PROFESSOR: Today we're going to talk about this topic of vibration isolation, which is a very practical use of knowing a little bit about vibration. So imagine a situation you've been in where there's an air conditioner in a window, and it's causing your table to shake, or where you're trying to work, or your bed rattles, or something like that.

How many of you have ever had an experience like that, something's kind of annoying, messing up a lab experiment, or whatever? Yeah, you've all experienced these things, right? So as clever engineers, are there simple solutions sometimes to fixing these problems? And that's what we're going to talk about today.

I have a little quick demo that I'm going to show you. You saw this the other day, where this is my little squiggle pen, and it's got a rotating mass inside. And we've looked at rotating masses a lot. It has some unbalanced, statically unbalanced rotating mass. Could be a fan blade with a hunk of chewing gum or something stuck on a blade or broken piece out of it.

Puts in a force me omega squared cosine omega t, basically an F0 cosine omega t kind of excitation. And if it happens to be a flexibly-mounted, mass-spring dashpot system, it'll vibrate. And so I showed you that the other day. We'll do this, and we'll need to lower the lights a little.

But today, I've set your bed. This is your microscope here, this little one. Think you can see it in the foreground. And this is the air conditioner, or the water pump, or whatever is causing the trouble. So this is running at about 28 hertz. If I set the strobe just right, I can absolutely stop the motion. It doesn't look like it's moving at all. That's because the strobe is at exactly the same rate as the squiggle pen.

Now I'm going to de-tune the strobe a little bit so you can see the motion. There is the motion of this main system. That's causing the problem. But it actually puts vibration into the tabletop. And next door over here, I have a little beam. And you can see that little piece of white moving up and down. It's just a little flat piece of spring steel with a magnet on the end as a mass and a piece of white tape so you can see it. But notice it's going up and down in synchrony with the other one.

So this is your microscope sitting on a lab bench some distance away. The problem vibrations are being created by the original imbalance in something, travels through the floor, gets to your table with the microscope on it. Now your microscope shakes. So the issue is, what can we do about it?

OK. Oh, I do-- yeah, no, I'll leave this for a second. I'll turn it off. And then I'm going to have you consider. So I want you to get in pairs and talk about this. I want, as a group, we're going to come up with at least three ways to reduce the vibration of your microscope-- relatively simple ways to fix it. How would you do it? So think about. Talk about it. And come up with three ways of fixing this problem.

I'm going to do one more demo on this in a minute and then put up the transfer function for the force, the one we had the other day, OK? The picture.

All right. Let's have some suggestions. How would you go about fixing this? All right. You had your hand up first.

AUDIENCE: We have it on suspension systems, and springs.

PROFESSOR: Yeah. So put some springs on what?

AUDIENCE: Like, have a table where your microscope is, and then have [INAUDIBLE].

PROFESSOR: So springs support the microscope. All right. So I had this little magnet here sitting on this beam. This is that spring-supported microscope. And I have done a heck of a lousy job. This thing shakes like crazy. So what do you mean? Not like that. How might you-- OK. So you think you could change the properties of this system so that it might do better? Let's think about that. OK, what's another idea?

AUDIENCE: Change the length of that spring could change its natural frequency.

PROFESSOR: Yeah. Are you talking about the microscope one, the receiving one? If you change the length of it to make it longer, it makes it softer, actually. Make it shorter, it makes it stiffer. So you would change its natural frequency. Now, the two ideas together, if you set the natural frequency correctly, the system on the receiving end, you can reduce its vibration. And I will demonstrate that in a second. What's another idea?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Acid damping. OK. Well, that's a very interesting suggestion. Damping, it helps under one circumstance, but not under some others. And we're going to explore that today. This is all basically-- we've come up with one of the three ideas. What else?

AUDIENCE: Attach something that vibrates 180 degrees [INAUDIBLE].

PROFESSOR: Oh, that's interesting. That's the fourth one. I mean, that can be a little expensive, but these are like noise-canceling headsets, right? Could we put in something else somewhere else on the table that cancels the vibration out where your microscope is? Yeah, you could do that. A little expensive. What else? Yeah.

AUDIENCE: You could cushion [INAUDIBLE].

PROFESSOR: Where would we put that?

AUDIENCE: You could do it underneath [INAUDIBLE].

PROFESSOR: Yeah. And so that's generally the same idea. So you're all treating the microscope. Can you treat something else in the system? So yeah, we'll fix the microscope end. But that might knock it down by a factor of 10. I want to knock it down by at least a factor of 100, if not a factor of 1,000.

