PROFESSOR: The purpose of these recitations, small group recitations, is so that we can get out the key concepts over the week and what I call the essential understandings-- what are the really important points for the week so that when the first quiz comes, you will know how to deal with it.

So let's start with that. But you're going to help me think through this. So take a minute or two, write down on a piece of paper two or three things that you think are the most important things that you heard, saw, read this week about this course.

Let's report out. I want one from a number of you. Who wants to volunteer here?

AUDIENCE: Using different reference frames?

PROFESSOR: Say it again?

AUDIENCE: Using different reference frames.

PROFESSOR: Using different reference frames. I'm going to write that, once I get the chalkboard. Using-- I'm going to write it as multiple reference frames. Close enough? What's your name?

AUDIENCE: Christina.

PROFESSOR: Christina? What do you have?

AUDIENCE: All points on a rigid, rotating object have the same rate of rotation.

PROFESSOR: She said, all points on rigid object that's rotating, all points have the same rotation rate. So this is rotation and translation of rigid bodies. I'm going to generalize what you said a little bit, because somebody else tell me, what can you say about translation?

So rotation, key point is, all points share the same rotation rate. How about translation? Two different points on an object-- what can you say about it?

AUDIENCE: They follow the same paths.

PROFESSOR: Parallel paths. They go through the exactly same parallel paths. So those are two key things we remember about that. How about another point?

AUDIENCE: [INAUDIBLE]

PROFESSOR: OK. This is actually quite important. I'm going to write it slightly differently. We need to talk about this. And that is that rotations-- to be absolutely correct, finite rotations are not vectors.

I want to come back to that in a minute. Lots of possible confusion around that. One more-- or actually, there's more.

AUDIENCE: The MLM strategy?

PROFESSOR: MLM. So I mentioned that last time. That's problem solving my way, which is first M is figure out the motion, describe the motion. That's kinematics. The second term is L. What is that? Laws. By the physical laws.

And the second M? Do the math. So motion, laws, and math. There's something else here. Well, you may have just decided it's going to encompass in that. But I want to go a little further than that. What did we talk a lot about yesterday in the lecture?

AUDIENCE: Different types of acceleration.

PROFESSOR: Accelerations and velocities and translating and rotating frames. Translating and rotating frames-- running out of room here but you get the point. All right, that's a pretty good list.

If I'd been coming up with a list on my own, what would have thought was important, that would've captured most of those things. Certainly this is really important this week. And we definitely need to learn how to use translating and rotating frames.

And you're absolutely in trouble if you don't know this. This is just sort of fundamental to the whole thing. And then this is a subtle point. Let's start right there for a second. Who has a textbook? It doesn't actually really matter. Let me borrow your notes.

Rigid body, got the print on the front. I'm going to rotate it twice. The x-axis and call this the z-axis. It comes out top actually pointing at you. So I did that right. So now I'm going to do the rotation. Now this one first. And then what was the other rotation?

AUDIENCE: Backwards.

PROFESSOR: Different answer, right? Totally different answer. You can't add angles as actors. Doesn't work. And it's just-- the way I think of it, mathematics is largely done to help describe the physical world.

Newton and all those people were figuring out-- needed calculus to describe the motion of the planets. Vectors were invented to do analytic geometry. And it doesn't work for angles. You just can't use them for angles. It's just the vector math that they figured out just wasn't quite clever enough to include angles.

However, vectors can be applied to positions, velocities, accelerations, and angular velocities and angular accelerations, but not angles themselves. That's the basic thing you need to learn from that. Let's use some multiple frames. We're going to do that today.

We're going to now apply this and this and this today to do some problems. And let me see where I want to go first with this.

So I have a problem that I wanted to do. And it's a circus ride. There's an arm. And that arm is rotating. Attached to the arm is a cross piece. And a passenger can sit in each one of these things.

And this is basically horizontal. You're looking down on it. So you'd be riding around in these cups at the circus and it's going around and around. And I want to know the velocity. What's the velocity of point B in the O frame?

