PROFESSOR: Let's get on with some dynamics. So the place I'm going to begin is just a comment about mechanical engineering courses. The first, and you may have heard this already in classes, you'll be taking subject 2001 if you're Course 2 majors through 2009, and if you're 2-A, most of the odd ones. But the subjects 2001 through 2005 are really basically engineering science subjects that are all foundational to mechanical engineering, and they all have a common or property through them. And that is that we make observations of the world, and we try to understand them. We pose problems.

Why-- 400 years ago, is the sun in the center of the solar system or not? And we try to produce models that explain the problem. So here's the problem, the question of the day. We try to produce models to describe it, and we make observations, measurements, to see if our models are correct. And if we feed that information back into the models, we try out the models, we test it against more observations, and you go round and round. And this is kind of the fundamental-- this is the way all of these basic first five subjects use, basically, this method of inquiry.

So in 2003, the way this system works, my kind of mental conception of this modeling process, is three things. And this applies to you. You have a homework problem. How do you attack a homework problem? You're going to need to describe the motion. You're going to need to choose the physical laws-- pick, I'll call it because it's short-- the physical law that you want to apply like f equals ma, conservation of energy, conservation of momentum. You got to know which physical laws to apply.

And then finally, third you need to apply the correct math. And that's really-- most dynamic problems can be broken down this way. That's the way I like to conceptually break them down. You might have another model, but this is the way I'm going to teach it.

Can you describe the motion, pick the correct physical laws to apply to the problem, and able to do the correct math, solving the equation of motion, for example. And all this is what fits in our models box. And we test it against observations and measurements and improve those things over time.

So I'm going to give you-- how many of you like history? I find history and history technology kind of fun and interesting. So I'm going to throw a little bit of history into giving you a little quick course outline of what we're going to do in this subject this term.

Because the history dynamics and what we're going to do in the course actually track one another remarkably closely. So if I ever gave you a bunch of names like Galileo, Kepler, Descartes, Newton, Copernicus, Euler, Lagrange and Brahe, which one comes first? Take a guess.

AUDIENCE: Copernicus.

PROFESSOR: Good. Copernicus.

So Copernicus was Polish, and the story starts long before then, but in about 1,500 Copernicus said what?

AUDIENCE: [INAUDIBLE]

PROFESSOR: The sun's the center?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Or the Earth is the center?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Which did say? Yes, so Ptolemy, back around 130 AD said, well the Earth's the center of the solar system. Copernicus came along and said, nope I think that, in fact, the sun's the center of the solar system. And it for the next 100 years-- more than 100 years, couple hundred years-- there was a really raging controversy about that.

So Copernicus, Brahe Kepler-- so I'm putting them in rough chronological order here. Now, I'm going to run out of board. Oh well. Galileo, Descartes-- I'm gonna cheat-- OK, Descartes, Newton, Euler, and Lagrange. So we're going to talk and say a little bit about each of them. And now that I'm-- like I told you, I haven't used this classroom before so I gotta learn how to play this game. I need to be able to reach this for a minute.

So Brahe, he was along about 1,600. Brahe was the mathematician that wrote-- the imperial mathematician to the emperor in Prague. And he did 20 years of observations. And he was out to prove that the Earth was the center of the solar system.

And then Kepler actually worked with him as a mathematician, and then took over as the imperial mathematician. And he took Brahe's data-- 20 years of astronomical data without the use of the telescope-- and used it come up with the three laws of planetary motion. And so his first and second laws were put out about 1609.

And one of the laws is, like, equal area swept out in equal time. Have you hear that one? That actually turns out to be a statement of conservation of angular momentum, which we'll talk quite a bit about the course.

Then came Galileo, and I'm not putting their birth and death dates here. I'm kind of putting in a period of time in which kind of important things happened around him. So 401 years ago a really important thing happened. Galileo, in 1609, turned the telescope on Jupiter, and saw what?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Four moons, right? And then they really started having some data with which to really argue against the Ptolymaic view of the solar system.

Descartes is an important figure to us. And in the period of about 1630 to 1644-- in that period Descartes began what is today known as analytic geometry. He was geometer, he studied Euclid a lot. But then he came up with a Cartesian coordinate system, xyz, and the beginnings of analytic geometry, which is essentially algebra, coordinates, and geometry all put together. And we are going to make great use of analytic geometry in this course.

Then came Newton, kind of in his actual lifespan, 1643. It's kind of interesting that he spans these people. And in about 1666 is when he first-- the first statement of the three laws of motion.

