PROFESSOR: --get started. What do you think are some important new concepts that we've been talking about in the last week? Make a list. Betsy.

AUDIENCE: Various ways to calculate kinetic energy [INAUDIBLE].

PROFESSOR: OK. All right?

AUDIENCE: Virtual work.

PROFESSOR: Virtual work. Something else. [INAUDIBLE].

AUDIENCE: The Lagrange equation.

PROFESSOR: All right. [INAUDIBLE].

AUDIENCE: [INAUDIBLE] generalized coordinates.

PROFESSOR: Generalized coordinates. Anything else on your list?

AUDIENCE: Generalized forces.

PROFESSOR: OK. Lagrange equations. That's a pretty good list. Now, we've talked about quite a few new things lately. So is there any-- did you come in here today with any questions about something that just isn't sitting right on these topics or anything else? And I think write up a couple extra questions that we may be able to cover them as we go along. Steven?

AUDIENCE: So when you solve the entire thing, everything in general is coordinates, right? But say you want to find the actual numbers. Do you calculate them as your [INAUDIBLE], or do you just [INAUDIBLE]?

PROFESSOR: So this is really how to use the equations you end up with in practical situations? OK. I'm just going to answer this one. So when you choose your generalized coordinates, you'll probably choose about the same ones you'd choose if you did the direct method. So if you did direct method and got equations of motion, would you be asking the same question?

AUDIENCE: I don't know.

PROFESSOR: All right? OK. So really, you want to choose coordinates that are going to be practically useful in the end. OK? And so it shouldn't matter whether you're doing it by Lagrange or doing it by the direct method to get to them, you're going to use them in the same way in the end.

Now, we have two methods to get equations of motion now. When you're doing tough problems, complicated problems, I would always do it one way and use the other way to check it. I'd end up doing it both ways. If it was really important, you do it both ways. And one will give you insight about the other.

Yesterday in lecture, I was talking about, does it make sense to have a Coriolis term in this equation? Would you expect it? And that kind of common sense checking. All right. Any other questions? Good question. Generalized coordinates or forces or anything? All right. Let's get on. Let's keep working here.

So I have an assignment for you, and it's here. This is a familiar problem that you've worked before by other methods. And you did, in fact, even find the kinetic and potential energies, I think, last week for this. So here's a cart with a rod, uniform rod. No dashpot at the moment, and no external forces. I'm going to give you coordinates to use. So here's our inertial system, deflection of the cart x, rotation of the rod theta.

We're going to break you into groups of about-- we got a big group here-- 5, 10. Well, I want you to break into three groups, kind of like five or six there, five or six there, and the same thing here. And one group each-- so this group in the front left, compute the kinetic energy of this system. And the group behind them, compute the potential. And this group over here, come up with the velocity of G and the G dot VG, the velocity squared. OK?

And this won't you take long because you've done the stuff before. And then when you get done, somebody from your group, just when you're done, come up and write it down. All right. Let's sort these out.

So let's start with the velocity. We're going to need the velocity to be able to finish out the kinetic energy. So the vector, right, and they've broken it into two pieces. And since the others, if you haven't worked on it, are you guys--

STUDENT: Can you switch the sine and cosine?

PROFESSOR: I was about to ask you about that. I think that that's better. So this is sine, right.

STUDENT: No, that's cosine.

PROFESSOR: I just erased it. That's cosine, sine. This is your l theta dot piece and you want it this way. And that's theta, so it should be sine theta j, cosine theta i. I think that's good for the velocity. And then velocity squared, just the square reach of pieces.

So we have v dot b. We need to put this into the kinetic energy expression, but one at a time here. The potential energy expression, 1/2 kx squared. Everybody good with that? Looks like the spring potential energy.

And m2g, 1/2 l is a position of the center of mass. 1 minus cosine l that times 1 minus cosine theta is the amount that it changes height from the reference. What's the reference position? What have you assumed for the reference position in this formulation? Yeah?

STUDENT: [INAUDIBLE].

PROFESSOR: So your reference position is center of mass when it's hanging straight down. Great. So there's your potential energy expression. Kinetic energy expression-- everybody comfortable with that? I'm not totally comfortable with it.

Let's talk about a minute about kinetic energy and say, what can we use, what formulas can we use here for t? There's totally general and then we can narrow it down. What do you suggest?