AUDIENCE: Fix the air conditioner.

PROFESSOR: Ah-ha-ha-ha-ha! You know, put another piece of gum on the other blade. Or clean it up, or balance the rotor, in effect. Rotors are not manufactured defective usually. They get rejected at QC before it goes out the door. So fix the rotor. Can be a little expensive sometimes, but worth it. They do maintenance checks. Actually, they have accelerometers built in to all expensive rotating equipment these days.

And it's called condition monitoring. And when they get outside of certain limits, they shut the thing down and rebuild it. In electric generator sets, gas turbines, jet engines on all aircraft are all incredibly carefully balanced in a way. They start getting out of balance, they stop and fix them before the things blow up on them and you have a $20 million problem instead of maybe a $20,000 tear-down. All right? So yeah, you fix the rotor.

Third idea. Well, could you-- this thing's shaking like crazy. What if you change the length of this beam? Could you stop this thing from shaking so much? And if you stop this from shaking so much, would it put so much excitation into the table? No. So fix the rotor, isolate the source, isolate the receiver. Now you get at least three ways. And the gentleman up here came up with the fourth way-- active cancellation. OK. Great.

I have a demo of one of these. Sometimes if you're desperate and you're trying to get some sleep-- it's your bed that's rattling or something-- you might want to try the following that you can do very quickly. So we'll dim the lights again. We'll turn it back on.

[VIBRATING]

Both are vibrating like crazy. All right. So one way to detune the receiver is to put a big weight on it and change its natural frequency by changing the mass. And now the little thing is hardly shaken at all, because I've detuned just by changing the mass. Accomplished the same thing as switching the length. Instead of messing with the stiffness, I've changed the mass of the system. This is still shaking like crazy. OK.

So now it's back to vibrating again. But over here is my source. I don't know if it'll work so well in this one, because the source is a lot more massive. But I'm going to put the same mass on it. Ooh, it made it worse. Ah. And that's another good demonstration. Excellent.

So we could come back up with the lights. You got to be careful of that. You go to mess with one of these systems, if you do it wrong, you make the matters worse. First consulting job I ever had back in 1977 or something like that, they had a vibration problem on a ship. And the first consultant in said, stiffen up. It was actually the exhaust stacks on these 5,000 horsepower diesel engines, and they were 30 feet tall and shaking like crazy. And the first guy said, stiffen up those exhaust stacks. And he did exactly the wrong thing. And it just shook worse than ever. OK.

So now what I'm going to show you-- what we'll put on the board today is a little bit of mathematics to back up how you go about doing the two things. One is isolating the receiver, or the other one's isolating the source. I'm going to start with isolating the receiver. But we're going to start with a little bit of math, a little math tool that we need that will make life a lot easier for us. Yeah, I'll work here.

If you recall last time, now would be the time to do it, we derived the transfer function for essentially this system, where we had an F0 cosine omega t. And we computed the response, x of t, as some x0 cosine omega t minus the phase angle. And we worked it all down to where we could plot it like that. But quite frankly, it was kind of a lot of lines of math. And it was sort of painful. I actually hated doing it on the board. But it was easy, because it was familiar. It's just trig stuff. It was all trig and a little bit of calculus. So we do that first, because it all makes sense to you mathematically.

But there's a vastly easier and quicker way to do this, which we'll address right now. And that's using complex numbers. So we need a couple bits of information here. One is Euler's formula. So if you have e raised to the power i theta, you can show that that is the same thing as cosine theta plus i sine of theta. That breaks into a real part and an imaginary part.

So if we wanted to express this excitation, F0 cosine omega t, in complex notation, we would say it is the real part, which I'll denote as Re, the real part of F0 e to the i omega t. And I'm going to just specify that F0 itself here, this is real and positive. So this is real and positive. So the real part's going to be F0 times-- and if you break down co ee to the i omega t into its-- by Euler's formula, it gives you cosine omega t plus i sine omega t. And the real part is the cosine part just according to this. OK.

Now, another little fact that I want to show you is if you have a complex number, a plus bi, I want to express it as some ce to the i theta. So you want to use Euler's formula to express a complex number. And if we draw it, the answer becomes pretty obvious. This is a point up here, a, bi. And this is the imaginary axis here and the real.

And this point, this side is a, and this side here is b. And this side here, the length of this triangle, is c. And the angle in here is theta. So now if I ask you, what's c, well, you say, oh, well, c is obviously the square root of a squared plus b squared. And theta is a tangent inverse of b/a, which is the imaginary part over the real part.