Then Euler, and he's 1707 to 1783, and that's his lifespan. Euler came up-- Newton never talked about angular momentum. He mostly talked about particles. Euler put Newton's three laws into mathematics. Euler taught us about angular momentum, and torque being dh dt in most cases. He's the most prolific mathematician all time, solved all sorts of important problems.

And then finally, is Lagrange. And Lagrange, in about 1788, uses an energy method, energy and the concept of work to give us equations of motion.

So the course, 203, stands on the shoulders of all these people. But with Descartes, we start with kinematics, really. This is analytic geometry. And that's where we're going to start today is with kinematics.

And very soon thereafter, we're going to review Newton, the three laws, and what we call the direct method for finding equations of motion. Conservation of momentum, fact that force-- some of the forces on an object equals mass times acceleration, or it's a time derivative of its linear momentum. And we use that to derive equations of motion.

So we're going to go kinematics into doing the direct method to getting equations of motion. And we go from there into angular momentum, and what Euler gave us-- the same thing, torque. We're going to do quite a lot with angular momentum.

Because I know you know a lot about f equals ma and you've done lots of problems

801 applying that. You've done some problems on rigid body rotations. But I think there's a lot more you need to understand about this, and we'll spend quite a bit of time on it.

And then near the last third the course we shift, because Lagrange said that if you just write down expressions for energy, kinetic and potential energy, without any consideration of Newton's laws and the direct method, you can derive the equations of motion. That's pretty remarkable. So there are actually two independent roots to coming up with equations of motion.

And in this course, about the last third of the course, we're going to teach you about Lagrange. And then all these things are going to be-- one of the applications that are important engineers is the study of vibration. So we'll be looking at vibration examples as we go through the course, and applying these different methods to first, modeling, and then solving interesting vibration problems.

Which brings-- ah, I have a question for you. So how many of you were in this classroom last May with Professor Haynes Miller, and I showed up one day and we talked about vibration? How many remember? I told you I was going to ask this question, right? Great. OK, it's good to see you here again, and we will talk about vibration in this course.

So there's kind of the subject outline built on the shoulders of these people in history that made important contributions to dynamics. Any questions about the history? If you want to know, one of my TAs compiled a pretty neat little summary.

Maybe I will see if I go back and find this. I just printed out and sent it-- how many of you like to know a little bit more about the history? These are like two liners on each person. Anybody want it? Is it worth my time to send this out? OK, it's kind of fun.

So let's do an example of this modeling describing the motion, picking physical laws, applying the math. And that'll get us launched in the course. And we'll do it using Newton and the direct method.

So last May, Haynes Miller and I talked about vibration. So I'm going to start with a vibration problem. And I brought one. So here's my couple of lead weights and a couple of springs. So really I just want to talk about-- this is the problem I want to talk about.

Now you've done this problem before. Haynes Miller and I did it last May. And you've no doubt it in other classes. OK, it's a system which has a spring, a mass, it exhibits something called a natural frequency. But let's see what it takes to just initially begin to follow this modeling method to arrive at an equation of motion for this problem.

So what do I mean by when I say, describe the motion? Really what that boils down to if you have to assign a coordinate system so that you can actually say where the object's moving. And I'm going to pick one here. So here's-- coordinate system going to be really important in this course. And I'll give us an xyz Cartesian coordinate system.

And I'm going to try to adopt the habit, for the most part, during the course that this o marks this origin, but it also names the frame. So we're going to talk about things in that are reference frames. And most important one that we need to know about in the course is an inertial reference frame, and when you can use it, and when a system is inertial and is not.

So I'm gonna say that this is inertial. It's fixed to the Earth. It's not moving. And we're going to use this coordinate x to describe the motion of this mass. And the motion is going to be-- this x is from the zero spring force position.

It's actually quite important that you pick-- that you have to say what's the condition in the spring of the system when x is 0 So we're going to say it's, when there's no force in the spring means it's not stretch, that's where 0 is. So we've established a coordinate system.

Second, we need to apply physical laws. Now, I'm going to do this problem by f equals ma, Newton's second law. Sum of the external forces is equal to mass times the acceleration. So that's the law I'm going to apply.

Sum of the external forces, it's a vector but we're just doing the x component only so we don't have to carry along vector notation, is equal to, in this case, mass times acceleration. So that's the law we're going to apply.