And so this has to do-- one of the things on this list might be to get the notation down. So this is the velocity. This is the point. And this is the frame. So what can you write down? Just take 30 seconds.

See if you remember. Write down the general velocity formula that was put up yesterday-- vector velocity formula for a point in a moving frame that's moving in a fixed frame. Came up with a general formula, had two or three terms in it. And we'll walk our way through it.

I realize I did something maybe slightly out of order. So hold that thought. You've written down what you've got. We have to do something before you can actually write that. We haven't actually picked our reference frames, have we?

So think about that for a second. How would you set up this problem? What would you make translating reference frames, your rotating, translating frame-- where would you assign it? Think about it for 30 seconds.

Who's got to take a shot at it for me? Where would you pick reference frames for this problem? What your name?

AUDIENCE: I'm Ben.

PROFESSOR: Ben.

AUDIENCE: O and along the cross?

PROFESSOR: Here, for sure. This is your inertial frame-- not moving, right? And?

AUDIENCE: Two axes on the cross?

PROFESSOR: So you would put one up here? OK. I'm going to line up with the cross, and I'm going to stick out here and call it x2. And then there'd be a y2 here. And it rotates with the cross? All right. That's good. Now go back to that equation.

Now give me the velocity, the general expression. I don't want you working out the details, just what set of terms would you plug things into now to get the velocity of B and O?

Then we'll evaluate the terms and talk about it, using now what we've decided here. OK, somebody help me out. What's on the right hand side of this equation? First term, Mary.

AUDIENCE: Velocity of--

PROFESSOR: What's your name? Steven?

AUDIENCE: Velocity of A with respect to O.

PROFESSOR: Velocity of A with respect to O. All right. That's the velocity at this point in this frame, right? What else do we need? What's your name? Andre?

AUDIENCE: Yeah. [INAUDIBLE]

PROFESSOR: I hear a velocity of V with respect to A. And what is that-- is that influenced by rotation? Can you describe what you mean by the velocity of v and A physically?

AUDIENCE: [INAUDIBLE]

PROFESSOR: So it's as if you were sitting on that frame, right? Does the rotation have anything to do with what you see? No. So I sometimes remind myself right here this is omega equals 0. And you can set the omega equal to 0, what you would see is what this term is. Do we need anything more? Name?

AUDIENCE: Christina.

PROFESSOR: Sorry, you gave it to me once before. It's going to take me awhile.

AUDIENCE: It's the rotational motion of B spinning around in there. So it has to do with the omega as seen in the reference frame, the origin, cross product with r from the in regards to the x2 xy.

PROFESSOR: And we have the name of that frame to help us out. This is then frame A, x2, y2, z2. If you really wanted to write [INAUDIBLE] We just call it frame A. so this is would be rB as in NA And these are all vectors and I often forget to underline them. Do we have it right? Anybody want to add to that, fix it? Correct it? Steven, right?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I left it vague on purpose. We need to figure that out. He asks, is it omega 2 or omega 1? Really important point we want to make today about what omega this is. We'll get to that. Yeah?

AUDIENCE: Well, if they're rotating in the same direction, wouldn't it be added in both omega 1 or omega 2?

PROFESSOR: Well, OK. Let's talk about it right now. Are we agreed that this is the right formula? Then let's set about figuring it out. And we can talk about this term first. So we want to know, this is the rotation rate of this arm out here in the base frame.

That's what the notation says. And we know that the rotation rate of this first arm in the base frame is this. And we know that the rotation rate of this thing with to-- now this has gotten a little complicated, because this isn't quite exact enough.

This is omega 2 with respect to this arm. That's what's given in this problem. So this is omega 2 with respect to the arm OA. Yeah?

AUDIENCE: So does that mean it's omega 2xz from coordiante system B?

PROFESSOR: No, coordinate system A x2y2 rotates. And if you're sitting in there, you wouldn't see it. So this is correct. It's the rotation rate as seen in O. So we need to figure out what that is. And I'm telling you in this case, you were given-- you might have been given the rotation rate in O.

