2.003 Engineering Dynamics
Problem Set 2 Solution

Problem 1: A small box of negligible size slides down a curved path defined by the parabola $y = 0.4x^2$. When it is at point $A(x_A=2m, y_A=1.6m)$, the speed of the box is $V_{box} = 8m/s$ and the tangential acceleration due to gravity is $dV_{box}/dt = 4m/s^2$.
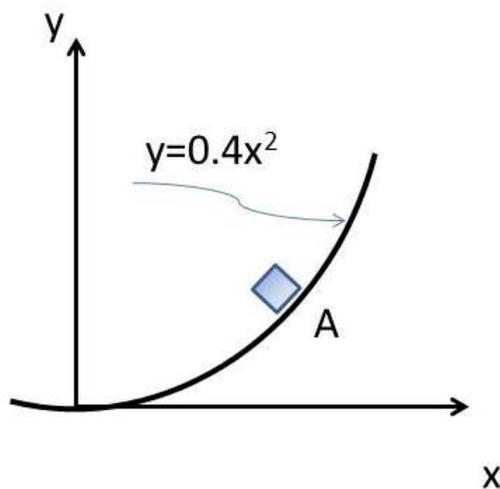
Figure 1

Concept question: When a particle travels along a curved path it experiences an acceleration given by the speed squared divided by the radius of curvature. In what direction is the acceleration?

    A. Tangent to the path
    B. Towards the concave side of the curved path (i.e. the inside of the curve, for example toward the center of a circular path)
    C. Towards the convex side of the path

Answer: The answer is B. The box is travelling in a curved path and experiences centripetal acceleration inwards along the instantaneous radius of curvature.

Problem 2: The 0.5-lb ball is guided along a circular path, which lies in a vertical plane. The length of the arm OA is given by equal to $r_{A/O} = 2r_C \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot\theta = 0.4$rad/s and an angular acceleration $\ddot\theta = 0.8$rad/s^2 at the instant $\theta = 30^0$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_C = 0.4$ft.

$\theta = 30°$
$\dot\theta = 0.4\text{rad/s}$
$\ddot\theta = 0.8\text{rad/s}^2$
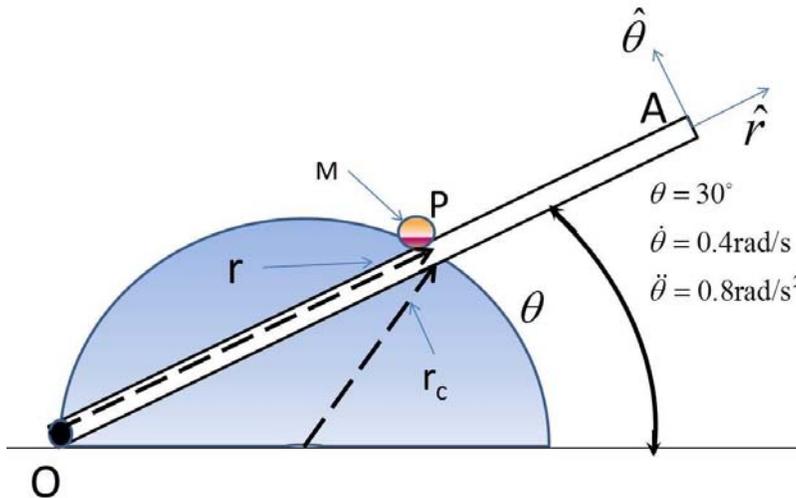
Figure 2

Concept question: In what direction is the force of the arm on the ball?
   A. Parallel to the arm
   B. Perpendicular to the arm
   C. Parallel to the radius of the circular arc, with radius r_C

Answer: The arm OA is frictionless and can only support a force normal to its surface.

Problem 3: Helicopter Blades Rotor blades of a helicopter are of radius 5.2m and rotating at a constant angular velocity of 5rev/s. The helicopter is flying horizontally in a straight line at a speed of 5.0 m/s and an acceleration of 0.5m/s^2, as sketched in the figure. The reference frames defined in the figure are as follows: O_{xyz} is fixed to ground; B_{x1y1z1} is fixed to the translating helicopter.
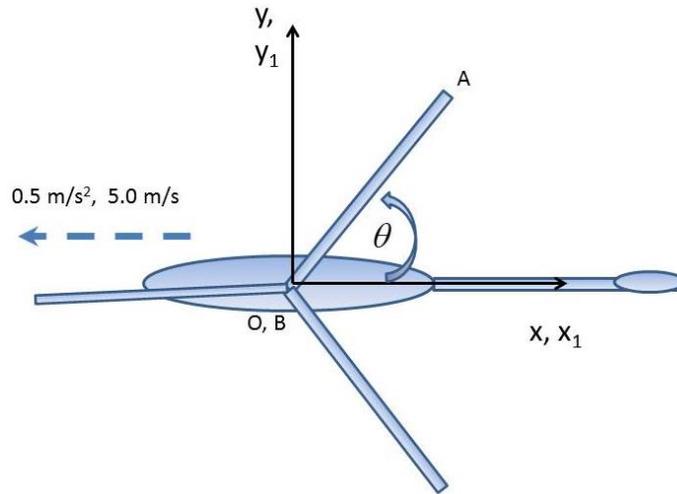
Figure 3