And then finally the math to solve the equation of motion that we find, that'll be the third piece. But part of applying the physics, in order to do this now, we need what I call an FBD. What do you suppose that is?

AUDIENCE: Free body diagrams.

PROFESSOR: Free body diagrams. You've used these many times before, so we're going to do those. And free body diagrams--

And I'm going to teach you, at least the way I go about doing free body diagrams, as things get more and more complicated, you're going to have to be more sophisticated in the way that you do these things.

So I just have some simple little rules to do free body diagrams that keep you from getting hung up on sign conventions. I think the thing people make most mistakes about is they get confused about signs.

So I'll try to show you how I do it. So first you draw forces that you know, basically in the direction in which they act. Seems obvious. So when you know the direction-- so this is a really trivial problem, but the method here is very specific.

So what's an example? Well, gravity. So we'll start our free body diagram. Gravity acts at the center of mass. It's downward. This is what I mean by the direction in which it acts. And it has magnitude, mg. OK.

Now the other forces aren't so obvious. The force that's put on by the stiffness and this damper in the spring, which way do you draw them? What's the sign? What's the sign convention?

So the convention, the way I go about doing these things, is I assume positive values for the deflections and velocities. So in this case, x and x dot. You just require that the deflections that you're going to work with are positive.

And then from the positive deflection, you say which way is the resulting force? So if the deflection in this is downwards, which direction is the force that the spring applies to the mass? Up, right?

What about if the velocity is downwards, which direction is the force is the damper puts on the mass? Also up, right? OK. So this allows-- this gives us-- so here's f spring and here's the f damper. And other any other forces on this mass? So spring force, damper force, and the gravitational force.

And so third, you deduce the signs basically from the direction of the arrows. First we need what's called your constitutive relationship. So the spring force, fs, well you've made x positive so it keeps things nice, the spring constant's a positive number, so fs is kx. Fd is bx dot.

And now we write the statement that the sum of forces in the x direction. We look at up here, we say well that's going to fs plus fd minus mg. So that's-- whoops, I wrote it the wrong way around. Minus, minus, plus. Because I'm plus downwards, right?

Well, spring minus fs is minus kx minus bx dot plus mg equals mx double dot. And I rearranged this to put all the motion variables on one side. mx double dot plus bx dot plus kx equals mg. So there's my equation of motion, but with a method for doing the free body diagrams, which will work with multiple bodies.

So you have two bodies with springs in between them. This is when the confusion really comes up. Two bodies with a spring trapped between them. What's the sign convention? You do the same thing. Both bodies exhibit positive motions, the force that results is proportional to the difference, and you work it out. And you'll get the signs right.

OK, so here's our equation of motion arrived at by doing the direct method. And if we went on to the third step, which we're not going to do today, and that is apply the math, it might because I want you now to describe the motion for me, solve for the motion. That means solving the differential equation. And that's what we did last may in Haynes Miller's class. We'll come back to this later on.

But for today's purposes, we don't need to go there. Got something else much more important to get to about kinematics. But I want to show you one thing, and that is just a little tiny introductory taste to this point.

So I've derived the equation of motion of this by Newton's laws. But I'm going to ignore Newton now and saw I'm going to drive equation of motion by another way. And it's an energy technique, and that is-- well let's talk about the total energy of the system. It's going to be the sum of a kinetic energy and a potential energy.

And we'll find that even with Lagrange, there's a problem with forces on systems that are what we call non-conservative, things that either take energy out of, or put energy into the system. And the dashpot does that. Dashpot generates heat and takes energy out of the system.

So I'm going to have to ignore it for the moment. So the sum of the kinetic and the potential energies in this problem is a 1/2 kx squared for the potential of the spring, plus a 1/2 mx dot squared for the kinetic energy of the mass, and minus mgx for the potential energy that is due to the object moving in the gravitational field.

And that's the total energy of the system. Now my problem, I've allowed no forces. There's no excitation on here. This is just free vibration only. That's all we're talking about, make initial displacement and it vibrates. If there's no damping, what can you say about the total energy of the system?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Say it again. I heard it over there. It's got to be constant, right? All right, well, so this must be constant. Therefore, the time derivative of my system, it better be 0. The energy is constant. Take it's time derivative, it's got to be 0. Apply that to the right-hand side of this, I get kxx dot plus mx dot x double dot minus mgx dot equals zero.