If you want to express a complex number this way, well, the magnitude is square root of a squared plus b squared. And the phase angle that you put up here is tangent inverse of the imaginary part over the real. OK.

So now we have the basic tools we need to take on the vibration problem. And so we have that system up there. And our output, from the way we derived it last time, the output is some x0 cosine omega t minus phi. And one of the reasons it was so painful doing it this way last time is you have to-- this is cosine is a function of both time and phase. And to break it apart takes a lot of work.

So we want to do the same thing, but with complex variables this time. So I want to express this then as the real part of-- I could say it's the real part of x0 e to the i omega t minus phi. And despite Euler's formula if this is real, just the number. Than this breaks down into cosine omega t minus phi and i sine omega t minus phi.

But here's the beauty of using complex notation and exponentials. This now becomes the real part of x0 e to the minus i phi times e to the i omega t. If I could separate these two. And this is what makes it so much easier to use this approach. And I'm going to call this part of it just some capital X. And it is a complex number. For sure.

If I break this up into cosine of minus phi and minus i sine phi, it's got an i sine phi. So this is a complex number-- a plus bi kind of form. So in general, this thing is complex. So this whole thing is the real part of some X, which I don't know now, e to the i omega t.

So now we can quite quickly do the derivation we did last time. We're talking about representing linear systems by some kind of black box-- has a transfer function in it, which we call H, in this case, x/F. Response x per unit input F. And remember, we're talking about steady state response only. And we have, as our input here, some F0 e to the i omega t.

And we have, as an output, some X e to the i omega t. And we know that we're going to use the convention that we care about we have to have real number answers. So we'll be eventually actually using the real part of the input and the real part of the output. But to get there, we're going to use complex notation first and then separate out the real and imaginary parts at the end.

So for our system, we know the equation of motion, so now it's some F0 e to the i omega t. There's our equation of motion. And I'm going to let x here be this unknown capital X e to the i omega t. And I'm going to plug it in. And the exponentials are particularly easy to deal with when you're taking derivatives.

So upon doing that, we immediately get minus omega squared M plus i omega c plus k. All of that times Xe to the i omega t equals F0 e to the i omega t. Immediately, I can get rid of the time-dependent parts. And I can solve for x/F, which is what we set out to do the other day to find this transfer function between input and output.

So if I solve for x divided by F, I'm going to get all this stuff and the denominator on one side. And I'll write it out here. It simply looks like that. And now remember, I can substitute in some things. I remember k/m is omega n squared. And zeta in c over 2 omega n M.

And I plug those things in and just rearrange it a little tiny bit. We should come up with something like we found before, so that x/F, 1/k, and the denominator, 1 minus omega squared over omega n squared-- not quite yet here-- plus 2 i zeta omega over omega n. That's what it looks like.

So you still have a complex denominator. And this basically looks like a number 1 over k times 1 over some a plus bi. There's your a term. Here's your bi term.

And the way you deal with something like this-- you have an a plus bi in the denominator-- you multiply the numerator and denominator by the complex conjugate in order to get this into actually standard a plus bi form. If I do that symbolically here, it comes out looking like a minus bi over a squared plus b squared. And that's 1/k e to the minus i phi over square root of a squared plus b squared.

Because now, see, the denominator's just a real number. So this whole thing is, in some form, c plus a di. You could break this into a real part, complex part. We could say that's equal to some magnitude times e to the i phi. To get the magnitude, you take a squared plus b squared square root. It cancels. This is squared the denominator, so you end up with this part, square root, in the denominator. This is what the-- we need to know what phi looks like.

Well, phi had better come out like before, where now phi is minus tangent inverse of the imaginary part over the real part. And the imaginary part has a minus here. That's why a minus pops up here. Imaginary part comes from this. The real part comes from there. The common denominator stuff all cancels out when you take the ratio.

So this is tangent inverse of two zeta omega over omega n all over 1 minus omega squared over omega n squared, as before. I've skipped a couple of steps, but we cranked this whole thing out before. This is the same steps that you would go through to do that. We're just doing this to get to the phase angle.

But this now is exactly the same thing we got before, which we have plotted up there. We work with magnitude and phase angle. So the magnitude of x/F is the same thing as saying the magnitude of the transfer function. And that transfer function looks like 1/k, the magnitude, all divided by 1 minus omega squared over omega n squared squared plus 2 zeta omega over omega n squared square root. That is the same transfer function magnitude that we derived last time, with a lot more work.