STUDENT: For the [INAUDIBLE] you can-- you have to take into account that it's not spinning about its center of mass. So you use the general mass formula one half omega dot, its angular momentum [INAUDIBLE].

PROFESSOR: OK, so we have some general formulas. And let's take a step by step approach to this. We have how many rigid bodies? Two. And you compute the kinetic energy for each one individually. That's the safest way to go about this.

So I would stay away initially from doing this. You lumping the two together. So I would take the two individually. So if you take the main mass, the block on wheels, what is its velocity? Its velocity is this x dot, right? And its kinetic energy is?

STUDENT: [INAUDIBLE].

PROFESSOR: So for the first block, you have m1x dot squared. And you're done. That's the first block. Now you need the second rigid body. So the second rigid body, you could do-- the full, general expression is one half m2 vg in o dot vg in o plus a half omega H, with respect to g.

Good with that, Vicente? You need it transposed? Good. And can we simplify that at all? For example, is it that rod rotating about a fixed point? It's not. The point it rotates about moves, right?

So you can't say it's just-- you can't use, for example, parallel axis theorem and just say it's 1/2 I with respect to that point, theta dot squared. Won't work. Can't use that one. You will find, that if you work this out, you can say 1/2 I with respect to g omega squared, in this problem it will work out.

It'll come out to that because this is a planar motion problem and there's only one component of rotation. So this will work out to be a 1/2 I with respect zz of m2 with respect to g omega z squared, in this case theta dot squared.

That's what this term will reduce to. But don't assume it just out of the box. And then you need this term. And that's why we need bg. So you need to do that. Put that piece in over there. So we need to-- so what is Izz about g for m2? For a rod, slender rod.

So m2 l squared over 12. That's what you need to put in here. We have an expression for v over there. We know everything now. So now let's apply our Lagrange equations. And I'm going to need to rearrange the board here a little bit.

I'm going to need that board space. So our T is 1/2 m1 x1 dot squared, or x dot squared plus 1/2 m2 vg.vg. And now I can cover this up. All right.

So the next task is, let's work out, do our Lagrange equations work? So how many generalized coordinates do we have and what are they? x and theta, that we've chosen. Two degrees of freedom-- they're complete, independent, polynomic.

And we can use Lagrange equations. We're going to come up with two equations of motion. And we're going to apply this. That's the Lagrange equation. We're going to apply it twice, where l is defined as T minus v.

If you just plug in l into this expression and just expand it, you get-- instead of two terms, you get four terms, because you have these two guys. And this term-- for mechanical systems, what can you say about this term generally? That's the derivative of v with respect to Q dots to velocities. Why?

STUDENT: Conservative forces?

PROFESSOR: No, just that you find it for mechanical systems, springs and gravity, you will never find that the potential energy as a function of time or velocity just isn't-- and if it's not a function of velocity, you take a derivative with respect to velocity you get 0's. So this goes to 0 for mechanical systems.

An exception for non-mechanical would be like a charged particle in a magnetic field. Then the forces get involved with velocities and so forth. It gets messy. 0 for mechanical systems. So we really don't have to deal with three terms-- that one, that one, that one, and then on the right hand side are generalized forces.

So we can break into four smaller groups. Two groups are going to do the x equation. You have to take these Qj's is this problem are Q1 is x and Q2 is theta. So for the x equations, we need to do these derivatives. And for the theta equation, we need to do these computations.

So let's have one group A do these. Group B do these and group C do these and a D group do those. So break yourselves into four groups and we'll do A, B, C here in the center, group here, four or five, four or five, or group here the C group, and D group over here.

Do these calculations and let's get our two equations in motion. And when you get your stuff done, so the A group, when you finish, come up here and right here, write your stuff. And the B group, write your answer here, and the C group and the D group. As soon as you get it done, come up and put it down.

We got this term, this term, this term, and this term. Did you guys check? Did you guys get a little time, B group to check on the A group? Which one was it? Who's checking on whom? You're checking on-- what do you think?

STUDENT: We got the same answer.

PROFESSOR: So main mass acceleration, the second mass, its total acceleration, these pieces, and there's an acceleration that's Eulerian and then there's an acceleration that is-- what's this term related to? You expect it to come up?

And the kx term, and all these are going to equal to the generalized forces of any non-conservative forces. So you're OK with this one. Let's move on to this one, then. Who's the check group here? What do you think?

STUDENT: I think they made [INAUDIBLE].

