PROFESSOR: All right. Let's start off a quick list of important concepts for the week. Just-- and I'm going to push you right along today. I want to get this done. What do you got?

AUDIENCE: Static Equilibrium positions?

PROFESSOR: Static equilibrium, OK. So EOMs from xstatic. OK, good. Another concept for the week?

AUDIENCE: Damping.

PROFESSOR: Damping, yeah. And what have we've been talking about all week?

AUDIENCE: Vibration.

PROFESSOR: Vibration. OK. So vibration, damping, OK. More?

What's the whole general subject that we've encased vibration in? It's an introduction to what? Have to make it. We've been making an assumption that we haven't made before. Yeah, linear.

We've been studying-- this has been an intro to-- linear systems. We can only do things like transfer functions, that sort of thing, by assuming linearity. Anything else? What have I just spent the last two lectures deriving for you and showing you? Transfer functions.

In fact, I've shown you three. All right? All right. That's beginning to get at what we've been talking about. So introduction to linear systems. We've been studying vibration as one implementation or application of linear systems.

We've been looking at single degree of freedom vibration problems only so far. And we've found frequency response functions. These frequency response function tell you the response of the system to what kind of an input? What are the characteristics of the input that we've been looking at? What do we call harmonics?

Single frequency cosine omega t-like input. Cosine omega t, sine omega t, e the i omega t. One frequency, repetitive, it's called a harmonic input. So for a linear system-- we've also we're talking about what we call the steady state response. What's that mean? OK. Put that into words.

AUDIENCE: [INAUDIBLE]

PROFESSOR: So you turn the motor on, the whole building shakes, settles down, and just has a steady vibration you're dealing with. So after any startup transients have died out, it's the steady state response. If you have a linear system and you're putting in some F0 e to the i omega t force, a real part of steady state response, what is the frequency of the response?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Right. So for linear systems, the steady state response, the frequency of the output always it's a property of a linear system. So a couple really important things to remember here. Good.

Now we're going to move onto a problem-- Oh. And then one other thing I usually do-- ask of you, is are there any major questions? Something left over from lecture. Something we've done in the last week that just isn't going down.

AUDIENCE: [INAUDIBLE] gravity [INAUDIBLE]?

PROFESSOR: How do you get gravity to go away in the equation? Actually, this is the most common question that's been asked to the TA's this week, of Professor Gossard, of me. So there still seems to be a little bit of misunderstanding about-- and this is the subject of static equilibrium, verses unstretched spring kind of positions. I think I'll just deal with that right away.

You're familiar with this problem? K, C, M. And most of you are comfortable with the notion of measuring x from the unstretched spring position, right? So that's what I've done right here. Now you add gravity though, and let it find its equilibrium position under gravity, it's going to be at some other location.

And I'm just going to call that xstatic. Static displacement. Now when you turn on the harmonic excitation, or even grab it and let it go from initial condition, where will it vibrate around? It's going to vibrate around the static equilibrium position.

Here's my system. Here's my free body diagram. Got all the gravities and Cx's and Kx's and x dots and so forth in it complete, right? I can write an equation of motion for that system, which you've done lots of times. And it has a gravity in it.

But now I'm going to say look, here's my x-coordinate, it stretches down xstatic. And on top of that I'm going to add a little dynamic motion. And I'm going to describe that as x equals xstatic a constant, plus the dynamic movement with respect to xstatic. Take the time derivative of it, because this is constant. It goes away.

And two time derivative. So x dot is x dot d, and x double dot is x double dot, the dynamic quantity. Take these three and plug them into here. Mxd double dot, Cxd dot. But the K term, you still have xstatic in it.

The original Kx breaks into two pieces. And on the right hand side is my Mg plus f of t, like before. But K xstatic must be equal to Mg in order for static equilibrium to have existed. Everybody agree with that?

Another way say that is let's say there's no dynamic motion. This term goes away. This term goes away. This term goes away. Turn off the excitation. You're left with that equals this.

And that's what says it sits in the static equilibrium position. If those two are equal, they cancel, leaving you with this. So if you had understood this in the beginning, you could have said let's make our inertial coordinate from the static equilibrium position. Just let the thing find its static equilibrium position, set the inertia coordinate measured from there, and you would get to this equation, with the possible confusion of having to deal with the free body diagram. Because the free body diagram still has on it Mg and the spring force that comes from the static position.

So if I were doing this, and just say I want to go directly to this equation, I would draw the free body diagram that would have Kxd Cxd dot, F MG, but then I would also just put in here-- there's one more force. K times xstatic or delta or whatever you want to call it. You know there's some xstatic, and you know-- because you're doing this-- you know this about the problem. You know this cancels that, and it won't appear in the equation of motion.