And this approach, using complex variables, you can use for any single input, single output linear system. And we're going to do it to derive right away the transfer function for the response of this to motion of the base. So if you follow how we used this complex variables in e to the i omega t's to get here, we can now apply the same tools to do other transfer functions to be a lot more efficient about it.

Before I jump to this one, remind you how, in practice, we use this. So if the statement magnitude of x/F equals everything on the right there. Then in the way we would normally use this is to say, well, if you want the magnitude of the response, you take the magnitude of the input force, multiply it by the magnitude of the transfer function, evaluate it at the correct frequency. That would give you the magnitude.

If you want the time dependence, x of t, well, that's the magnitude of the force, magnitude of the transfer function times the real part of e to the i omega t minus phi. And this gets us back to when you work this out, this is your x0. And this is your cosine omega t minus phi.

So once you know what the excitation force is and its frequency, you put the force in here. You evaluate that thing on the left at the correct frequency. And you write out the answer directly. In one of the homeworks for today, the question just had you go through the exercise of figuring this out at three different frequency ratios, like 1/2, 1, and 3, or something like that, would put you to the left of the peak at 1/2, on the peak at 1, and way off to the right out at the right edge at 3. And you'll get three different response amplitudes and three different phase angles that go with it. All right.

So that's how you review. Did the same thing a different way. And I'm going to move on to base motion. But any questions about this now? Yeah.

AUDIENCE: Was the e to the negative ib included in your F of x?

PROFESSOR: Yes.

AUDIENCE: OK, so why did the negative b appear again after your final [INAUDIBLE]?

PROFESSOR: The very top expression up there, it says x/F. It says we're trying to cast it in the e to the minus i phi form. That's my goal. And I did that, because we started over here with the problem that we had done before, where that's the way we decided to write the answer. And it turns out that it's just a convention in vibration engineering that authors and people have adopted to express the phase angle as minus phi. They could have done it as plus phi. The plots like this are phi.

AUDIENCE: Right. But I guess what I'm wondering, isn't [INAUDIBLE] x/F.

PROFESSOR: Oh, I see what you mean. It's in there before you take its magnitude. So the Hx/F, when it is-- this here is left in complex notation. And this is Hx/F of omega. And it is complex. We take its magnitude. Then the magnitude is not complex, right? And so we take its magnitude. We get that expression.

But when we take its magnitude, we've thrown away the phase information. So we have to keep it and put it somewhere. And so we put it in the e to the i phi form. And I guess what I should have done here is now this is-- I've taken-- this is Hx/F, same thing as x/F, in complex form.

And I've said, OK, if I write it this way, I have just said it is a magnitude. Times its phase information. I've separated its phase information from its magnitude by writing it this way. OK. And the phase then is that. And its magnitude is that. Good question. All right.

So now let's see if we can kind of pretty quickly do the same problem for base motion. So this is our microscope now, idealizes a mass spring system. So this is our microscope. Has some mass stiffness damping motion, x of t. And how do you suppose-- where would you measure that motion x of t from? Like, to define your coordinate here is a major point in the last homework.

Is gravity involved? But only as a constant term, mg in the equation of motion. It's only there depending on if you write the equation of motion in the less desirable way. Where is this measured from do you guess? Equilibrium position? Static equilibrium position? Or 0 spring force position? How many suggest 0 spring force position? How many suggest static equilibrium? OK. You got the message.

This is from equilibrium, because you don't have to deal with the mg term. So this is measured from equilibrium. That's the deflection of the microscope support. This is the deflection of the floor that's driving it. Then we know we've got that table shaking like crazy. That's what's causing this to vibrate.

And we need a free-body diagram. And we approach free-body diagrams just like before. You imagine positive motions of x and x dot, positive motions of y and y dot, and deduce their forces. So positive x gives you a kx opposing. A positive x dot gives you a cx dot opposing. A positive y gives you what? A force that results on this. Positive motion of the floor.

Positive or negative force? How many think positive? How many think negative? How many aren't sure? How many aren't awake? OK. Look.

If I push up on this-- and now this is fixed when you do this mental experiment. You fix this momentarily. You cause a positive deflection here. It compresses the spring. Does the spring push back or not? So if I'm moving upwards, which way is the spring pushing? All right. But if I'm pushing upwards, which way is the spring pushing on the mass? Up.

