PROFESSOR: In a minute, we're going to go through this exercise of key concepts for the week. But for you early birds, I'll offer, and we may get to some of this. Is there just something from the weak or the problem set? Just some topic that's still bugging you that you just don't get and you'd like me to put on a little list and I might get to it?

AUDIENCE: So if we solve for velocity, is it enough to just take the time derivative of that velocity.

PROFESSOR: Yes. So if you have all of the terms, the full velocity expression in vector notation, and you want to know the acceleration of that point, just take the derivatives taking care of all the rotating unit vectors and all that stuff. Yeah?

AUDIENCE: When you take the derivative of the velocity, where does the Coriolis term drop from?

PROFESSOR: Where does it fall out of?

AUDIENCE: Yeah.

PROFESSOR: I can't answer it in words. I can't remember exactly. But Coriolis term-- curiously, it comes from two different places. It's two omega v rel kind of term. And actually, it comes from two different places when you grind out the derivatives of the velocity.

AUDIENCE: [INAUDIBLE].

PROFESSOR: So in terms of using the general-- so I remember. Now what I want you to do and, partly, what this session is going to do is get you to use the acceleration formulas enough that you have those just committed to memory.

Now we're going to let you have crib sheets when you go into quizzes. And I would always have the velocity and acceleration formulas in polar coordinates and in full vector form. If you identify each of the terms correctly, that formula will work just fine.

Getting the right components of rotation rates can be a little tricky, as you might have found in that homework problem from this week. In that time derivative of a rotating vector, the omega cross the vector. What omega is that? That's tricky. That you have to be careful with.

But really remember. If you remember the velocity and acceleration equations, you can trust them. They will work. So OK, we can start now.

And I'll go back to here. So take that minute, like we did last time. Write down on a piece of paper two, three, four. What you think are the key concepts that were important in the past week. By important, you need to know them to do a good job on the quiz coming up.

Important concepts. OK, let's make a list.

AUDIENCE: Did you say polar coordinates?

PROFESSOR: Polar coordinates. And I'm going to generalize that to call it choosing coordinate systems. There's an art to that. So when you to use them. It's [INAUDIBLE] of that, yeah.

AUDIENCE: The equation for some of the parts?

PROFESSOR: Oh, yeah. The torque equation. Some of the external torques vectors is dh/dt. And torques are always with respect to some point. So dh/dt. Angular momentum is with respect to some point.

And there's a second term to this, though. And it is velocity of the point in the inertial frame cross the linear momentum in the inertial frame. That one, you haven't had to use much yet. But that's one of the two really important Newton law kind of things that we use in the course.

Sum of forces equals mass times acceleration. Sum of torques equals that. OK, I have another one.

AUDIENCE: Derivative of rotating unit vectors.

PROFESSOR: Derivative rotating vectors. And taking derivatives rotating vectors always boils down to, eventually, it's down to you got to do the unit vector. So that's part of it. How about another one?

AUDIENCE: [INAUDIBLE].

PROFESSOR: OK, so I'll generalize that to, essentially, being able to find equations of motion. So from sum of forces and-- all right. OK, anything else?

I'll tell you one really important one that you've been. Maybe you think we did it last week. I'd say being able to find accelerations and moving coordinate systems. We did mostly velocities last week. But finding velocities and accelerations in rotating and moving frames.

OK, so that's a pretty good list. [INAUDIBLE] ask me what I thought the important things from the week were? That hits the important stuff. OK. What we were talking about as people were arriving is and I might not get this today if we run out of time.

But does anybody just got a burning question about some concept that just didn't work out for you or a problem set that came up? Yeah?

AUDIENCE: The derivatives for [INAUDIBLE] vectors and [INAUDIBLE].

PROFESSOR: So that's really the derivative of rotation rate vector.

AUDIENCE: Yeah.

PROFESSOR: That was, kind of, a nasty problem the first time you hit that. That was new, right? Let me write that one down. And rotating frames and all that. I get it. Does somebody else have another one?

AUDIENCE: [INAUDIBLE] of that. [INAUDIBLE] just like which omega is it when you're looking

[INAUDIBLE].

PROFESSOR: Right. Oh, yeah. Right. OK, anybody else a burning question? I think I'm actually going to start there. We're going to spend just a minute on this one. And as an example problem yesterday in the lecture, I basically did the same. I had a rotor here. It could have a disk on it just to make it clear.

