2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*

Spring Term 2008

Problem Set 6

Assigned: March 28, 2008 Due: April 4, 2008

Reading:

- Nise 4.1 — 4.6

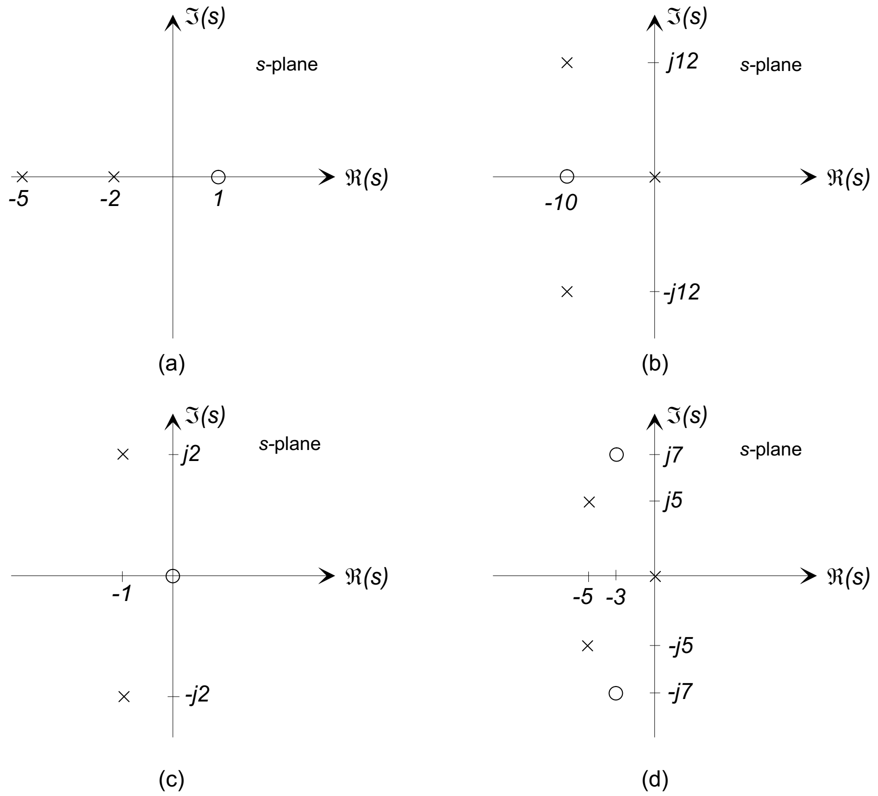
Problem 1: Determine the system poles and zeros for each of the systems described by the differential equations that follow. Also write each transfer function in factored form, with its poles, zeros, and gain constant. Make an s-plane plot of the poles and zeros.

(a) $$\frac{dy}{dt} + 3y = \frac{du}{dt} + 3u$$

(b) $$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = 2\frac{du}{dt} + u$$

(c) $$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 7\frac{dy}{dt} = \frac{du}{dt}$$

Problem 2:



For systems represented by each of the above four pole-zero plots

(a) Determine which ones could have a steady-state step response $y_{ss} = 2$.

(b) Determine the transfer function (as far as is possible).

(c) Determine the modal response components as real functions.

Problem 3:

(a) Show that for a system with a zero at $s = z$, the forced response (particular solution component $y_p(t)$) to a system input

$$u(t) = Ae^{zt}$$

(where A is a constant) is zero.

(b) Generalize this result to show that the particular response component $y_p(t)$ of a system with a complex conjugate zero pair

$$z_1, z_2 = \sigma \pm j\omega$$

to a real input

$$u(t) = Ae^{-\sigma t}\sin(\omega t + \phi)$$

is zero.

Problem 4: A first-order system, at rest at $t = 0$, with transfer function

$$H(s) = \frac{a}{s + a}$$

is subjected to an exponential input $u(t) = e^{-bt}$ for $t > 0$.

(a) Find the response $y(t)$.

(b) What happens to $H(s)$ as the value of the exponent in the waveform approaches the value of the pole? What is $H(s)|_{s=-a}$? What happens to the particular solution component as $b \to a$?

(c) Does the overall system response become unbounded when $b = a$? Consider $\lim_{b\to a} y(t)$ in the solution to (a) to find the response of the system to the input $u(t) = e^{-at}$. (Hint: L'Hospital's rule may be useful.)

Problem 5: A unity gain second-order mechanical system has poles $p_1 = -2$, and $p_2 = -4$. A closed-loop proportional controller with gain K_p is used to regulate the output.

(a) Find the plant transfer function $G_p(s)$.

(b) Find the closed-loop transfer function $G_{cl}(s)$ as a function of K_P.

(c) Find the closed-loop poles for the values $K_p = 1, 2, 3, \ldots, 10$. Yes, I know it's tedious - use whatever automated method you like.

(d) Make an s-plane pole-zero plot with 1) the open-loop plant poles, and 2) all of the poles found in part (c), and "connect-the-dots" to draw a continuous curve showing how the closed-loop poles migrate away from the open-loop poles as the proportional gain K_p increases.

Congratulations! You have just drawn your very first *root locus* plot! Frame it and hang it on the wall (after it is graded!) The root locus is a very important tool in control system design.

(e) From your root locus plot estimate the value of K_p that will create a critically damped closed-loop system (a coincident pole pair).