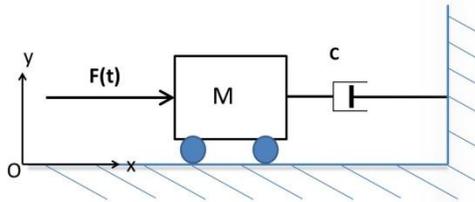


## 2.003 Engineering Dynamics

### Problem Set 7—Questions with answers to concept questions

#### Problem 1:

A cart of mass 'M' is connected to a dashpot with constant 'c'. A force  $F(t)$  acts on the mass, where the force is given by  $F(t) = F_o \cos(\omega t)$ .



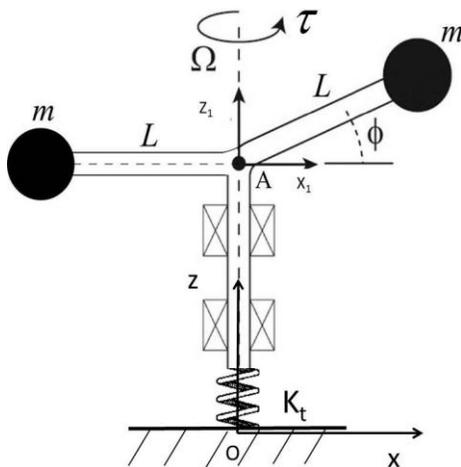
a). Use Lagrange's equations to derive the equation(s) of motion for the system.

**Concept question:** A long time after applying  $F(t)$  the mass will exhibit what kind of motion? a). oscillate and move steadily to the right, b). oscillate and move steadily to the left, c). oscillate about a mean position, d). not move.

**Answer:** c). The mass will oscillate about a mean position.

#### Problem 2:

Two identical masses are attached to the end of massless rigid arms as shown in the figure. The vertical portion of the rod is held in place by bearings that prevent vertical motion, but allow the shaft to rotate without friction. A torsional spring with spring constant  $K_t$  resists rotation of the vertical shaft.



The shaft rotates with a time varying angular velocity  $\Omega$  with respect to the  $O_{xyz}$  inertial frame. The arms are of length  $L$ . The frame  $A_{x_1y_1z_1}$  rotates with the arms and attached masses. A time varying torque is applied to the rotor about the  $z$  axis, such that  $\tau_{/A} = \tau_o(t)\hat{k}$ . Note that the angle  $\phi$  is fixed.

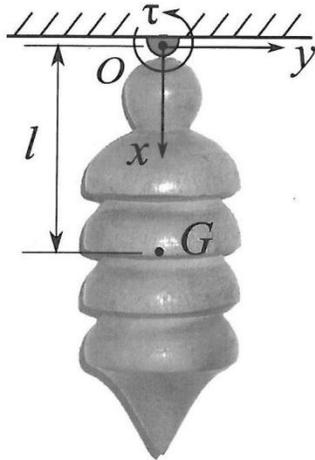
a). Derive the equations of motion using Lagrange equations.

**Concept question:** What are the units of the generalized force in this problem. a). N, b). N/m, c). N-m, d). other.

**Answer:** It depends on the choice of generalized coordinate. Since this is a rotation problem, the choice of coordinate is most like a rotational coordinate, such as  $\theta$ . In order for  $\delta W_\theta = Q_\theta \delta \theta$  to have the appropriate units of work or energy, then  $Q_\theta$  must have units of torque.

**Problem 3:**

A pendulum of mass 'm' is allowed to rotate about the z axis passing through point O in the figure. The center of mass is at a distance 'l' from O and  $I_{zz/G}$  is known. An external time varying torque,  $\tau(t)$  is applied to the pendulum at O.



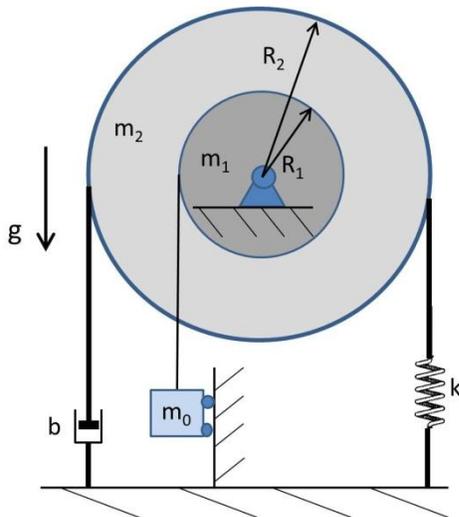
a). Derive the equation(s) of motion using Lagrange equations.

