac{Y_3 F_{act}}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3} \end{aligned}$$

Now use Cramer's Rule to solve for v_b :

$$\begin{aligned} v_b &= \frac{\begin{vmatrix} Y_1 + Y_2 & F_{act} \\ Y_2 & F_{act} \end{vmatrix}}{\begin{vmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{vmatrix}} \\ &= \frac{-Y_1 F_{act}}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3} \end{aligned}$$

The relative velocity $v_{rel} = v_{m_1} - v_{m_2} = v_a - v_b$

$$v_{rel} = \frac{(Y_3 + Y_1) F_{act}}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3}$$

Substitution for the admittances gives

$$G(s) = \frac{v_{rel}(s)}{F_{act}(s)} = \frac{(m_1 + m_2)s^3 + B_1 s^2 + K_1 s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\begin{aligned} \text{where } a_4 &= m_1 m_2 \\ a_3 &= (m_1 + m_2) B_2 + m_2 B_1 \\ a_2 &= (m_1 + m_2) K_2 + m_2 K_1 + B_1 B_2 \\ a_1 &= K_1 B_2 + K_2 B_1 \\ a_0 &= K_1 K_2 \end{aligned}$$

- (b) 1. If the actuator force is proportional to the relative velocity between the two masses, that is

$$F_{act} = K_p(v_{m_1} - v_{m_2})$$

it will “look like” a damper with $B_{act} = K_p$. Therefore the effective damping between the two masses will be $B_2 + K_p$. It is interesting to note that if $K_p < 0$ the overall damping coefficient may be reduced.

2. If integral action is used so that $F_{act} = K_i v_{rel}/s$ then

$$F_{act} = K_i(x_{m_1} - x_{m_2})$$

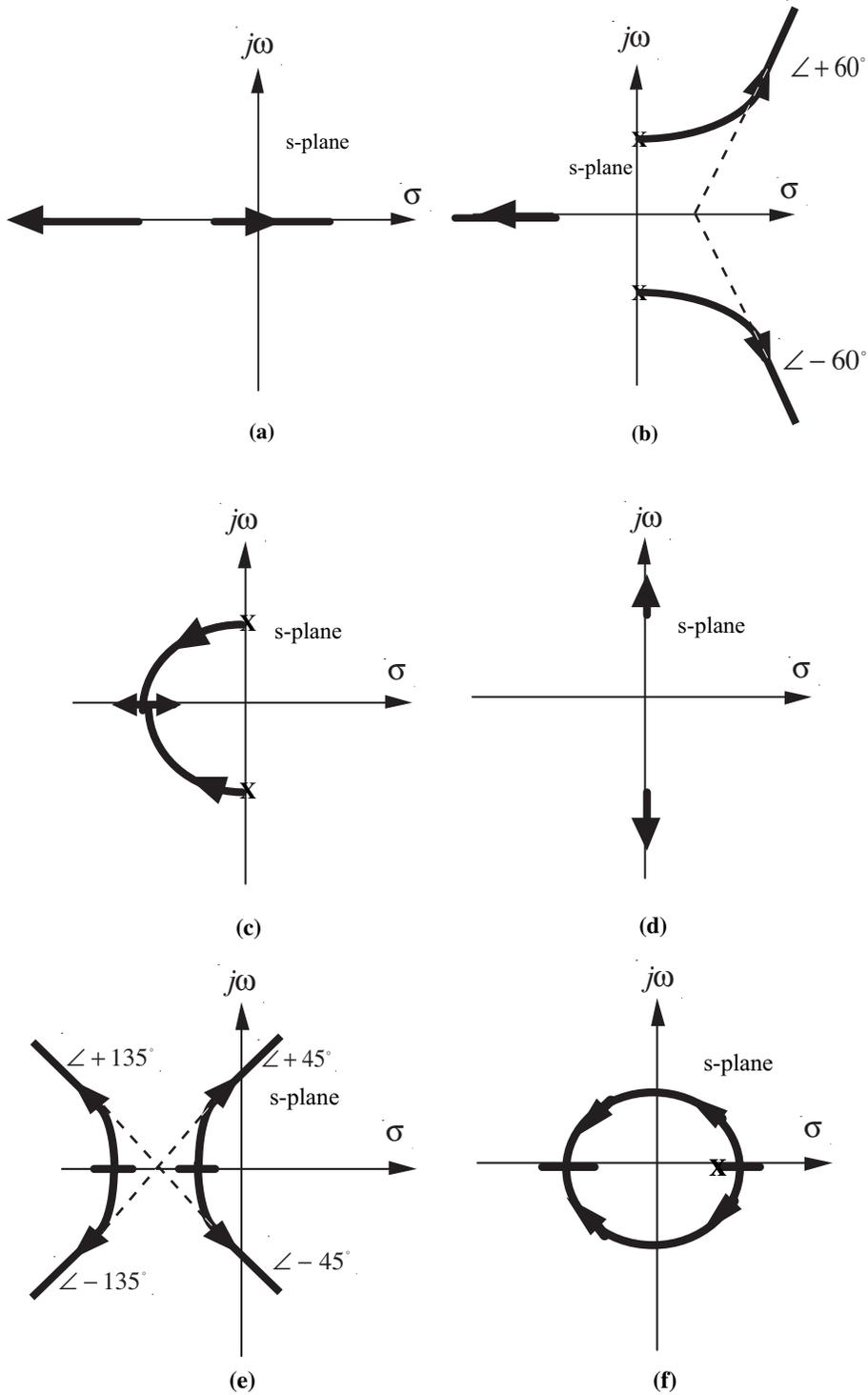
so that the force is proportional to the relative displacement of the two masses. Thus the actuator has the same effect as a spring with stiffness $K = K_i$. This stiffness will act in parallel with the physical spring K_2 , producing a total stiffness $K_2 + K_i$ between the two masses.