So this one gives me a ky up. And the dashpot does a similar thing-- cy dot up. And there's also an mg here. But there's also a kx static, if you will, up. And they cancel. We know that. So we don't have to deal with the mg terms.

So now we can write our equation of motion. And the equation of motion for this system is the mass times the acceleration. That's got to equal to the sum of all the external forces-- one, two, three, four of them. And I'll just save a little time and board space. I'll put them on the correct sides of the equation. So these are the x-- put the x terms on the left side. cx dot plus kx. And on the right-hand side, I get ky plus cy dot. This is my excitation. That's the floor motion. And this is my response on the left-hand side.

So I'm going to let y of t, the input, be some y0 real positive times e to the i omega t. And I'm going to assume that the response is some x, probably complex, e to the i omega t. So this is x of t here. Equals some x I don't know e to the i omega t. And I'm going to plug those two into this equation.

If I just do that directly, x is on the left side. y is on the right side. Then I find minus omega squared m plus i omega c plus k, just like before, xe to the i omega t equals k plus i omega c y0 e to the i omega t. And nicely, I can for now get rid of the time-dependent part. And I can solve for the response that I'm looking for-- x over the input is real and positive, amplitude of vibration of the floor. And that I will call Hx/y of omega, a transfer function, probably complex, that I can then deal with like I did above.

And when I finish manipulating things, substituting in zetas and omega n squareds and that kind of thing, this becomes-- well, first, I'll write it this way. I can write this as a magnitude times an e to the minus i phi again. That's where I want to go. And when I do that, 1 plus-- a little messier-- 2 zeta omega over omega n squared square root.

This is just the numerator. And the denominator is just the same as the other single degree of freedom things. 1 minus omega squared over omega n squared squared plus 2 zeta omega over omega n squared square root e to the minus i phi.

So now it's the transfer function as before except the denominator's a little messy. And there's no 1/k. And I am going to have a messier expression for phi here. So there is something wrong with one of the boards this morning.

Kind of messy, complicated. Do I ever use it? Rarely. What's important in these things and what isn't-- really what's important when you're just trying to get a quick solution to vibration isolate something, you really want to know what this is going to come out looking like. You're trying to make the response x small compared to the input. That's the whole objective.

Right now the table might be moving a half a millimeter or something like that, but this thing's moving out here five or six or seven millimeters, 5 or 10 times that. And what we'd really like is if the table's moving a millimeter, you'd like this thing out here moving 1/10th of a millimeter. So the real objective here is to make this small. It's the magnitude you care about. Phase you rarely even want to know or need to know.

So we're going to do a sample calculation. Let's give an example here. So the source is at 20 hertz. So your unbalanced pump, your unbalanced rotor. Yeah.

AUDIENCE: How do we know in the previous thing that the frequency of oscillation has to be the same? Like, why wouldn't it be twice that?

PROFESSOR: OK. That's a great question. And I haven't mentioned this before, and I intended to. These systems that we're looking at are linear systems, which is where we started the other day. Linear systems have some interesting and very useful properties that we depend upon. One was, I said, force one gives you output one, force two gives you output two. Force one plus two gives you the sum of the outputs.

The other feature of a linear system is steady state response after the transients have died away. If the frequency of the input is at 21.5 Hertz, the frequency of the output is at 21.5 Hertz, period. Linear systems, the frequency of the input is equal to the frequency of the output. That's a really important little factoid to remember.

So I turn on the pump, the pump's running at 20 Hertz. 20 Hertz times 60 is 1,200

RPM, very common motor speed. So the pump's running at 20 Hertz. So that fan, it's got an imbalance. So that means you're putting excitation into the floor at 20 Hertz. And I want to reduce the vibration at the microscope by 90%. What that really means is that my goal is that the magnitude of x/y is 0.1. And that's the magnitude of this transfer function, Hx/y. So I want this transfer function to be 0.1.

So just look at the picture. Can I get that answer to the left of the peak? And what this plot shows you is this magnitude of the transfer function, for a variety of values, a damping. And of course, the lower the damping, the higher the peak gets at resonance. Right? So no matter what the damping is, what is the curves all go to in the left-hand side? They go to 1.

And that's really saying the static response of this system is if you deflect the floor an inch, the table moves with it. Everything has to move together when you get down to 0 frequency input. So everything goes to 1 on the left. You go through resonance at omega equals a natural frequency. But out to the right, as the excitation frequency gets higher than the natural frequency, the response drops off below 1.