You weren't. You were given the rotation rate relative to here. So I'll write it as W2 with respect to this arm OA. So how do you get-- we need omega in O is what? Help me out here.

AUDIENCE: Is it O-- should there be a small b at the bottom [INAUDIBLE]?

PROFESSOR: Good. But what is it? Let's deduce it. If my arm here, this is the first arm. And this is the at AB link. Now if omega with respect to this arm, this thing weren't moving, no rotation rate relative to this, the whole thing would be straight, right?

And it's going around like this. What's the rotation rate of the link out here? Omega 1. And now this arm's not moving, but this is rotating relative to it at omega 2. What's the rotation rate of the link out here? Just omega 2. If I put the two together, what is the rotation rate of this arm, this second link?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Omega 1 in, certainly in O plus omega 2. It's not with respect to A. I'm just going to call it with respect to the R maybe. Even this notation is failing a little bit.

But you get what I mean. It's omega 1 plus omega 2. And let's just write it as omega 1 plus omega 2. And what direction is it in? It's a vector. So one of the things we have to pay attention to are unit vectors. Yeah?

AUDIENCE: [INAUDIBLE]

PROFESSOR: So this is capital I hat here and capital J hat there and coming out of the board, K hat. Now, this is certainly K hat, capital K hat. This one, though, is relative to-- it's the rotation rate of this thing. Here is a reference frame.

What's sticking out this way? A little k2, right? But is it parallel to capital K? Always parallel to capital K? So they're the same thing. If unit vectors in this are parallel, they amount to the same thing. So we can put capital K, lowercase k, anything we want here and it's correct.

Now we've got an answer for that. So when you're given-- when the one thing's attached to another and you're given the-- if out here you are given the rotation rate in the base frame, you're done.

But if you're given the rotation rate relative to some other moving part, then you have to add them up to get the true rotation rate. That's the bottom line message. All right. So we started-- we're trying to figure out this expression here.

And we started with one of the harder terms. And we need to figure out-- to finish it, though, let's do this over here. We have velocity of A in-- and we have the velocity of B in A with no rotation.

And we have omega B in O. And let's finish that. We know what omega is now. Whoops. It's not omega. The third term is omega B and O cross RBA. So we've gotten the first bit of this. Let's finish the problem.

This is omega 1 plus omega 2 times k hat cross with what? We need a length. I'll call this L. It's L long. So what is RB respect to A? Yes?

AUDIENCE: L X 2 hat?

PROFESSOR: L X 2 hat. And I'll call that Lj2. The coordinate is x2. The unit vector would be I2, not a j, an i. The unit vector is i2. OK, great. Now what is k cross i2?

j2. So we get omega L omega 1 plus omega 2 j2 hat. That's that term. And we need to figure out our other two terms. What's this term? Remind yourself of the meaning. This is the velocity of point B with respect to the A frame, which is attached to it. It's on a rigid body.

AUDIENCE: [INAUDIBLE]

PROFESSOR: He said omega 2 times L. She says 0. Any other? I hear another 0. Why 0?

AUDIENCE: Because it's rigidly attached into the ride, if you're moving around. It's not moving on the ride versus strapped in.

PROFESSOR: Right. So this term is always from the point of view of a person riding on the frame. Riding on that frame-- so you won't ever see rotation from inside the frame. You're just moving with it. So that's called, in the Williams book, he calls this term the rel. It's the relative velocity between these two points and no rotation.

So what is that in this case? I hear 0. Everybody agree it's 0? It's a rigid link. Two points don't move. So now we're just left with this one.

And now, one of the points I really wanted to drive home today is in fact this problem is one that, depending on how you set it up, you can think of as actually having multiple rotating frames.

And if you do that, what's the correct way to add up the parts so you get to the right answer? Because we've left this one for the last. And I want to make sure you go away knowing a formula you can always use, and it's going to work.

And the formula we can always use is the one that's of this form. Every one of these problems, including multiple links and things, you can build up by doing a sequence of this problem again and again and again, until you get the whole answer.