So this thing is rotating at some omega 2. And it was on a merry go round going at some omega 1. And I'll pick at coordinate system here. I think it was yxz that I wrote in the lecture notes. And that's the rotating one. So you have an o and an a here. And you have a fixed frame that but the y ones, the little ones, attach to the platform. It's going around. OK?

So the rotation rate of the platform here is what? In magnitude and unit vector. Pardon? Mega 1 k hat. And does it matter which k because they're parallel. So the little k and the big k.

OK, what's the rotation rate of this shift, in terms of a magnitude and a unit vector?

AUDIENCE: Omega 2 lower case y dot.

PROFESSOR: OK, omega 2. And I'll call it at j1 here. It's hard to distinguish my handwriting on the board between uppers lowers. So with z1's, x1's. OK, so it's in the J1 direction, right? And I've just intentionally made it positive. If it were in the other direction, have a minus there.

OK, so now at the end, the problem asked for the time derivative of the rotation rate of the spinning. The problem that in the homework, this thing was actually inclined, basically.

It's easy to just say the total rotation rate of the shaft. Let's get that settled first. In the fixed inertial frame, the total rotation rate of this is what?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, and you can sum rotation rates. So it's omega 1 k hat plus omega 2 j1. And I want to take the time derivative of this omega chafed total. So it's d by dt of these things. So I can start to do that.

So d by by dt is, in this case, is k change a direction. No problem. Could that change? Could it? Yeah, sure. I mean, it could be accelerating, right. If it is, then you get a term here that comes from this one that would be an omega 1 dot k hat plus. And now we need to take the derivative of this piece. All right.

And that's now a rotating vector. And we've said that the derivative of any rotating vector dq dt is the derivative of t as if the rotation is 0. It's its magnitude increasing, not its direction changing. Plus omega cross q.

And I've intentionally left sub-scripts and server scripts off here because that's the issue here. Which ones go where? All right, so this is what we need to apply to taking that derivative. We took this derivative and found this term was what gave us the omega 1.k.

And when taking derivative of this, what was the answer here?

AUDIENCE: Zero.

PROFESSOR: Zero because the rotation rate is omega 1. You're crossing it. Well omega 1k crossed with omega 1k. That goes with zero. There's no change in direction. So we applied this formula once to this. Now we're going to apply this formula here.

So I'll do it this way. So this is term one. Term one gave us this. Term two is going to give us this. So take this. Take the derivative of-- where we go here-- this guy.

Now when I say this is the-- it's the partial derivative of this thing, ignoring the contribution from the change of direction. So it's as if some rotation were zero. Which rotation? Right.

AUDIENCE: Omega 1?

PROFESSOR: Omega 1. Good. So what is the answer to this piece of that derivative?

AUDIENCE: Omega 2.

PROFESSOR: Omega 2 dot what direction? Same direction, right. Now we want to apply this piece. So now the question comes down plus some omega cross with the q. And what's the q in this case?

Two. So this is omega 2 j1 hat. That's coming from here. That's what this piece is. So now we need what's crossed with it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Hm?

AUDIENCE: Omega [INAUDIBLE].

PROFESSOR: I'm trying to think of a clear way to help you. What is causing its direction to change?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Right. That's the one you care about. What's causing its direction to change? What's causing this thing to come up? And its direction is changing because the platforms are moving. So it's omega 1. K cross j. So k cross j gives you?

AUDIENCE: [INAUDIBLE].

PROFESSOR: K cross j negative i 1 in the rotating frame, right. So this whole thing. Omega 1 dot k plus omega 2 dot j1 plus. No, this is a minus, actually. We said, minus omega 1. Omega 2. K cross j is i 1 hat. OK?

And finally, how do you deal with the fact if this rotor had been on an inclined angle? Some phi, which now this is the exact problem that was on homework. So propose a way of attacking that. Changes this problem just a little bit.

AUDIENCE: Can you change the reference frame [INAUDIBLE]?

PROFESSOR: No, you don't change the reference frame. You still have a coordinate system. Work with the coordinates you got.

AUDIENCE: [INAUDIBLE] to its components?