Which one drops the fastest? As you increase omega over omega n beyond 1, there's a whole mess of curves to the right that blend together. And they differ only in damping. Can you tell which one is the-- let's say if you go to-- at three, there, the response is at 0.1 for the lowest curve on that curve, right? And that's the one with no damping. It's a little counter-intuitive, right? All right. Well, let's come back to it. Damping does help, but not at this point.

So we need to find a value of omega over omega n which is greater than 1 that satisfies this. That's what we're after. And this is kind of messy to work with. And since I know the one that works the best is the one with no damping, we'll solve the no damping one first, because it makes the algebra really easy. And then we can go back and say, now, what happens if you add some damping? So for the case there's no damping, the numerator goes to 1. The denominator goes to just 1 over 1 minus omega squared over omega n squared.

So it becomes that. That simple. And because I want to work with this ratio bigger than 1, I don't want this to be negative. And I want to mess with-- keep carrying along absolute value signs. This is the same thing as 1 over omega squared over omega n squared minus 1. I just reverse this, because I know we're going to deal only with the ones greater than 1 here.

And I need this to be equal to 0.1. And that's just algebra. You could solve that. This implies that omega over omega n equals root 11, I recall. And that is 3.31. So this is saying on that curve, if you go out to omega over omega n equals 3.31 right about where that arrow is, the curve for zero damping drops down to 0.1. And now if, at that frequency-- ah.

So that means we have to design the spring support such that omega n is equal to omega over 3.31. But omega-- where'd we start? So F equals 20 Hertz. Omega equals 2 pi f. Do I have that number here? No, but-- so this tells me that I need a natural frequency that is omega over 3.31, or I need an fn that is f over 3.31 is 20 Hertz over 3.31. And that number I do have. 6.04 Hertz.

So I need a support whose natural frequency is 20 Hertz divided by 3.31. I need a support whose natural frequency is 6.04 Hertz. And that's how you go about designing a flexible base to isolate something from vibration of whatever it's sitting on. All right.

So my f here, 20 Hertz. But my fn needs to be 6.04 Hertz. That implies multiply by 2 pi. I'm looking for 37.96 radians per second. And that's equal to square root of k/M. So now what's the M? Well, it's whatever the mass of the microscope plus its base. Whatever is being supported by the springs will have that mass. You have to choose the k.

So let's say that M total for this system is 20 kilograms. Solve this equation for k. And that implies that k is 28,827 Newtons per meter. OK?

So if we were to design this system-- and it really mounts up to in the case of this. Let's see. Beams. The stiffness of a beam-- ah, that's a good. We'll do this. We have a cantilever here. And we've got a mass on the end. But most of you have been taking 2001. If you put a force out here, P, what's the deflection at the end of a cantilever?

AUDIENCE: [INAUDIBLE].

PROFESSOR: OK. So delta is PL cubed over 3EI. And the load, this force, is equal to some k equivalent times delta, right? This is just a spring. And k times the displacement is the force it takes to do it. So P's my force. The spring constant is somehow associated with the rest of this stuff. So if I solve for P over delta, I get 3EI over L cubed. OK?

So if I'm running right at the natural frequency here and I want to reduce this to a 1/10th of its motion, I need to change the spring constant of this cantilever by a factor of-- well, I need to change the natural frequency by a factor of 3.31. So my k equivalent here is 3EI over L cubed. And that's what would go into this equation.

But I know that I have a natural frequency right now. I want it to go down by a factor of 3.31. So that means I need to decrease k such that the square root of k goes down by the factor 3.31. So how much do I have to change the length? Probably something like the square root of 3.31. Roughly 2.

So if I double the length of this thing, do you think it's going to work? If I double the length of this thing and turn it back on, then we shouldn't see much motion out of this.

[VIBRATING]

That's moving a lot. It's moving a tiny, tiny bit. So it works. So that's one step of vibration isolation.

Now I'm going to show you a vibration engineer trick, which is a very handy thing to know. Where's my strong magnet here? So I've got another beam just like this one. I've got a pretty massive magnet on it. So it makes another cantilever beam just like I got over there. OK?

So I claim that with just a ruler, if I clamp this down at some length, I claim, with just a ruler, I can predict the natural frequency of that. Take a couple of minutes and see if you could figure out how to do it. Think about that. Just a ruler. Measurements that I can make. I don't know how long it actually is. I don't know how thick it is. I know it's steel, but you just don't have enough information to compute 3EI over L cubed. But simply with a ruler, I'm going to be able to do this. Talk about it. Think about that while I set up the experiment. OK.