So we've actually done what I would call the outer problem first. We've worked out this thing. We have to do the inner problem now. We could have done it in a different order, but I need to know the velocity of this point. And just to get you in the habit of using the vector equation, that we have, I want to know the velocity of A in O.

And I'm going to attach a rotating frame to this arm, x1, y1. It rotates with this arm at that rate. And I want you to use that frame to solve for the velocity of this point. And that would be-- velocity of point A in O would be the-- this frame now is an O little x1, y1, z1. It's a rotating frame, right?

Because the O's are going to get confusing. Better not call it O. We'll call this rotating one-- in Williams, he uses a lowercase o, but it's hard to do on the board. Let's call this c. So this is a frame, C, x1, y1. So this frame will be my c frame.

So I want to know the velocity of point A. It's the velocity of what? If you get stuck, use that top formula up there, put in the right points. So what's the first term mean? It's the velocity of the-- this time the rotating frame, does the rotating frame translate?

We have a rotating frame. Does it have any translational velocity? No, but you still have the right to turn it down and set it equal to 0. So what's the right term? How do you write it? Right? It's the velocity of my reference frame.

It's the transitional velocity of that reference frame in the O frame. And that's what it is. And in this case, it's 0 plus velocity of A with respect to c. And I'll remind you again.

It's as if you were now rotating with it, and you're sitting at c, looking at A. What's its speed? 0. Plus omega-- what omega? Seen where? Measured from where? Measured with respect to what frame?

I hear on O, cross with-- we need length? We'll make this length capital R, scalar. So what's the cross product here? What's the unit vector? Correct unit vector? Not x. x is the coordinate. The unit vector is--

AUDIENCE: [INAUDIBLE]

PROFESSOR: Right? That turns 0. That turns 0. This turns 0. Omega 1, And what's the unit vector associated with this omega 1? k cross i 1 j hat. And so we have R omega 1 j1 hat.

And now, we should be able to write out the full answer of the velocity of B in O is the velocity of A, which is R omega 1 j1 hat plus velocity of B with respect to A, which was 0 plus this term, which we figured out as L omega 1 plus omega 2 times j2 hat.

So we have two or three sub-- this kind of a hard problem, actually, for the first time out. Because it has a number of subtle concepts built into it. You actually have two rotating bodies. How do you deal with them? Well, you do sequential applications of that vector velocity formula. Yeah?

AUDIENCE: So I was wondering why we made another coordinate system that's rotating with the arm to solve for the velocity of A [INAUDIBLE].

PROFESSOR: She asked, why did we bother to make this another frame. The problems are going to get nastier and nastier. I could have asked you, when I walked into class, what is the velocity of point A. And you would have said, well, obviously R omega.

Why are we going to all this trouble, when everybody knows from high school physics that it's just R omega? And the answer is is because we're going to get to doing really nasty problems. And I want to make sure you understand all the subtleties about how we get these.

So we started simple, but I did it the long, hard way. Because later on, if I'd walked in at the beginning and just asked you right off the bat, what's the velocity of this point-- go for it-- you guys would have failed miserably.

It's not much harder, but it takes two sequential applications of what you think is obvious when you walked into class. So that's why. We're just doing it the hard way, so that you get all the little nuances. Yeah?

AUDIENCE: So why [INAUDIBLE] there for omega 2, we have it with respect to arm AB [INAUDIBLE] with respect to arm OA?

PROFESSOR: Why is it that way?

AUDIENCE: [INAUDIBLE] with respect to arm AB and when you wrote [INAUDIBLE] with respect to arm OA?

PROFESSOR: When I wrote the equation for--

AUDIENCE: For the omega 2. There you're saying it's with respect to OA, and there you say its respect to AB.

PROFESSOR: Oh, I see. Because this is wrong.

AUDIENCE: [INAUDIBLE]

AUDIENCE: My question deals with j1 and j2-- are they the same?