**Concept question:** Is it appropriate to use the principal axis theorem in this problem? a). yes, b) no.

**Answer:** Yes, This is a good opportunity to use the parallel axis theorem.

**Problem 4:**

Two uniform cylinders of mass  $m_1$  and  $m_2$  and radius  $R_1$  and  $R_2$  are welded together. This composite object rotates without friction about a fixed point. An inextensible massless string is wrapped without slipping around the larger cylinder. The two ends of the string are connected to a spring of constant  $k$  on one end and a dashpot of constant  $b$  at the other. The smaller cylinder is connected to a block of mass  $m_0$  via an inextensible massless strap, which is wrapped without slipping around the smaller cylinder. The block is constrained to move only vertically.



a) Find the equations of motion using the Lagrange approach.

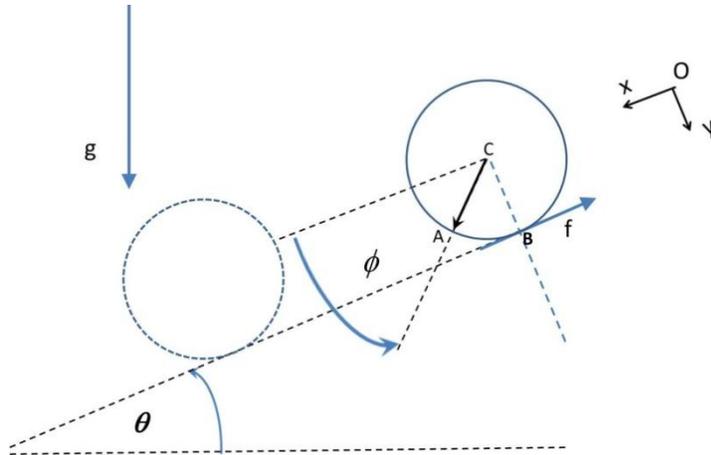
**Concept question:** For small values of the dashpot constant,  $b$ , if this single degree of freedom system is given an initial displacement from its static equilibrium position, will it exhibit oscillatory motion after release? a)Yes, b)No.

equilibrium position, will it exhibit oscillatory motion after release? a)Yes, b)No.

**Answer:** a). Yes, this system will exhibit decaying oscillations about a its static equilibrium position.

**Problem 5:**

A wheel is released at the top of a hill. It has a mass of 150 kg, a radius of 1.25 m, and a radius of gyration of  $k_G = 0.6$  m.



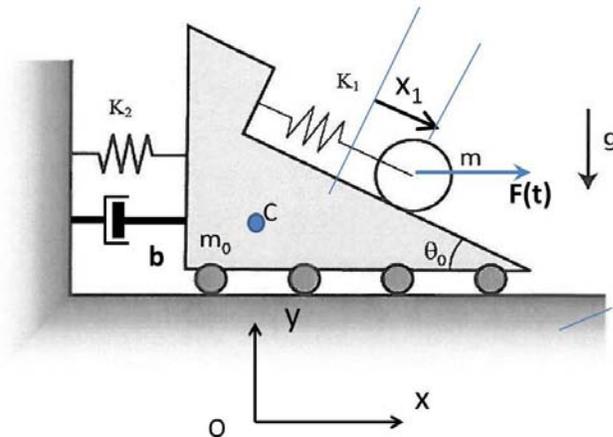
a). Assume that the wheel does not slip as it rolls down the hill. Derive the equation of motion using Lagrange equations.

b). Assume the wheel does slip, and that the friction force between the wheel and the ground is  $\vec{f} = -3N\hat{i}$ . Derive the equations of motion using Lagrange equations.

**Concept question:** For which case,

a) or b) will friction appear in the equations of motion as a non-conservative generalized force? A. in a) only, B. in b). only, C. in a) and b), D. Never. **Answer:** In b). only. When there is no slip, no work is done by the external force.

**Problem 6:** The cart shown in the figure has mass  $m_0$ . It has an inclined surface as shown. A uniform disk of mass  $m$ , and radius  $R$ , rolls without slip on the inclined surface. The disk is restrained by a spring,  $K_1$ , attached at one end to the cart. The other end of the spring attaches to an axel passing through the center of the disk. The cart is also attached to a wall by a spring of constant,  $K_2$ , and a dashpot with constant 'b'. An horizontal external force,  $F(t)$ , is applied at the center of mass of the disk as shown.



The cart is also attached to a wall by a spring of constant,  $K_2$ , and a dashpot with constant 'b'. An horizontal external force,  $F(t)$ , is applied at the center of mass of the disk as shown.

a). Derive the equations of motion for the system using Lagrange's equations, including the non-conservative forces. Recall in PSet6  $T$  and  $V$  were found.

**Concept question:** If the coordinate 'x' represents the horizontal motion of the cart, what is the generalized force associated with  $F(t)$  due to a virtual displacement  $\delta x$ ? a).  $F(t) \cos \theta$ , b).  $F(t)$ , c).  $F(t)\delta x$ , d). 0. **Answer:** The generalized force is b).  $F(t)$ .

MIT OpenCourseWare  
<http://ocw.mit.edu>

2.003SC / 1.053J Engineering Dynamics  
Fall 2011

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.