And I now cancel out the common x dot terms go away. And I'm left with-- and I've essentially solved for the equation of motion of this system without ever looking at conservational momentum, Newton's laws, only by energy considerations.

OK, so that's a very simple example of that you can use energy to derive equations of motions. But you then have to go back and fix it to account for the loss term, the damping term. And that you still have to consider it as a force, we'll find out. Even was Lagrange you have to go back and consider the work done by external forces.

OK. So you've just kind of seen the whole course. We've described the motion, we've applied to Newton's laws, the physics to the direct method to derive the equations of motion, we have gone to a direct method, and have derived the equations of motion that way.

And that's basically what you're going to do in the course. But now you're going to do it with much more sophisticated tools. You'll have multiple degree of freedom systems. The description describing the motion, is maybe going to be for some of you, the most challenging part of the course. And this is a topic we call kinematics. And that's what we'll turn to next.

So reference frames and vectors. That's the topic. This is now that we're talking about kinematics, and this is all about describing the motion. So Descartes gave us the Cartesian coordinate system, and we'll start there. So imagine this is a fixed frame-- we'll talk about what makes an inertial frame the next lecture.

But here we have an inertial frame. And it's the frame we'll call O-xyz or O for short. And in this frame, maybe this is me, and up here is a dog, and I'm going to call this point A and this point B. And I'm going to describe the positions of these two points by vectors.

This one will be R, and the notation that I'm going to use is point and it's measurement with respect something. Well, it's with respect to this point O in this inertial frame. So this is A with respect O is the way to read this.

There's another vector here. This is RB respect to A And finally, R of B with respect to O They're all vectors on the board. I'll try to remember to underline them in the textbooks and things. They're usually-- vectors are noted with bold letters.

And vectors allow us to say the following. That R, the position of the dog and the reference with respect to O, is the sum of these other two vectors. R of A with respect to O plus R R of B with respect to A.

And mostly to do dynamics we're really interested in things like velocities and accelerations. So to get the velocities and accelerations, we have to take a time derivative of our RBO dt. And that's going to give us what we'll call the velocity, obviously you write it as V. And it would be the velocity of point B with respect to O. And no surprise, it'll be the velocity of point A plus the velocity of B with respect to A.

And finally, if we take two derivatives, dt squared, we'll get the acceleration of B with respect to O. And that'll be the sum of A-- the acceleration of A with respect to O plus the acceleration of B with respect to A. All, again, vectors.

Now, just to look ahead-- this seems all really trivial. You guys are going to sleep on me, right? If these are rigid bodies, this is a rigid body that is moving and maybe rotating. And B is on it, and A is on it, and O isn't on it. It starts getting a little tricky.

And this, the derivative of a vector that's attached to the body somehow has to account for the fact that if I'm-- the observer's on the body, this other point's on the body. Say it's, I'm on this asteroid, and I've got a dog out there, and the dog's run away from me. The speed of the dog with respect to me, I can measure. But if I'm down here looking at it, it'll look different because it's rotating. So how do you account for all that?

So taking these derivatives of vectors in moving frames is where the devil's in the details. And that's part of what I'm going to be teaching you. OK. I'm still learning how to optimize my board use. I haven't got it perfect yet, but because I'm having to move around a lot here and improvise. But we'll persevere.

You need to remember a couple things about vectors, how to add them, dot products. If you've forgotten these things, you need to go back and review them really quickly. There's usually a little review section the book, so you need to practice that sort of thing.

Couple other little facts you need to remember. So the derivative of the sum of two vectors is just the sum of the derivatives. And quite importantly, we're going to make use of this one a lot, is the derivative of a product of two things.

One of them be in a vector, some function maybe of time and a here is derivative of f with respect to t times a, plus the derivative of a with respect to t times f. That we'll make a lot use of. So just your basic calculus.

So now, I want to take up-- let's talk about the simplest form of being able to do these derivatives and calculate these velocities, when everything's described in terms of Cartesian coordinates. Now I'm going to give you a little look ahead because I'm going to try to avoid confusion as much as possible here.

The hardest problem is when you have a rigid body, you got the dog on it, you've got the observer on it, it's rotating, and translating. And to take this derivative, you end up with a number of terms. The simplest problem is just something in a fixed Cartesian coordinate system. So we're going to start with a simple one, and build our way up to the complicated one, OK?

But let's now, we're going to do the really, the simplest one. We're going to do velocity and acceleration in Cartesian coordinates. And basically I should say fixed Cartesian coordinates, not moving.