Who's got it figured out? Anybody want to take a shot at this? So there's my beam. I put the weight on it. What does the beam do statically? Bends a little, right? kx static equals Mg, right? Has to. So x static is what I'm calling delta here. So k delta equals Mg. k equals Mg over delta.

Natural frequency equals square root of k/M. Incredibly simple, huh? So what's the experiment that I would-- what measurement would I make? Delta, right? Put my ruler up there. I measure its static position like that. Then I put my mass on it, and I measure the static position again. I measure the delta. And I get a prediction. And I did this in my office.

And the delta that I measured-- I actually set it at a particular length. It was 18 centimeters. Delta measured, I think, 0.5 centimeters, or 0.005 meters. And if you compute omega n then equals the square root of 9.81 over 0.005.

And I want this in Hertz. So I can divide by 2 pi. This comes out as 7.05 Hertz. And Fn measured was 6.57. Pretty good but not perfect. And it's because I've made an approximation that I glossed over pretty quickly.

What has been left out of this system that would cause the measured natural frequency to be lower than the predicted? What's been ignored? Yes.

AUDIENCE: Damping.

PROFESSOR: Damping. Ah. Maybe. How much damping do we have in this system? Probably at least 10 cycles to the k halfway, right? Certainly less than 1%. The damped natural frequency is equal to the natural frequency of the square root of 1 minus theta squared. So this is something like way less than half a percent difference. So that wouldn't account for it. That's considerably more than half a percent. So damping couldn't do it. Yeah.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Ah, the mass of the bar. Does this flexure have mass? Yeah. It's probably on the order of if you stack them all up and compared to that, it might even be as much as half the mass of the end. And as it vibrates back and forth, does it have kinetic energy? Yeah. We've ignored the kinetic energy of the mass.

And in fact, that's the principal error here. We've left out the mass. There's actually a pretty simple way to-- using energy and just thinking in Lagrange terms, you can account for the energy of the mass in this single degree of freedom system and get a very accurate prediction. We won't do that today. But I think we'll do that before the term's out. OK.

This applies to any simple mass spring system in the presence of gravity. So here's a mass. And actually, we're doing the problem today where I'm moving the base. So here's its base. So this is the table moving. And if I do this, it clearly makes that move. If I do this really fast, it doesn't move very much. If I do it close to the natural frequency, it moves a lot.

If I move it very slowly, as I go up one unit, this follows me exactly. That's why that plot goes to 1. At very, very low frequency, the support and the mass move exactly together. At very high frequency-- if I can stop the transient-- I can't do it very well. The mass doesn't move much. The base moves a lot. And at resonance, it goes nuts. OK.

The unstretched length of this spring is about seven inches. The square root of g over delta, I ought to be able to predict this. So I did a quick calculation on that. It was like 1% error. I measured it at 7.36 radians a second. And I predicted it at 7.43 measured 7.36. Same kind of thing-- ignoring the mass of the spring a little bit. So g over delta is a great little thing to remember. OK.

So we have done all but one. Everything we've started out with today, we've said there's three ways to fix this, and came up with a fourth way. So in this case, soften the spring support a lot, so that the natural frequency is way less than the excitation. We said, what about spring supporting, softening, flexibly amounting this source, so that it doesn't put vibration on the table? That's the piece we haven't addressed. So let's look into that problem now.

So here's our source, some rotating mass eccentricity causing an excitation. So this has a force F0 e to the i omega t, which is coming from the rotating mass. And it applies to the floor, through the dashpot in the springs, some FT, I'll call it, F transmitted to the floor, e to the i omega t. And I want to know-- I need the H force transmitted per unit force input transfer function. That's what I'm looking for.

So now free-body diagram. Now we're going to make an assumption. We're going to assume that the motion of the floor, which we'll call y of t, assume that y is much, much less than x. It's generally true. Whatever's shaking like crazy, the table's not moving much underneath it. So I'm going to assume, for the purposes of calculating forces, that this is 0.

So for the motion x, what is the force applied to the floor? So F of t. If you have a positive displacement x, the force is kx. You have a positive velocity x, the force pulling up on the floor through the dashpot is cx dot. So the other way of saying that is here's our free-body diagram. Here's our F0 e to the i omega t pulling up. It responds at some x. And the resulting forces through the spring and the dashpot we know are kx and cx dot opposing the motion x.

Well, by third law, if these are the forces on the spring and the dashpot, then down here on the floor, you better have some equal and opposite forces, kx and cx dot. So this force produces a motion x. The motion x produces forces in the mass in the spring, which make the force on the floor, the spring force, and the dashpot force. OK.