PROFESSOR: In which reference frame?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, the moving [? one. ?] That makes it easy because this thing now has a vector omega 2 like that. And you can break it up into a piece like that and a piece like that. So now this is phi. Omega 2 cosine phi is the side, right. Omega 2 sine phi is that side. What direction is it? What's its unit vector direction? This piece.

AUDIENCE: K?

PROFESSOR: K, OK. And we're going to end up crossing it with this term, right. So k cross k. Nothing. So in fact, this bit doesn't change in direction as it's rotating. Does it? So it isn't going to contribute to that second term.

Only piece you have to be concerned with is this one. So down in here in this part of it, all you would do is say omega 1 k cross. And now you just put in the q full omega 2, which is omega 2 sine k plus omega 2 cosine. Sorry about the writing here.

And do the cross product. You find out that term disappears. And you get an omega 1 omega 2 cosine. And this term here is still the answer. But you end up with a cosine phi in here because it's just the component of omega 2 that's in the j1 direction now that contributes.

OK, great. Good. That help? I'll leave this. Let's get on to the problem for the day. The problem for the day is you've got a rotating arm. You have a mass sitting on it. The arm is rotating at some rotation rate omega.

You might have an acceleration omega dot theta double dot. This mask can slide. So the problem is this thing is-- I'll start here. So it's rotating like this. And eventually, the thing begins to slide.

And in fact, if I do it, this actually works a little better here because I have a little tiny groove in this. It helps a little bit. So it's rotating like that. It eventually begins to slide. And if the rotation rate is slow, it slides down. If I made the rotation rate fast enough, it slides up.

And the angle at which it happens clearly depends on rotation rates, accelerations, the angle, friction coefficients, and things of that sort. So what I want you to start with is here's, basically, a drawing of it. And I don't want to give you any excess information.

But what I want you to do is each, by yourselves, take just a minute, and draw a free body diagram of this problem. And then, after that, I'm going to have you get in groups and improve on it. But start yourself. Draw a free body diagram that will describe this mask on this thing, plank.

OK, we've got a sketch with a little free body diagram on it. What I want you to do now, we're going to do this two or three times today. Get in groups of four five here. Compare notes, and come up with a group free body diagram.

So you're pretty close. Let me look. Where was your guys solution again? There is maybe about as good as it gets. So here's our mass. I think everybody had an mg. Almost everybody had a normal force. Everybody had a friction force.

Some discussion about what direction the friction force ought to be in. I'll draw it uphill here. And I'll call it ft for tangential, here. So some had about that much. There's a very important missing piece, which actually several groups had. What is that?

Can anybody guess what I'm-- three of your groups had something on here that I don't have up here yet. I hear coordinates. This one, you can almost get away with not putting it in the coordinate system first. But you, usually, need to have a coordinate system in order to determine the direction of the forces. OK?

And we'll get to that in a second. So everybody that I saw had written down, I saw, a number of the little x, y, z frames sitting out here like that. Maybe one was lined up.

Not sure. Most not. Most like this.

Is polar coordinates a good choice for this problem? Why not?

AUDIENCE: [INAUDIBLE].

PROFESSOR: So the question is whether or not r theta and z can completely describe the motion in the problem.

AUDIENCE: You have tangential forces, so it's better if you use rectangular.

PROFESSOR: The radius is changing. But that's known as r dot. r dot can handle the motion to this this way. r double dot can handle acceleration this way. Theta dot can handle this motion. Theta double dot can handle that acceleration.

Actually, is there any z motion or z-forces? No. Actually, the polar coordinates will actually work here just fine. So you definitely have to pick a coordinate system. So we're going to have an that system and theta hat in this direction because in this problem, whether you use x and y as a rotating frame here or r theta, you want to pick your coordinate system so that these things break down easily.

In this case, the normal equation will be in the theta hat direction. The sliding direction will be in just one unit vector direction. So you don't have multiple components in this direction. If you use that coordinate system to describe this motion, you have x and y terms. It makes it harder.

So this is a pretty good coordinate system to use. So that's a really important missing piece is the coordinate system, if you don't have it. And as an aside about free body diagrams-- free body diagrams, except my d is turned around-- how do you know that the friction force is in that direction. You had to assume something.

This is such a trivial problem you can kind of figure out in your head. I gave the demonstration. It slid down. But also, I said , you do it fast enough, it goes up. So you don't really know. So you have to have made an assumption about it. So the general rule is to assign or assume positive values for all motions.