If PI feedback is used, the actuator will appear as a passive parallel spring and damper. This allows us to “tune” the tuned-mass damper electronically and find an optimum set of values for B_2 and K_2 to meet some performance criterion.

Problem 2: Nise Ch. 8, Problem 1 (p. 417 5th Ed.).

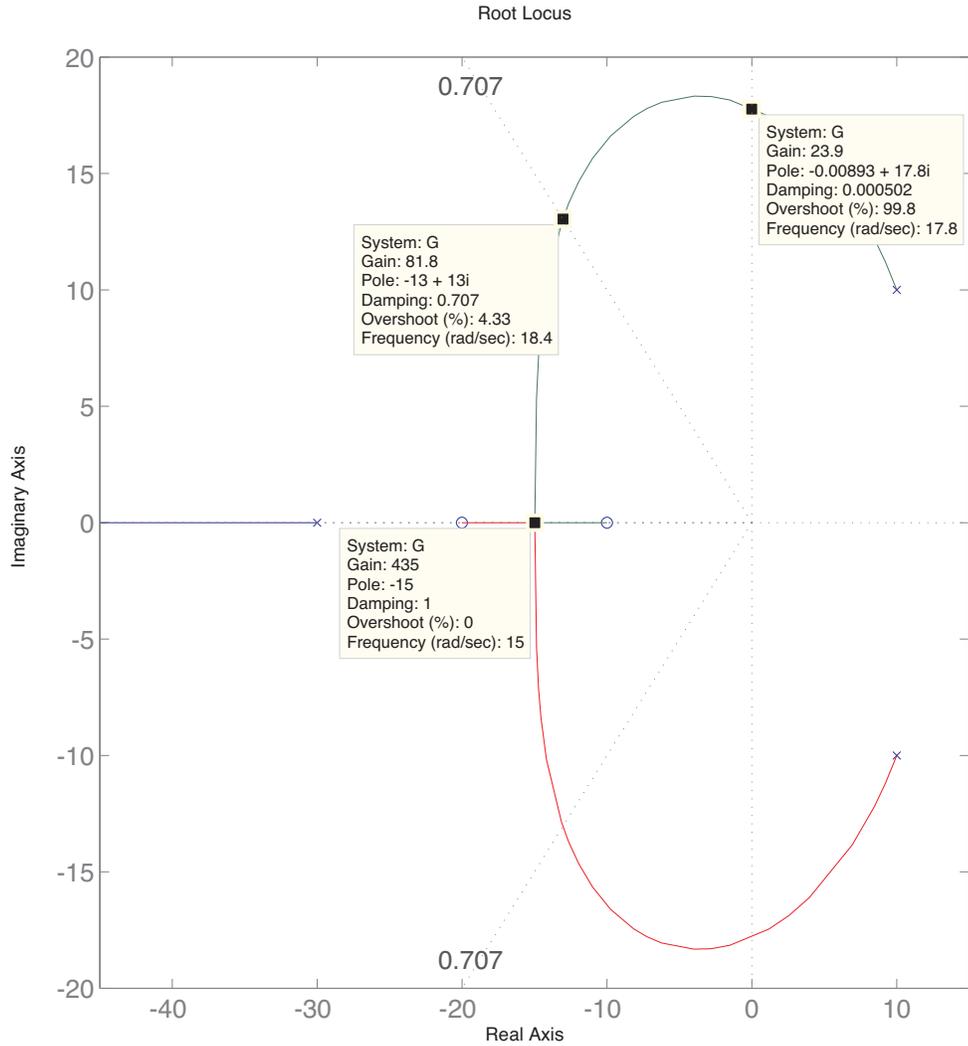
- (a) No: Not symmetric; On real axis to left of an even number of poles and zeros.
- (b) No: On real axis to left of an even number of poles and zeros.
- (c) No: On real axis to left of an even number of poles and zeros.
- (d) Yes.
- (e) No: Not symmetric; Not on real axis to left of odd number of poles and/or zeros
- (f) Yes.
- (g) No: Not symmetric; real axis segment is not to the left of an odd number of poles.
- (h) Yes.