PROFESSOR: Do you think there's a problem here?

STUDENT: There might be.

PROFESSOR: Can you give me an alternative?

STUDENT: It may be that their dt d theta there should be in the time derivative.

PROFESSOR: All right. So we need d, the derivative of this with respect to theta dot. The first term doesn't give me anything, the mx dot squared. The third term gives you-- should give you an Izz g theta double dot, eventually, right? So for sure, this d by dt, the partial of T with respect to theta dot.

So we just run through it. The first term gives us nothing. The third piece gives us, with respect to theta dot Izz, theta dot. And no one wipes out the 1/2. The time derivative makes it theta double dot. So the third term's going to be an Izz g theta double dot.

And it's the second term that needs a derivative of this with respect to theta dot. So both terms are going to yield some stuff, right? A lot of stuff. All right, I'm going to write down how this should work it out, rather than try to grind it out real time here. Izz

All right. These are the terms that should appear. This is the piece about g. This is the-- no, that's not quite right. That's the piece about g. This should be about-- and then this term.

That's how it actually should check out. Do you agree with me? Looks OK? And then if we add to that the m2 g l over 2 sine theta, which is our gravitational potential energy, all of that added together ought to be equal to q theta.

So let's move on to looking at the generalized forces for this problem. So don't know where you guys went wrong on this. But if you have any questions-- we can talk about this for a minute.

STUDENT: [INAUDIBLE].

PROFESSOR: d t d theta? So you got to take the partial derivative of this expression with respect to theta dot. This piece gives you a contribution that will be Izz theta dot. And you take its time derivative. So that's pretty obvious why it gives you the first piece.

STUDENT: I think the problem is understanding in the first place that they did [INAUDIBLE].

PROFESSOR: Oh.

STUDENT: [INAUDIBLE].

PROFESSOR: Then the second one, when you take the derivative of this expression with respect to-- this is T with respect to theta dot of this part, this expression, you get 2 times what's inside times the derivative of the inside with respect to theta dot.

And that'll give you another L over 2 cosine theta. And I think you're done. This times this stuff, right? 2 times this times the derivative of the inside, which is-- the derivative of the inside with respect to theta dot should be L over 2 cosine theta.

So you get an x dot plus L over 2 theta dot cosine theta 2 times that times the derivative of the inside with respect to theta dot. This is the only term that contributes is that. And then we've already done the derivative of this with respect to theta dot.

Wait a minute. We haven't. This one, now we got another term here. So this one gives you 2 times the expression times the-- this would give you L theta dot sine theta. But now you have to take the derivative with respect to theta dot, which gives you what?

STUDENT: [INAUDIBLE].

PROFESSOR: Another L over 2 sine theta? Something like that. So you end up with the theta dot L, L squared over 2, theta dot L squared sine squared. And you probably get a theta dot L squared cosine squared over here.

And those two add together to give you a theta dot L squared over-- theta dot L squared, I guess. Those collapse together. Those come together to give you the other piece of this.

STUDENT: [INAUDIBLE] one of the coefficients. But where is the derivative [INAUDIBLE].

PROFESSOR: No, the derivative with respect to theta only comes in in the potential energy term.

STUDENT: So what's number two?

PROFESSOR: OK. All right, yep. You need that. And so T with respect to theta, and you do have theta pieces in there. And it does kick out more pieces.

STUDENT: [INAUDIBLE].

PROFESSOR: No, I didn't. Haven't even done that piece yet. So you do that piece, a couple things cancel. And you end up with-- so I don't have time to work it out, to write it all up on the board. But the complete solution for this is posted.

So Professor Gossard, who teaches the other three recitation sections, writes these up and posts the answers. And so they're on Stellar. So you get the gory details of each of these pieces. Let's go on to talk about generalized forces, while we have a few minutes.

The way it was set up, were there-- what are the right hand sides? Are there any generalized external non-conservative forces, the way the problem was first posed? None. So let's put in a couple. Let's add dashpot, b here, and an external force here. Call it F1 of T.

So now, what's Qx and Q theta? That's an exercise I think you can all go through, but just check with your groups. Figure out the generalized forces. And do it by imagining-- for this one, for the x equation, say OK, you have a small, virtual displacement delta x. What's the virtual work that's done?

Your delta w, then, will be thing you're looking for, delta x. And the same thing. This is the x1. And you get a similar x thing when you do they one for theta. It would be Q theta, delta theta. Figure out the work done and that'll tell you what Qx and Q theta are.