And then, you deduce the direction of forces. And I'll take another quick aside here to illustrate this in a problem that works better than the current one for this. An obvious coordinate system for this little mass on rollers inertial frame. Here's an x.

And I asked you to do a free body diagram of that. Well you would show me a normal force. You'd show me an mg. But now I ask you to tell me the direction to draw the spring force.

So if you adopt the rule that for each body you're working with, you assume that it has positive x, positive x dot, positive y, positive y dot then you can deduce what direction of the forces would result [INAUDIBLE]. So in this case, the only motion that generates a force up here is what?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Is x. x dot. x double dot. The only motion that will generate a force is displacement because it generates a force where? In the spring. If they add a dash [? pod ?] here, them velocity generates a force, too.

But displacement, if you assume the displacement is positive, then which direction is the spring force? OK, so then you have spring force this way. And that's value kx. And when you write it in the equation of motion, you put a minus sign to account for the direction of that error.

OK. So in this problem we kind of-- and if you ever do it wrong, if you pick this direction wrong, what happens in the answer? Will you still get the right answer? If you're consistent, you'll get another negative sign popping up to fix it. OK.

Next. So I think we're in good shape now. I'm going to draw the friction force uphill because I'm going to assume this thing's going to slide down. And I'm going to simplify the problem a little bit for you. And I'm going to say that there's no angular acceleration. Constant angular rate. The distance that the mass starts off up the slope is at one and a half feet.

So it's going up like this. And it, eventually, reaches 50 degrees. And at 50 degrees, it begins to slide downhill. So it's going up a constant rate and starts to slide. So there must be some friction coefficient that provides a system with a property such that it slides at 50 degrees.

So the problem here is define the coefficient of friction. So now you're in your groups. Solve the problem. And what I really want you to do is come up with an expression. mu equals in variables and constants. Don't bother plugging in numbers in that.

Well if you're comfortable using polar coordinates, that'd be the way to do it. But you need to write some equations now. And I suggest you need to think in terms of equations [? in ?] motion. I think it's time to come back together here and work on this.

Two, three, or four groups are, sort of, struggling with this. And it's because you're not really going to first principles and doing the problem step by step. You're, sort of, jumping to the answer because it's a trivial, simple, Mickey Mouse problem that you did in high school. So I'm, kind of, pounding on you a little bit.

We give you a simple problem so we can do them in a short period of time. But you need to learn to do them in the rigorous way so that you learn the real fundamental stuff that you have to know. So none of you-- you all are sort of thinking about, well, we got sum of the forces here. We've got an acceleration there.

But nobody is just sitting down and saying that some of the forces is equal to the mass times the acceleration and working the problem out. OK, bad dog. Just because this problem is almost the statics problem doesn't mean you-- when things are statics problem, it just means that acceleration goes to 0. And the sum of the forces is now 0. And you solve the problem.

Start with Newtons, in this case, it's Newtons second law. It helps to know an expression for acceleration, so you don't have to grind it out. So polar coordinates works pretty well in this problem.

And I want you to commit to memory two acceleration equations. One in polar coordinates, and the one the general vector 1. So in polar coordinates, the one that I memorize is a with respect to o plus r double dot minus r theta dot squared r hat plus r theta double dot plus 2 omega r dot theta hat.

Coriolis, Eulerian centripetal, and your linear acceleration. What's this term? What's it doing there?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, but the reason I remember it is, in this course, we break down every dynamics problem we're confronted with into sub problems that can be solved as the sum of a translation of a body plus a rotation. So this expression allows you to write down the acceleration of a point on a body, which is both translating and rotating.

What this term accounts for. The acceleration contributed by what? So I've got this general-- let's see. I got a merry go round that's on wheel and is rolling along here. And I have an inertial frame.

And I got a point a here. But now I've also got a system in here, which I'm describing with an r theta connected to this. Thing can have translational acceleration. That's that term. This problem doesn't happen to have that. It goes to 0. In this problem, that's a 0.

In this problem, we can now apply this equation to that problem. How many sub equations are we going to get? And why no three.

AUDIENCE: But I said two.

PROFESSOR: I know you said two. But I'm asking why not three.

AUDIENCE: Because [INAUDIBLE] theta.

PROFESSOR: And a z, which you need because it describes theta dot. So you have a z direction. But is there any forces in this problem in the z direction? Are there any accelerations in the z direction? No, so you get a trivial answer there.