Problem 3: Nise Ch. 8, Problem 2 (p. 417 5th Ed.).



Problem 4: Nise Ch. 8, Problem 22 (p. 420 5th Ed.).

- (a) Approximate values can be computed from incoming graph. High precision values, given in the next parts, are computed analytically.



(b) $K > 23.93$.

(c) $K = 81.83$ for $s = -13.04 \pm 13.04j$.

(d) At the break-in point: $s = -14.965$ and $K = 434.98$.

MATLAB Command – line :

```
>> g1=zpk([-10 -20],[-30],1)
```

Zero/pole/gain:

(s+10) (s+20)

(s+30)

```
>> G=series(g1,tf(1,[1 -20 200]))
```

Zero/pole/gain:

(s+10) (s+20)

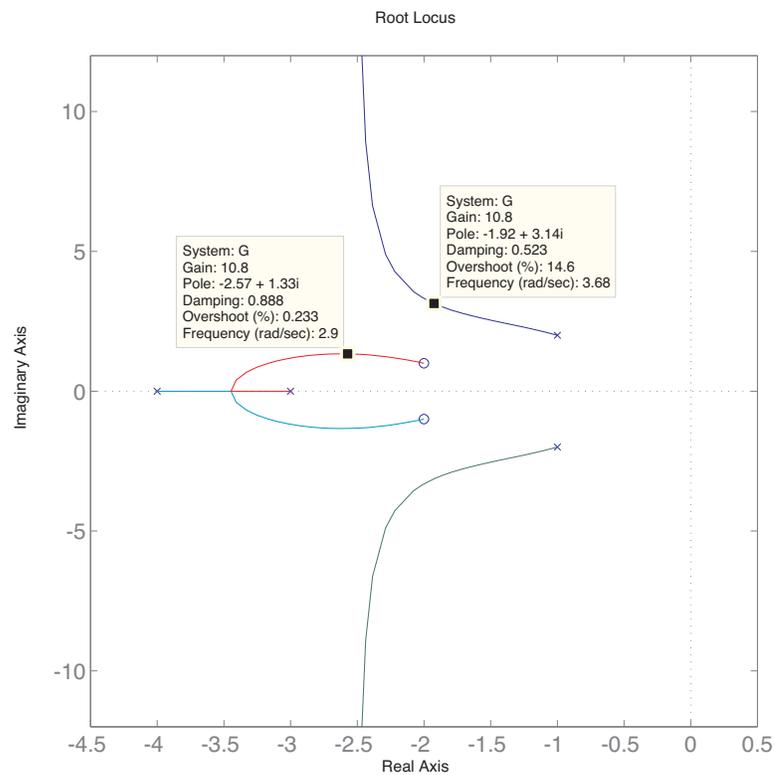
 (s+30) (s^2 - 20s + 200)

```
>> rlocus(G)
```

```
>> sgrid(.707,[])
```