PROFESSOR: So he has a question. And that's the final, subtle point I want to get to today. Good question. He's saying, are these the same? Are they different? How do we deal with it?

So a question for you. In general, if you're asked or given a problem like we just did, and you arrive at a solution, is it OK to give an answer where you had unit vectors in multiple frames? And neither of these unit vectors are in the base frame.

And yet, the answer we're claiming is that this is the velocity of B in O. And here we've got unit vectors that are not in the base frame. Is it a legit equation or not? What do you think? See a lot of no's out there. I think we better figure it out.

So we have unit on the arm. This is my c and O here. On the arm, I have frame that rotates with it that has unit vectors in the direction of the arm of i1 and j1. So here's i1. It's unit long. Here is the angle theta.

Here's j1 and the angles. And I want to know-- this is i1 and this is j1-- can I express i1 and j1 in terms of capital I, capital J, the unit vectors in the base frame?

I want to express them in terms of unit vectors that are in this rigid, non-moving, non-rotating, inertial frame. So down here, this is the i hat direction and this is the j hat direction, right-- not moving. So this is just a unit thing, unit long.

Can I project it onto its i component and capital J component? All right. So i1, it looks to me like cosine theta I hat plus sine theta J hat. Do you agree? Just standard trick, right? And this one, takes me a minute to figure this out.

Which is the theta here? This is theta. That's 90 minus theta. So this must be theta, right? There's a theta here. And if this is unit long, what's that? That projection there is-- so j1 has two components, minus sine theta I plus cosine theta J.

And I highly recommend you write that one down. Make sure you can drive it yourself. You're going to need it again and again and again and again.

Now, could we do the same thing for-- could we convert J2 to the base frame? And it rotates, so this x2 can be at any arbitrary position. But in order to do the problem, you have to pick a position.

And then you'd have to do draw an angle. And then you'd have to apply this formula. And so you're going to end up with an i2 and some cosine phi capital I plus sine phi capital J. And the same thing, j2 is minus sine phi i plus cosine theta J.

So we'll do a trivial example, solve a trivial case. What is the instantaneous velocity at the moment that the coordinate system is lined up as we see, and B is sitting right here?

So we've got to go look at our answer. Where was our final answer? Velocity of this guy here, right? What would be the contribution of this term?

We have to take each term and convert it to the base system and capital IJ terms, right? You do it one term at a time and add up the components. So how do you break this one down and put it into capital I, capital J components?

AUDIENCE: Substitute?

PROFESSOR: Yeah, what's the answer? So j, if it's lined up like this, j2 is importing in what direction? Up. And what is that in this system? Just capital J.

At this instant in time, that's just capital J. Trivial calculation, because this angle is 90 degrees. Plug in 90 degrees, this term goes to 0, this term goes to 1. J2 is capital J.

And what about the other term, J1? You just got to-- it's J1, right? So you just gotta go with the flow. It is is. You'd substitute this in for J1 right here, and you'd have R1 omega 1 cosine theta sine theta and j and k terms, plus this thing, capital J.

And you have just converted the answer, which was in terms of unit vectors in rotating to different rotating frames. You've converted it all down to the base frame.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Oops, I'm sorry. I just made a mistake. You guys got to get better catching me. That now make sense? So phi is the angle that the j2 unit vector makes with the inertial frame, right? And theta is the angle that the j1 or i1 make with the inertial frame.

Yes?

AUDIENCE: Phi is 0, though, right?

PROFESSOR: In this case, phi is 0. Does that still work out over there? Sine of phi is 0 and cosine of 0 is one and you get j.

AUDIENCE: Then why did we didn't plug in anything for j?

PROFESSOR: We did. There isn't a simple answer for it. And so you have to use the full expression. I just got lazy and didn't want to write it out. The answer is this. Stick in 30 degrees if you want, and then you'll get numbers.

So real important point that we discovered is that the answers are correct, expressed in rotating unit vectors, expressed in different unit vectors, different rotating ones. This is correct, because you can take this and you can reduce it down to the base frame.