So you could just write that one down. Mass times acceleration equals zero. And there's no forces. So there's three possible equations. Only two of them are meaningful in this problem. And we have one that we can summon the that component and one in the theta hat.

So the sum of the forces in the theta hat direction for this problem are what? From your free body diagram.

AUDIENCE: The normal force?

PROFESSOR: The normal in the positive theta hat direction and?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Minus mg. And I think it's [? percosen ?] theta. And that's your theta hat. And what is the acceleration in that direction? Well you go to acceleration formula. And now it's inspect the terms.

This one is 0 because the platform is fixed. r double dot? 0 because we're just waiting. It's just trying to calculate when it begins to slide. r theta dot squared. Not 0. r theta double dot.

Well it might be. But I said omega dots 0 constant. So that one's 0 for this problem. And this term, Coriolis. 0 because our dot is 0. We're really treating this like a statics problem. It hasn't started to move yet. So we're only left with one term here.

So the sum of the forces in the normal direction, which is the theta hat direction are 0. And that allows us to solve for the normal force. So you need Newton's law to find the normal force to start with. And you need the normal force to find the friction force.

So now let's do the sum of the forces in the that direction. And maybe, let's do the acceleration first. It's the mass times the acceleration in the that direction. And that's the mass. And now, what's the acceleration?

These are the r directed terms. We have one term, right. Minus r theta dot squared. No other terms. And those are going to be equal to the external forces in the radial direction. And what are they? From the [INAUDIBLE] diagram.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Plus or minus?

AUDIENCE: Plus.

PROFESSOR: Plus mu n. But we know n is mg cosine theta. OK, what else? Minus mg sine theta. All right.

You know everything in this expression. You know given theta dot. You're given r. You know mg. You know [INAUDIBLE] given theta. You could solve this expression for mu. All right?

And the mg goes to the other side. And you have a minus r theta squared. Notice the M's cancel all the way through, right. So mu would looks like it equals, to me, g sine theta minus r theta dot squared all divided by what? g cosine theta.

Break these two apart, this gives me a tan theta minus r theta dot squared over g QoS theta. That has units of acceleration. g has units of acceleration. This is dimensionless. The answer has got to be dimensionless.

So there's your answer. But you really had to use the equations in motion. OK now there's another thing I want to dress because I heard it pop up two or three times. Do not confuse accelerations with forces.

Newtons second law makes it really clear where each one goes. The sum of the external forces go one side of the equal sign. The mass times the acceleration goes on the other. Don't get the two mixed up. Solve for the accelerations, if you can.

Plug them in. And multiply them by mass. And now you have that one side of the equation. The free body diagram. The only vectors that should show up on a free body diagram are what?

AUDIENCE: Forces.

PROFESSOR: Forces. The real forces in the problem. So in that problem that free body diagram, there is no r theta dot squared term. It doesn't belong there. And it'll keep you from making these sign errors and things like that.

So the business about this thing comes up minus because it's minus right out of the acceleration equation. All right, what would happen if I had not made the-- we're a go? This term 0. I allow this to have some angular acceleration to it.

That's what's got to happen to. Well you don't have to do it. I can't do it constant rate. But if you can make this constant rate fast enough, it goes up. So even if it's constant rate fast enough, this thing will slide up the thing, immediately, when you start it. Almost immediate.

You start down here, the g. The friction force is pretty high. And the friction force, of course, diminishes as you get further up. And eventually, it takes off, right. That's because that minus r theta dot squared, that's a centripetal acceleration inwards. And if you don't provide the force that causes that centripetal acceleration to happen, it says, it says I want to leave town.

Now how about, though, if I had angular acceleration, as well? If I allow angular acceleration to this problem, how does it change the two equations that we wrote? What does it change? Does it change the forces? Does it change the free body diagram?

Not a bit. It changes the acceleration side of the equation and what term it now turns up that you didn't have before. This one. So now you have a dynamic term in the theta hat equation. And it comes into this expression for some of the forces in this direction.

And you would end up equals to m r theta double dot. Now it's a entirely different problem. It gets a little harder to solve. OK, but really good fundamental practices. f equals ma. And write out both sides carefully. And then, you won't make sign mistakes and so forth. Thanks. Good to see you.