Problem 5:

(a)

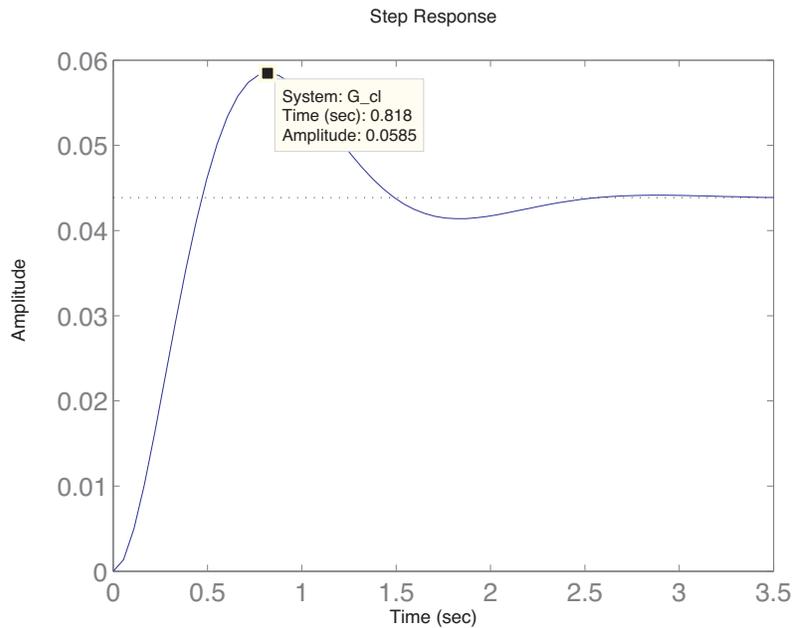


(b)

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{\pi}{\omega_d} \Rightarrow \omega_d = \pi \frac{rad}{sec} \Rightarrow K = 10.8 \text{ for } s = -1.92 \pm 3.14j$$

(c) See the attached code.

- (d) The actual peak time is 0.818 sec, less than expected 1.0 sec value. This can be due to the presence of nearby faster poles. We have predicted the peak time from the dominant pole, while the next pole has a real value (decay component) of -2.57 compared to -1.92 of the dominant pole, and hence can have a substantial effect.



MATLAB Command – line :

```
>> g1=zpk([], [-3 -4],1)
```

```
Zero/pole/gain:
```

```
1
```

```
-----  
(s+3) (s+4)
```

```
>> G=series(g1,tf([1 4 5],[1 2 5]))
```

```
Zero/pole/gain:
```

```
(s^2 + 4s + 5)
```

```
-----  
(s+3) (s+4) (s^2 + 2s + 5)
```

```
>> rlocus(G)
```

```
>> axis equal
```

```
>> G_cl=feedback(G,10.8)
```

Zero/pole/gain:

$$(s^2 + 4s + 5)$$

$$(s^2 + 5.147s + 8.404) (s^2 + 3.853s + 13.56)$$

>> step(G_cl)

Problem 6:

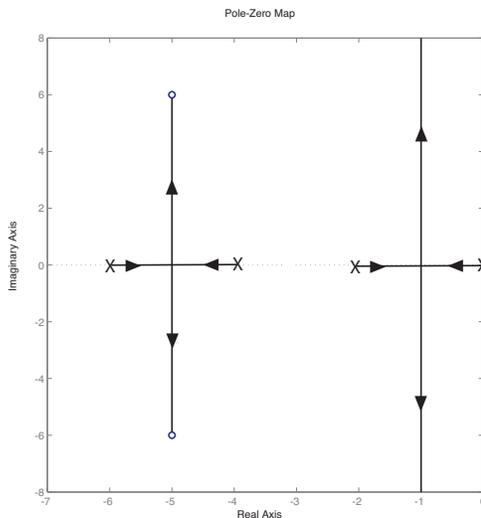
- (a) There are 4 poles and 2 zeros. We have 2 branches going to infinity making an angle of $\pm 90^\circ$ with real axis and have a common intersection at $\sigma_a = -1$. The part of the branches in the real axis can be determined exactly, however the branch behavior from break-in point toward zeros or infinity asymptotes is not clear and we have plotted arbitrary straight lines for that section.

