2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

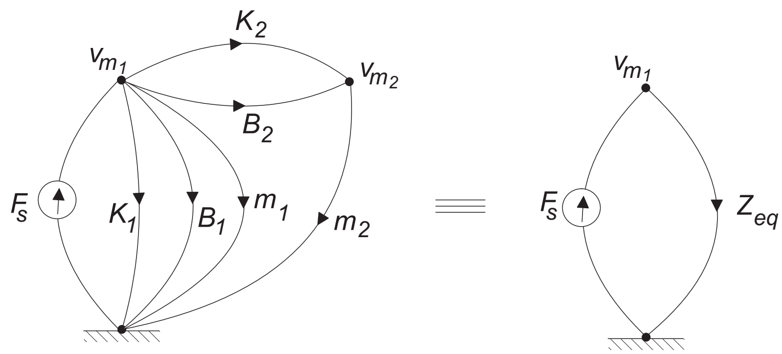
2.004 *Dynamics and Control II*
Spring Term 2008

Solution of Problem Set 7

Assigned: April 4, 2008 Due: April 11, 2008

Problem 1:

(a) The impedance graph below may be reduced to a single impedance as shown:



The required transfer function is

$$G(s) = \frac{V(s)}{F(s} = Z_{eq}$$

It is more convenient to work with admittances (since there are a lot of parallel elements):

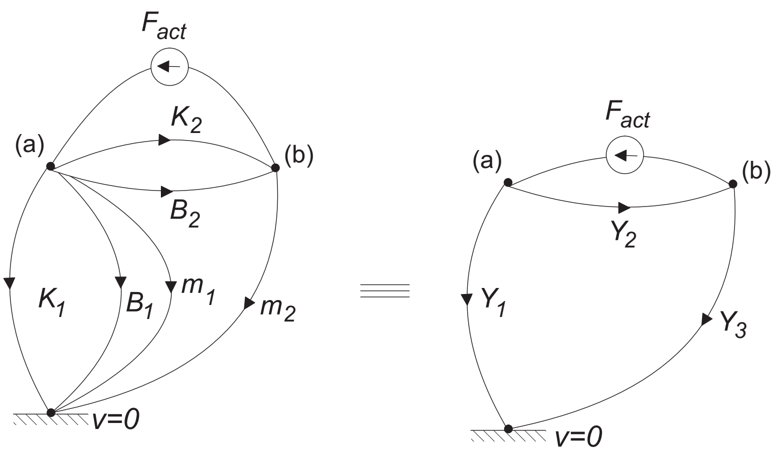
$$\begin{aligned} Y_{eq} &= \frac{1}{Z_{eq}} = Y_{m_1} + Y_{B_1} + Y_{K_1} + \frac{(Y_{K_2} + Y_{B_2})Y_{m_2}}{Y_{K_2} + Y_{B_2} + Y_{m_2}} \\ &= \frac{(Y_{m_1} + Y_{B_1} + Y_{K_1})(Y_{K_2} + Y_{B_2} + Y_{m_2}) + (Y_{K_2} + Y_{B_2})Y_{m_2}}{Y_{K_2} + Y_{B_2} + Y_{m_2}} \end{aligned}$$

Then

$$\begin{aligned} G(s) &= \frac{1}{Y_{eq}} \\ &= \frac{Y_{K_2} + Y_{B_2} + Y_{m_2}}{(Y_{m_1} + Y_{B_1} + Y_{K_1})(Y_{K_2} + Y_{B_2} + Y_{m_2}) + (Y_{K_2} + Y_{B_2})Y_{m_2}} \\ &= \frac{m_2 s + B_2 + K_2/s}{(m_1 s + B_1 + K_1/s)(K_2/s + B_2 + m_2 s) + (K_2/s + B_2)m_2 s} \\ &= \frac{m_2 s^3 + B_2 s^2 + K_2 s}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \end{aligned}$$

$$
\begin{aligned}
\text{where} \quad a_4 &= m_1 m_2 \\
a_3 &= (m_1 + m_2)B_2 + m_2 B_1 \\
a_2 &= (m_1 + m_2)K_2 + m_2 K_1 + B_1 B_2 \\
a_1 &= K_1 B_2 + K_2 B_1 \\
a_0 &= K_1 K_2
\end{aligned}
$$