Concept question: When is the magnitude of the acceleration at Point A the greatest?
A. $\theta = 0°$
B. $\theta = 90°$
C. $\theta = 180°$

Answer: Point A is subjected to the vector sum of the forward acceleration of the helicopter plus the centripetal acceleration at point A on the blade. This sum is largest when the two vectors point in the same direction which is when $\theta = 0$.

Problem 4 - An Amusement Park Ride A ride at an amusement park consists of a rotating horizontal platform with an arm connected at point B to the outside edge. The arm rotates at 10 rev/min with respect to the platform. The platform rotates at 6 rev/min with respect to the ground. The axes of rotation for both are in the z direction, out of the plane of the page as shown in the drawing. Gravity acts in the z direction. A seat is located at C at the end of arm BC, which is 2m long. The radius from A to B is 6 m long. The mass of the passenger is 75kg. Note that gravity acts. Find the velocities and accelerations of the passenger.

The reference frames defined in Figure 4 are as follows: O_{xyz} is fixed to ground; A_{x1y1z1} is fixed to the platform at the axis of rotation. $B_{x2\,y2\,z2}$ is fixed to the arm BC at B and rotates with the arm.
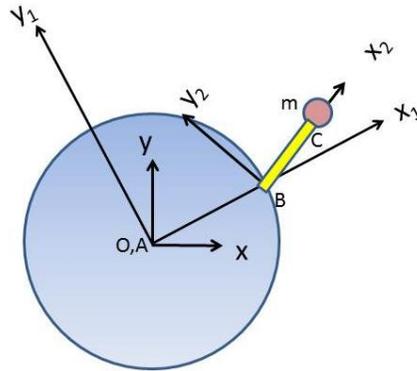
Problem 4

Concept question: What is the rotation rate experienced by the passenger with respect to the fixed coordinate system attached to the ground?
A. 4 rev/min,
B. 10 rev/min,
C. 16 rev/min.

Answer: The rotation rate of the arm BC and the passenger sitting on the arm at C is the sum of the rotation rate of the arm relative to the platform and the rotation rate of the platform, C.

Problem 5 – Cart and roller system: A block of mass M_b is constrained to horizontal motion by rollers. Let the acceleration of the block in the inertial frame O_{xyz} be designated as $\vec{a}_{A/O} = \ddot{x}\hat{i}$ which is assumed to be known. Point A is fixed to the block. A massless rod rotates about point A with an angular rate of $\dot{\theta}$. A point mass, m, is attached to the end of the rotating rod at point B.
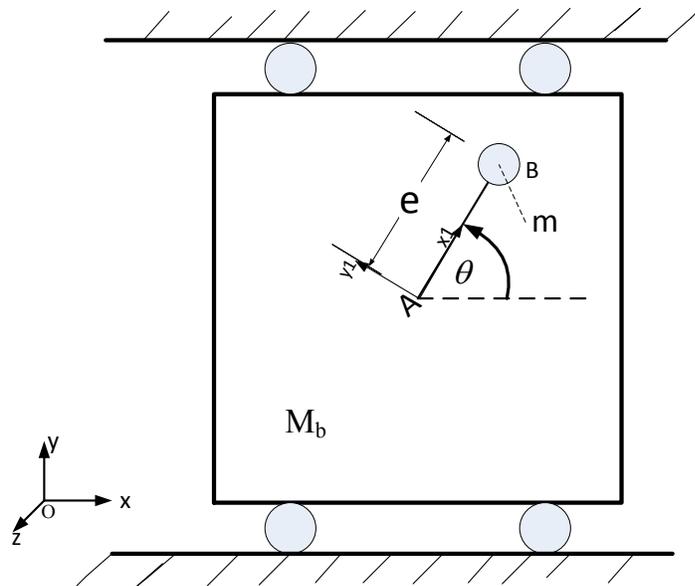
Problem 5

Concept question: When considered as a two particle system, the block and the rotating mass have a center of mass which one could calculate as a function of the position of the rod as it rotates. When viewed from a fixed inertial frame external to the block, the center of mass of the system

A. moves left when the small mass and rod are to the left of point A

B. moves right when the small mass and rod are to the left of point A.

C. does not move left or right

Answer: There are no external forces in the horizontal direction on the three body system, which consists of the block, the massless rod and the mass attached to the rod. Therefore the center of mass of the system in the horizontal direction does not move, C.

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