All right, so now let's consider the dog out here, and his position in the Cartesian coordinate system. And I could write that and you'll, without any loss of generality here, you'll know what I mean if I say RBx component. And I'm going to stop writing the slash O's, because this is now all in this fixed reference frame. And it's in I-hat direction. And I've got another component, RBy in the J-hat, and an RBz in the K-hat.

And I want to take the time derivative-- I was looking for the velocity. I want to calculate the velocity. So the velocity here of BNO is d by dt of RBO. . And now this is now the product of two things, so I've got to use that formula over here. Product one turn times the other, and so forth.

So I go to these, and I say OK, so this is R dot Bx times I plus R dot By times J plus R dot Bz times K. And then the other-- the flip side of that is I have to take the derivatives of I times RBx, the derivative J and so forth.

But what's the derivative of, let's say, I? Capital I is my unit vector in the fixed reference frame, my O-xyz frame. 0 So it's a constant. It is unit length, and it points in a direction that it's fixed. So what's its derivative? It's going to have a 0 derivative. So the second part of this-- second bits of that is zero. So that's the velocity in Cartesian coordinates of my dog out there running around.

And the acceleration, in a similar way, now to get the acceleration, you take another derivative of this. And again, you'll have to take derivatives of I, J, and K, and again they're going to be 0. So you will find that the acceleration then, is just R double dot x term in the plus R double dot By in the J plus r double dot Bz in the K. That would be our acceleration term, and it's easy.

Now imagine that we are doing this in polar coordinates, unit vectors in polar coordinates. Let me check, last year the students told me that in your physics courses, you use unit vectors R-hat, theta-hat, and K. Is that right? So I'll use those unit vectors so they look familiar, because in polar coordinates people use lots of different things.

But think about it, in polar coordinates, theta-- it's a fixed, maybe, coordinate system, but now theta goes like this and R moves with theta, right? So the unit vector is pointing here, but over time it might move down to here.

And unit vector has changed direction, and its derivative in time is no longer 0. So it starts getting messy as soon as the unit vectors change in time. And so that's one of our objectives here is to get to that point and describe how you handle those cases.

So a quick point about velocity. You need to really understand what we mean by velocity. So here's our Cartesian system. Here's this point out here B. And now, this is the dog running around, and the path of the dog might have been like this.

And right in here he's going this direction. And in a little time, in delta t, he moves by an amount delta RB with respect to O. And that's what this is. He's moved this little bit in time delta t. And he happens to be going off in that direction.

So this then is R prime, I'll call it, of B with respect to O, and this is our original RB with respect to O. So we can say that his new position, RB with respect to prime is RBO plus delta R. And these are all vectors. And the velocity of B with respect to O is just equal to this limit of delta RBO over delta t as t goes to 0.

So what direction is the velocity? The velocity is in the direction of the change, not the original vector, it was in the direction of the change. And in fact, if the path of the dog is like this, at the instant you compute the velocity, you're computing the tangent to the path of the dog. So that's what velocity is at any instant time is a tangent to the path. And that's a good concept to remember.

So we're still in this fixed Cartesian space, and I have of couple of points. I'll make it really trivial here. Here's B, and here's A, and the velocity of B-- where's my number? We'll make this 10 feet per second. And it's in the J-hat direction. And A, this is the velocity of BNO. The velocity of ANO, we'll say is 4 feet per second, also in the J direction.

And I want to know what's the velocity of B with respect to A. So now I'm chasing the dog, he's running at 10, I'm running at 4. How do I perceive the speed of the dog? Well, to do this in vectors, which is the point of the exercise here, is we have the expressions we started with over there. And we're going to use these a lot in the course.

So the velocity of B with respect to O is the velocity of A with respect to O plus the velocity of B with respect to A. And if I want to know velocity of B with respect to A, I just solve this. So velocity of B with respect to O minus the velocity of A with respect to O, and in this case that's 10 minus 4 is 6 in the J.

Point of the exercise is to manipulate the vector expressions like this. So take whatever known quantities you have and solve for the unknown one. In this case, I want to know the relative velocity between the two, and it's this.

If I'm here, and I'm watching the dog, that's how I perceive the speed of the dog relative to me, right? 6 feet per second in the J direction. What's the speed of the dog from the point of view of over here? The speed of the dog relative to me. So it's again the velocity of B with respect to A, but from a different position in this fixed reference frame.