So this Ft is-- I want to write it here. That's all that is-- positive. And I'm going to assume a solution that we know to work for x, which is xe to the i omega t. We've plugged it in before. So I plug that in here, I get a k plus i omega c, xe to the i omega t. So I can just express my force on the floor in terms of the motion x.

And I'm looking for a transfer function for force transmitted over force in. But force transmitted is my k plus i omega c, xe to the i omega t. And the force in is F0 e to the i omega t. Cancel out the time-dependent part. And it says the transmitted force over the input force is this little complex expression times the response x over F. But we know what that is. That's the transfer function Hx/F. So this is k plus i omega c times Hx/F of omega. So this gives us a slightly different transfer function.

Ooh, look at this. Before, when we did x/y, we ended up with k plus i omega c y e to the i omega t. And when we did then x/y, we got the same ratio as this. Exactly the same thing. So I could write all this out, but-- and let's say I'll do this. Hx/y-- no, no, I won't do that. What I'm going to tell you-- if you just work through this now, you will find that H force transmitted over force in is exactly the same as Hx/y.

And that what we really care about is what the magnitude is. So the magnitude of these two things are the same. And in fact, just work out to that same expression as before, the 1 plus 2 zeta omega over omega n squared square root all over the usual big denominator.

So conveniently, for vibration isolation, the solution to the two problems are exactly the same. So if you have that one, you have the transfer function xy that was projected on the screen a minute ago, it is also the force transmitted to force in transfer function. So you just have to remember one. And if you now want to-- we said, let's say, doubling the length of this just about accomplished the reducing the vibration of the microscope by this factor of 10.

So if I doubled the length of this one, I would roughly do the same thing. I would change the natural-- this thing is right on the natural frequency of this beam. That's why it shakes so much. And so it is this system. It's shaking like crazy, putting force into the table. The table is vibrating, causing the other one to move.

So now if I change this one, then the same kind of idea. Maybe roughly double its length. Natural frequency diminishes by a factor of 3 or so. The vibration of this ought to go way down. And actually, our little beam out here is picking up more than the other one. Shh. So this thing is hardly moving at all now.

So by doing that, we've essentially detuned it. This is no longer running at the natural frequency of this base. So it's no longer resonant. You're way out on the curve to the right. So the response of this isn't very much. That means it doesn't transmit much force to the base, maybe down by a factor of 8 or 10. That means the table vibration amplitude drops by that factor.

Means that the base motion over here is now a factor of 10 smaller than it was to begin with, so that we get a reduction of 10 here. And we get another reduction of 10 here, because we detuned it. So you might get a factor of 100 reduction by working on both, you see. You've treated the source and you've treated the receiver. But fortunately, they use the same curve.

So damping. When you do vibration isolation, you're trying to get well out on this curve to the right. So there's a couple of practical engineering things that limit how far you can go. To get further out on the curve to the right, what do you have to do to the spring in the system to get stronger or softer?

You're trying to make the natural frequency-- see, the excitation frequency doesn't change. In order to get omega over omega n to go bigger and bigger, the excitation's staying the same. You're having to reduce the natural frequency. And so what do you have to do to the spring constant? Decrease it. What is the practical limit of decreasing the spring that supports your pump, or your washing machine, or your air conditioner?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pretty soon it's just going to-- if it's too heavy-- you put it on there, it's just going to squash the springs, right? So you can't-- there's limits to how soft you can make springs to support heavy machines. So there is a practical limit to how far to the right you can go. But normally, you get out there as far as you can.

And then if the real system has damping, does it improve or degrade the performance of your vibration isolation system? Well, the more the damping you have, the higher up you are on those curves. So the damping decreases the performance. But every system has to-- when you first turn on that motor, the system has to spin up.

And you're going to have to go through that resonance, so that you want some damping. Because if you've got your scanning electron microscope or your laser interferometry system set up on a spring-supported table, if that table has no damping and you walk in the door and bump it, it is going to sit there and vibrate all afternoon at its natural frequency due to the initial conditions.

So you need some damping to prevent problems, either response to initial conditions, or bumping it, or whatever. Or even as the system turns on and speeds up, it'll have to go through that resonance. And it'll vibrate like crazy as it does, and then finally settle down at the higher frequency. So you need some damping. But damping does degrade the steady state performance. And I'm out of time. And we'll see you in recitation. Thanks.