So to be able to do-- anytime you're confronted with a problem that has gravity, which causes a static deflection, or results in a static equilibrium position, it might be 0 in your coordinate-like pendulum. You look at it, and you say, now in the equation of motion that you derived from the unstretched spring force position is the Mg term a function of the motion coordinate? And in this case it is not. So it doesn't matter what x becomes, this force stays constant. It doesn't enter into the dynamics of the problem.

If you ever write an equation of motion, and you come to an Mg term that doesn't have a theta or an x or any variable in it, you know that there's another way to write that equation about static equilibrium, and you'll be able to get rid of it, and simplify the math basically.

OK, now we want to move on to the problem of the day. And the problem of the day, I've written up here. That's the last class's list.

We've got a motor, fan or something. It's got a rotating imbalance. It's got a flexible base. It's got a lot of vertical vibration. Driving you crazy.

The rotation rate-- this is a pump or something with a rotating-- some rotating mass inside. Has some eccentricity. So it's causing some external force. Causing a force that results in vertical vibration. The rotation rate of that fan is 1750 RPM.

The weight of this motor sitting on this frame is 500 pounds. When you set this motor and bolt it down on that frame, the frame deflects 0.026 inches. And this thing's vibrating like crazy. Your assistant comes in and says, I got a solution for this.

We're going to put braces in here, and those braces will double the K. The spring constant of the thing. And he or she asserts that he thinks it will cut the amplitude of response in half. Cut the vibration amplitude in half. So you're going to do the calculation today to see whether or not this is a good idea.

And we're going to begin. So assignment 1 is come up with a simple lump parameter model-- masses, springs, dash, pots-- of this system so that you can model it. So we've only been doing single degree of freedom systems, it's probably going to look something like that.

So come up with a lump parameter model of that, and we've got what? 4, 8, 12. We could do four groups easily here of four or five each. So break into four groups. Come up first with the-- and group 1-- we'll put group 1 right here. And group 2 here, and 3 and 4 there.

So come when you get-- when your group figures this out. And we got to move right along because there's several parts to this. Just come up, sketch your model that you're going to do it with, and then part 2 right after that is to compute the natural frequency of the system.

We're going to eventually have to compute the transfer functions and then try out the fix. And see if the fix makes things better or worse. First assignment, come up with a lump parameter model and find the natural frequency. Form your groups and get talking.

AUDIENCE: Who's is this? OK, guys, what about the force? And what is it?

PROFESSOR: And it's really some me omega squared cos omega t. And this comes from the equivalent little mass on some arm, e going around and around inside. This is the equivalent unbalanced mass here. So keep coming up. Put your models up here. Yeah?

AUDIENCE: You get the [? mpd ?] [INAUDIBLE].

PROFESSOR: Well, let's go back. We've done this problem about four times over the course of homework. And the term block with an arm with a mass on the end. The e is the length of the arm, and the mass on the end is essentially the location of the center of mass of the rotating part.

So the other three groups need to draw a model up and come up with the natural frequency. One of the annoyances of this problem is it has nasty English units. You all remember what g is in English units? I want radians per second.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, but why do-- convert English inches to feet and you're done. Because you know g in feet per second squared. So rather than do three conversions, do one.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well that's one, and then you know g. So you can do it that way. Sure. I don't care. I just want the result.

Which is group 1? You guys are 1? You got a number you can give me? How many MIT students does it take to convert from inches to feet?

AUDIENCE: We got that, but we're not sure if it's giving hertz or radians afterwards.

PROFESSOR: Well, in this equation, does it give you Hertz or radians? Sure. And you just made this substitution in here. So it hasn't done anything to change to Hertz. Write that down. What a struggle! OK.

Good. That's good. You can leave it. All right. We've all converged to about 121. That was quite a struggle.

Was it the units, or was it-- partly was Hertz? And you guys haven't had-- you're dealing with nothing but equations for weeks. And to actually have to do real engineering numbers has thrown you a little bit. But so it's about 121. And that g over delta thing is really handy.

Now I want to know what is the response of this system to this input given the frequency of the input is 1750 RPM? And RPM, N is often written, how is it related to frequency in Hertz or cycles per second? Anybody know? This is revolutions per minute.

What would be revolution per second? Is that same thing as Hertz? So revolution is something going around. One full time around is 2 pi. It's one cycle.

So if you do 60 cycles in a minute, how many do you do in a second? One. So this RPM divided by 60 equals f in cycles per second. Or Hertz. RPM divided by 60 gives you revolutions per second.

AUDIENCE: That says RPS.

PROFESSOR: Well, excuse me. I was talking about-- get rid of that. That's the relation you want. And what's the relationship between omega and f? For every cycle, you get 2 pi radians.

So now compute-- I want you to give me an estimate-- find-- post my problem here. So part 1 you've done. Part 2 here and to get an estimate of the magnitude of this steady state response. And we've been talking about transfer functions. So it's probably a replica applying a transfer function here.