$$G_c(s) = \frac{K_p s + K_d s^2 + K_i}{s} = K \frac{((s + 5)^2 + 6^2)}{s} = K \frac{(s^2 + 10s + 61)}{s}$$

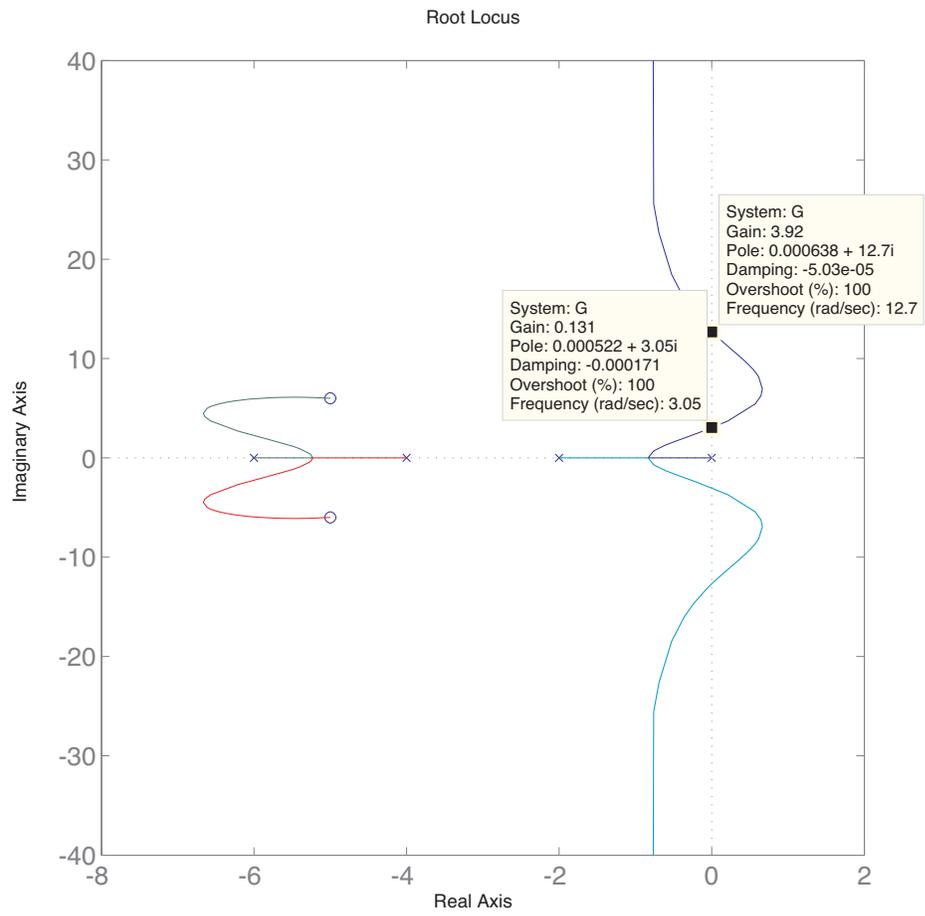
$$G(s) = G_c(s)G_p(s) = \frac{48K(s^2 + 10s + 61)}{s(s^3 + 12s^2 + 44s + 48)}$$

$$G(s) = 48K \frac{(s + 5)^2 + 6^2}{s(s + 2)(s + 4)(s + 6)}$$

$$\sigma_a = \frac{(0 + (-2) + (-4) + (-6)) - ((-5) + (-5))}{2} = -1$$



(b)



(c) Unstable for $0.13 < K < 3.92$.