Really important point, actually. This is a really important conceptual point. Somebody be bold. What's the speed with respect to O? The velocity of B with respect to A seen from O, as computed from O, measured from O. Got radar down there, and you're tracking them.

AUDIENCE: [INAUDIBLE]

PROFESSOR: In what direction?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah. It's the same. The point is it's the same. If you're in a fixed reference frame, a vector of velocity is the same as seen from any point in the frame. Any fixed point in the frame of velocity is always the same.

And in fact, in this case, the velocity-- this is a moving point and the velocity of him with respect to me this is different six feet per second. And I, from here, say the velocity of that guy with respect to this guy is still 6 feet per second.

Any place in that frame or even any point moving at constant velocity, you're going to see the same answer. So it doesn't matter where you are to compute the velocity of B with respect to A. That's the important point. OK.

OK, we got to pick up with, and I may not quite finish, but I am going to introduce the next complexity. OK. So what we just arrived at a minute ago is that the velocity as seen from O is the same as the velocity as seen from A. And A is me, and I'm moving, and I'm chasing the dog. So I'm a moving reference frame, I'm what's called a translating reference frame.

So now we're going to take the next step. We had a fixed reference frame before purely, and now I want to talk about having the idea, the concept of having a moving reference frame within a fixed one.

So this is the reference frame O capital XYZ. And this little reference frame now is attached to me, and it's A, and I call it x-prime y-prime. So just so you can-- it's going to be hard to tell this X from this X if I don't do something like a prime.

So that this is the concept of a translating coordinate system attached to a body, like a rigid body, for example. We're going to do lots of rigid body dynamics here. And within this coordinate system, I can compute the velocity of B with respect to A, and I'll get exactly the same answer. I'll get that 6 feet per second in the J direction.

So it's as if-- so this concept of being able to have a reference frame attached to a body and translating with it, you can measure things within it, get the answer, and then convert that answer to here if you're using a different coordinate. You could use polar coordinates here and rectangular here, but they still can be related to one another. We'll do problems like that.

OK. So now what I'm doing is I told you like in the readings, the end game is to be able to talk about translating and rotating bodies, and do dynamics in three dimensions with translating and rotating objects. And we're going to get there somewhat step by step. But I want you to understand the end game so you know where we're going. And you need to have a couple of concepts in mind.

So the first concept is that this is a rigid body now. And you can describe the motion of rigid bodies by the summation, the combination of a translation and a rotation. And of the rigid body, if you can describe its translation, and you can describe its rotation, you have the complete motion.

So you got to understand what do we mean by what's really the definition of translation. So translation-- so I've got this-- I'll call it a merry-go-round. We'll use a merry-go-round example in a minute. And you're observers in a fixed inertial frame up above this merry-go-round looking down. OK. But so you can see it, I got to turn it on its side.

So here's my merry-go-round. And if it's not rotating, but let's say it's sitting on a train, on a flat bed and moving along. It's translating. And when you say a body translates, any two points on the body move in parallel paths.

So two points, my thumb and my finger-- if I'm just going along with this, those two paths are traveling parallel to one another. If I got Y pointing up, the body does this, is it rotating and translating?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Are any two points on a moving in parallel paths? Right? OK. When it goes through curved things, it's called curvilinear translation. But it's still just translation. OK, so I'll stop and hold steady. The train stopped, and the thing-- let it rotate. So that's pure rotation.

And the thing to remember about pure rotation is that anywhere on the body rotates at the same rate. If this is going around once a second, the rotation rate is one rotation per second, 360 degrees, 2 pi radians per second is its rotation rate. Every point on the body experiences the same rotation rate. That's a really important one to remember.

If I'm holding still, merry-go-round's going round and round, it has a fixed axis of rotation, right? But do rotating bodies have to have fixed axes of rotation? So if I throw that up in the air, not hanging onto it, it's got gravity acting on it, it's rotating. What's a rotate about?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Center of mass, OK. Is the center of mass moving? So this is clearly-- this is an example of rotation plus translation. It rotates about an axis but the axis can move. That's another important concept that we have to allow in order to be able to do these problems. But this is now general motion, it's a combination of translation and rotation, and we figure out each of those two pieces, then we can describe the complete motion of the system.

All right, where we'll pick up next time is then doing that. And it would help actually, if you go read that reading, especially up to chapter 16, we have to get into to taking derivatives of vectors which are rotating, and come up with a general formula allows us to do velocities and accelerations under those conditions. See you on Tuesday next.