I would put your symbolic answer up first would be a terrific idea. Think you all-- when you think you have kind of a symbolic approach to this. Go write it up, and that way I can know where you need a little help, if any. That's OK. So you don't know K.

So just leave that has an undetermined quantity. F0-- what is-- you guys keep working for a second. Then we'll talk about it. You don't know K, so just leave it as an unknown. Don't do that. Just call it F0.

But now plug in some numbers for what you do know. So one quantity you're going to need to make over omega n, so write that one down when you get it. What's omega over omega n, and then move on from there.

Now there's a little pitfall here that you're all running into. And that is you're taking the positive square root as written. It's that whole quantity down there. Squared square root, and you're writing it 1 minus omega squared. And this thing's going to turn out negative on you if you're not careful.

But you can flip it around. It's really the absolute value of that quantity, because it a-- quantity squared square root, so it can be plus or minus. So you got 1.54-- 1.5 what? 1.5 is your ratio?

OK, write it down. Just write the ratio. That's what you need. 1.5. And you've got-- this is 4/5 here? That's what you guys have worked out? OK, so 0.8.

AUDIENCE: How did they get 1.5?

PROFESSOR: They calculated omega over omega n. You have 183 divided by-- what was your omega n?

AUDIENCE: Oh, 60. OK.

PROFESSOR: Not 60.

AUDIENCE: Well, no. I mean it's like 360 over 260.

PROFESSOR: Yeah. Well, you've got to divide 1750 by 60 to get F. And then you got to multiply by 2 pi. I'm doing it for the benefit of everybody in the room here. 0.8 F0 over K, all right. Now we're getting somewhere.

0.8. 0.8. What do you guys have here? You got 1 over 1.3. How does that work out? So about 0.8. OK, so you've come to the conclusion that the magnitude of that study state response is about 0.8 F0 over K.

And we call that F0 over K-- in lecture, I call it the static response. The book calls it the static response. Is it the same static response as we're talking about here? Don't confuse the two. This was the effect of gravity on the system.

The static response we're talking about here is if the frequency of this excitation went to 0, then the response-- this would stretch that spring by the amount we're calling xstatic, F0 over K. All right.

So now you know that you're operating at a frequency. What's the frequency ratio that you found? Omega over omega n. That was an intermediate step here. All right, 1.5.

All right, now you're going to-- the proposed fixed is to double the stiffness. Double the stiffness. Now we want to compute this result if you double the stiffness. And the person who proposed that, thinks it's going to half the response.

That's the proposal. So double the stiffness. And in order to get finished here, what effect does changing the stiffness have on this system?

AUDIENCE: Decreases the [INAUDIBLE].

PROFESSOR: Well, you jump into an answer-- intermediate step here. What does it do to the natural frequency? There's a natural frequency change if you change the stiffness. OK, so the natural frequency is normally some kind of K over m. Now we also know it's g over delta.

But we're going to double the stiffness, so what does that do to the natural frequency? Does the m change? OK, so what's the-- this is omega n before. Omega n after equals right. OK. And what does that do to your frequency ratio?

So the effect of changing the stiffness changes the natural frequency, and that changes the ratio of the input frequency to the natural frequency, and that will change this calculation here, right? So come up with a new value for this. Anybody got a number? What's the frequency ratio after?

AUDIENCE: Uh, 1.1-- never mind. I was doing the squared term.

PROFESSOR: I wanted to know the frequency ratio. This is after is equal to the frequency ratio that was something to do with root 2 here. The natural frequency is going to go up or down by a factor of root 2. If you double K, the natural frequency will go up by a square root of 2. So the frequency ratio after-- the natural frequency goes up by the square root of 2.

That's the same as root 2 over 2 here. So the whole thing drops down. So 1.5 times square root of 2 over 2. What is it? All right. 1.07.

Now what is-- figure out this quantity. Pardon? 3.4. And did you put in a 2 here? Right. So this is the correct expression to be using.

But now in this new problem-- this was before. In this problem afterwards, x is steady state is F0 over 2K if double the stiffness, all over this quantity omega squared over omega n squared minus 1, but with the new frequency ratio. This is now 1.07. You do this number, what do you get? 3.-- what did that say? 3.4 roughly,

F0 over K. So you have apples and apple together. You want to compare the F0 over K before and after. End it was a factor of 3.4. So should you make the change? No. That's a bad idea.

You ran into a situation like this before, going to all that trouble of cutting steel and getting out the torch. The welding system. Take yourself a great big weight and walk up, and thump some weight on top of this thing, and see if it gets better or worse. Because if I double the mass of the system, what would happen?

It's going to go the other direction. This will go up by a factor of root 2. And then this denominator-- you're pushing it further out. And instead of being at 0.8, you'll be down further.