So you will be-- usually in problems that you're given, you'll be asked to express the answer in terms of unit vectors in the base frame. Or you'll be told you can leave it in whatever is your comfortable set of unit vectors.

Most of the time, the first ones you'll arrive at are the ones in terms of the rotating coordinates that are easier to use. The more natural answer falls out in terms of these. Good.

All right, and we've got three, four minutes left. What have I confused you with here? So key concepts-- what have we-- what hasn't been clear or maybe we didn't cover it yet-- another point.

AUDIENCE: So the reason we chose those as the starting reference frames instead of I hats and theta hats?

PROFESSOR: Only because at the beginning of class, we talked about it-- which frames do we want to use, and then we chose those. Could we have used a polar coordinate system to do this problem? Sure. Twice You do it once in each--

AUDIENCE: Is there a way to know up front which one would simplify down to the inertial i hats and j hats more simply?

PROFESSOR: The easiest way? Is there a way to know upfront? No. That's just experience. Work lots of problems, and you get good at picking frame. We can probably, with time as we meet and talk about these things, we'll come up with some sort of general insights about how to do that. Yes?

AUDIENCE: Is this picture up in parentheses supposed to be those coordinate systems?

PROFESSOR: This picture is the coordinate system of that first arm.

AUDIENCE: OK, so is that supposed to be phi up there?

PROFESSOR: Yeah. Wait a minute. No. This is the first arm. That is theta and these are ones, right? The 2 system would be phis

AUDIENCE: OK. I was just wondering if that was the ride or if that was not.

PROFESSOR: No. This is point A, if you will. Well, it could be. It's lined up with point A. This is A.

AUDIENCE: Because I thought we decided that the phi was--

PROFESSOR: This is point A, right? That is point A. And this is arm CA.

AUDIENCE: So is this one here the origin of this one?

PROFESSOR: Well, look at whatever the unit vector is. The unit vector in this system is lined up with that arm. So this is just a breakdown of these unit vectors so I could draw the angles and figure out the sines and cosines. You could draw a similar picture for i2 j2's. And then it would be phi's. Good question. Yes?

AUDIENCE: So, since you can choose between Cartesian and polar coordinates, could you set one in Cartesian, one in polar, you can mix and match it or-- is that beneficial in some problems?

PROFESSOR: Polar is-- I don't have time to show you today. But for planar motion problems, which are things confined to a plane, they rotate, axis of rotation's always in the k direction, which is all the problems that you ever did in 801 Physics. You didn't do general things actually.

But for planar motion problems, cylindrical coordinates, actually, you still need the k to describe the rotation, right? Polar coordinates, cylindrical coordinates are oftentimes really convenient. And they're easy to use because you've learned them a long, long time ago.

And you know the relations. But you can make it a rotating x1, y1, z1 rotating system and it will all work out. We came up with this little formula here, right? This could just as easily have been r hat.

And this could have just as easily been no difference whatsoever in a planar motion problem, when you attach an xy system that rotates with it, or I call it r and theta. These are the same direction. R is in the direction of i1. Theta is in the direction of j1.

So use it when it's convenient, and it's convenient a lot of times, especially that nasty acceleration formula. In polar coordinates, it reduces down just to the set of five terms. Memorize it and just tick them off-- Coriolis, centripetal. You see them right away. You know what they are. But there are certain problems, even in planar motion problems that polar coordinates don't work for-- doesn't work for.
[INAUDIBLE]

And think about that. It's actually a simple problem. Put a dog on a marry-go-round. The dog's running in a random direction on the merry-go-round. And the merry-go-round is turning at some rate. And you only want one rotating coordinate system, r and theta.

You can't do the problem with polar coordinates. Think about it. Go away, think about why not. I'll tell you the answer in words. You go figure it out. You can't describe the velocity of the dog in polar coordinates.

The dog is running around. If the dog's fixed on the rotating thing, than polar coordinates work. If the dog's running, you can't do that velocity. So you need a more sophisticated coordinate system.