(b) Reduce the system graph to a reduced impedance graph as shown below:



where

$$
\begin{aligned}
Y_1 &= \frac{1}{Z_1} = m_1 s + B_1 + \frac{K_1}{s} \\
Y_2 &= \frac{1}{Z_2} = B_2 + \frac{K_2}{s} \\
Y_3 &= \frac{1}{Z_3} = m_2 s
\end{aligned}
$$

Use node equations:

$$
\begin{aligned}
\text{At node (a)} \qquad & F_{Z_1} + F_{Z_2} = F_{act} \\
\text{At node (b)} \qquad & F_{Z_2} - F_{Z_3} = F_{act}
\end{aligned}
$$

Substitute admittances

$$
\begin{aligned}
v_a Y_1 + (v_a - v_b) Y_2 &= F_{act} \\
(v_a - v_b) Y_2 - v_b Y_3 &= F_{act}
\end{aligned}
$$

and express in matrix form

$$
\begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ Y_2 & -(Y_2 + Y_3) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} F_{act} \\ F_{act} \end{bmatrix}
$$

Use Cramer's Rule to solve for v_a:

$$v_a = \frac{\begin{vmatrix} F_{act} & -Y_2 \\ F_{act} & -(Y_2+Y_3) \end{vmatrix}}{\begin{vmatrix} Y_1+Y_2 & -Y_2 \\ Y_2 & -(Y_2+Y_3) \end{vmatrix}} = \frac{Y_3 F_{act}}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3}$$

Substitution for the admittances gives

$$G(s) = \frac{v_a(s)}{F_{act}(s)} = \frac{m_2 s^3}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\text{where} \quad \begin{aligned} a_4 &= m_1 m_2 \\ a_3 &= (m_1+m_2)B_2 + m_2 B_1 \\ a_2 &= (m_1+m_2)K_2 + m_2 K_1 + B_1 B_2 \\ a_1 &= K_1 B_2 + K_2 B_1 \\ a_0 &= K_1 K_2 \end{aligned}$$

and we note that the denominator is the same as in (a) above.

Problem 2: Nise Problem 4-23 (p. 207).

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Problem 3: Nise Problem 4-29 (p. 208).

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Problem 4: Nise Problem 4-55 (p. 212).

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Problem 5: Nise Problem 4-62 (p. 214).

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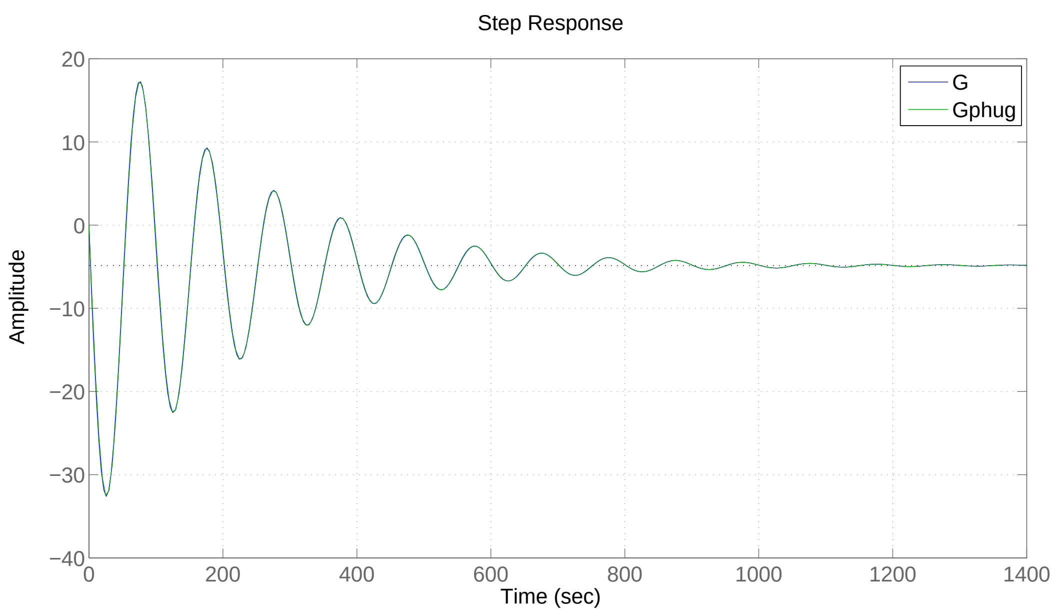

Both responses are indistinguishable.

<u>Problem 6:</u> Nise Problem 4-67 (p. 215).

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