So the total work of the non-conservative forces through these virtual displacements, you can just add them up. So there's a contribution that comes from a delta x. And there will be another contribution from the delta theta. And we can figure out each piece. And you assign each piece to the equation it goes with.

So if you do a small virtual deflection in the x direction, how much work is done? Somebody give me a term here. So work, remember, is F dot d dot dr. And this dr is a function of our delta x's delta theta's and so forth.

STUDENT: F of T dx?

PROFESSOR: Delta x? That'll be some work done. So that force, external force moves through the full delta x. And what else?

STUDENT: Minus dx [INAUDIBLE].

PROFESSOR: So that suggests then we have here, this is F1 of T in positive I direction minus dx dot in the opposite direction times delta x is a virtual work done by-- as you do that. Now how about the delta theta?

Somebody else-- how much virtual work is done by these forces F and minus bx? They're the only non-conservative external forces in the problem are the dashpot and the F, whatever it is. How much work is done by those forces in a virtual displacement delta theta?

Hand up back there? I hear a bid for 0. What do other people think? Do those forces move at all because you make motion delta theta? Is there any dr here that results because of delta theta in the direction of any of these applied forces? At the point of application of these forces, do they move?

So this force is right here. Does it move because you do a delta theta? Nope. And this force, applied right here, does that point move because of delta theta? No, so in this problem then this piece here is 0 delta theta. And total virtual work done is that.

And you assign each piece to its appropriate equation. So Qx, this is Qx right here, delta x. Qx belongs up here. The sum of these things equals Qx. And in the second case, for the beta equation, it's equal to 0.

So let's make the problems a little bit harder for a second. Let's put a force, apply a horizontal force here. We'll call this F2. So now, what is the work done in this system? We now have an additional force. Is there any work done because of delta x?

This is the real-- you understand this piece, then you really begin to understand how you do these generalized forces. Qx delta x-- is the generalized force associated with delta x, is it affected by this new force? Is any work done? So I now cause this little delta x to happen.

Is that force doing work, assuming no other deflections aren't happening right now? Why? So is there-- it's not allowed to move. Whatever instantaneous position it's in, it's frozen there in its coordinate.

But if there's an x component, it moves in x. So it's angled like this off the cart. But now delta x does this to it? Is that force F doing work? How much?

So then, when you add that one to it, we end up with a-- and is it in plus direction, and it's exactly in the same direction as delta x. There's no components. The dot product of F2 is in the I direction dot delta x, which is also in the I.

So you get an additional contribution of F2 delta x. And so now this generalized force has an F2 in it. How about the other direction, though? How about this now Q theta delta theta? What does it give you?

Now, if you now freeze x, and allow a slight angular variation delta theta, does this guy do any work? How much, and is it F2 delta theta? That's wrong in two ways. Dimensions are wrong.

Something else is wrong. So let's draw it. Here is this thing. Here's F2. Delta theta causes the motion of this point in what direction? Perpendicular to this, right? So you can think of it like this. And this would be L delta theta is the actual distance that it moves.

And so can either say the component of that motion in the direction of the force, or you can say the component of the force in the direction of that motion, dot product between this and that. And this is theta. So what is the component of L delta theta in the direction of the force?

That's this right here. And that's now in the I hat dot F2 I. So our delta w in the theta direction is Q theta delta theta equals my F2 L cosine theta delta theta.

And to solve for Q theta, now these can disappear. And Q theta is F2 L cosine theta. Are the units right? What are the units of this generalized force, meaning you got to think about that equation. The theta equation-- we talked about this before-- has units of what?

What are the dimensions of term that look like that? This comes from, when you do it the direct way, the summation of external torques. This had better have units of torque, right, which is units of force times distance. So force times distance had better be the units of this equation.

And therefore that had better be a torque. Is force times length a torque, of units a torque? Sure. So that's the correct unit. And over here, for the x equation, do we come out with the correct units? Yeah, come out force.

This naturally works, because the delta theta is dimensionless. So the length dr, the length piece that comes in here stays with this to give you a torque. Over here, delta x has length in it. And you're just left with the force. So you get a force-- this generalized force for the x direction is a force.

The generalized force in the theta direction is a moment, a torque. All right? Good. So we're officially done. But if you have any last questions on this, I'll stick around and we can chat about it.