Massachusetts Institute of Technology
Department of Mechanical Engineering

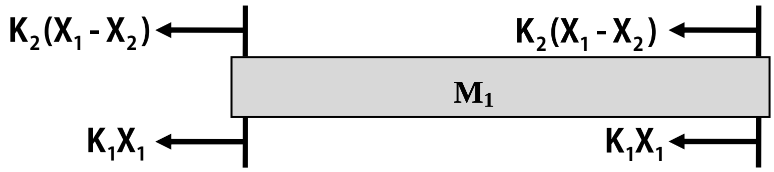
2.003J/1.053J Dynamics & Control I

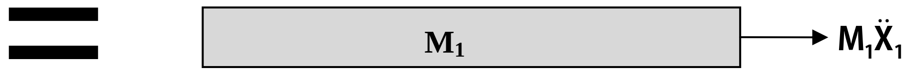
Fall 2007

Homework 9 Solution

Problem 9.1 : Equation of the lateral vibration of a 100 story building.

i)

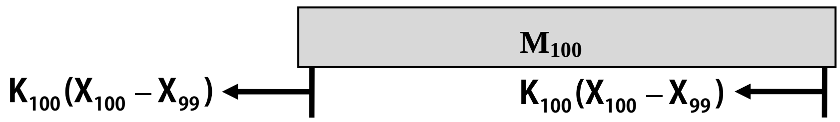


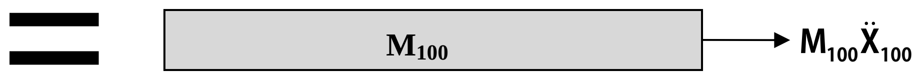


$$-2K_2\left(X_1 - X_2\right) - 2K_1X_1 = M_1\ddot{X}_1$$

$$M_1\ddot{X}_1 + 2K_2\left(X_1 - X_2\right) + 2K_1X_1 = 0$$

ii)

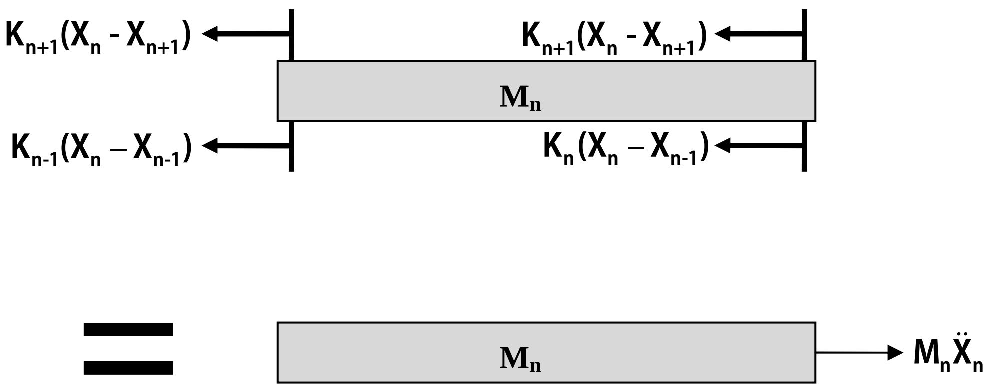




$$-2K_{100}\left(X_{100}-X_{99}\right)=M_{100}\ddot{X}_{100}$$

$$M_{100}\ddot{X}_{100}+2K_{100}\left(X_{100}-X_{99}\right)=0$$

iii) Derive the generalized equation of motion of n[th] floor ($2\le n\le 99$)



$$-2K_{n+1}\left(X_n-X_{n+1}\right)-2K_n\left(X_n-X_{n-1}\right)=M_n\ddot{X}_n$$

$$M_n\ddot{X}_n++2K_{n+1}\left(X_n-X_{n+1}\right)+2K_n\left(X_n-X_{n-1}\right)=0$$

Problem 9.2 : Generating code to calculate vibration modes of a 100 story building

First, the mass matrix can be calculated, based on equations in problem 9.1.

$$[M]=\begin{pmatrix} M_1 & 0 & \cdots & 0 & 0 \\ 0 & M_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & M_{99} & 0 \\ 0 & 0 & \cdots & 0 & M_{100} \end{pmatrix}$$

Next, stiffness matrix can be also obtained, based on equations in problem 9.1.

$$[K]=\begin{pmatrix} 2(K_1+K_2) & -2K_2 & & & & & \\ -2K_2 & 2(K_2+K_3) & -2K_3 & & & & \\ & & \ddots & & & & \\ & & -2K_n & 2(K_n+K_{n+1}) & -2K_{n+1} & & \\ & & & & \ddots & & \\ & & & & -2K_{99} & 2(K_{99}+K_{100}) & -2K_{100} \\ & & & & & -2K_{99} & 2K_{100} \end{pmatrix}$$

Therefore, $[M][\ddot{X}]+[K][X]=0 \quad \textit{where} \quad [X]=[X_1 \quad X_2 \quad \cdots \quad X_{99} \quad X_{100}]^T$ and

$[\ddot{X}]=\lambda[X]$ if the solution of $[X]$ is harmonic.

$$\lambda[M][X]+[K][X]=0$$

The above equation is represented as the form of eigen values problem. Now, we can applied MATLAB function `eig` to it. The m-codes are shown as below.

```
function v=building(nvm)
% function v=building(nvm)
%  nvm: vibration mode number you want to generate
%  v: eigen vector for nvm-th vibration mode

% Define several parameters
m=300;  % Mass for each floor
k=800;  % Spring constant for each wall
n=100;  % Number of floors in the building
% Initialize mass and stiffness matrix
M=eye(n);
K=zeros(n);
% Generating mass matrix
M=m*M;
% Genrating stiffness matrix
% [2k_1+2k_2 -2k_2]
K(1,1:2)=2*k*[1 0]+2*k*[1 -1];
for i=2:n-1
   % [2k_n-1 2k_n-1+2k_n -2k_n]
```

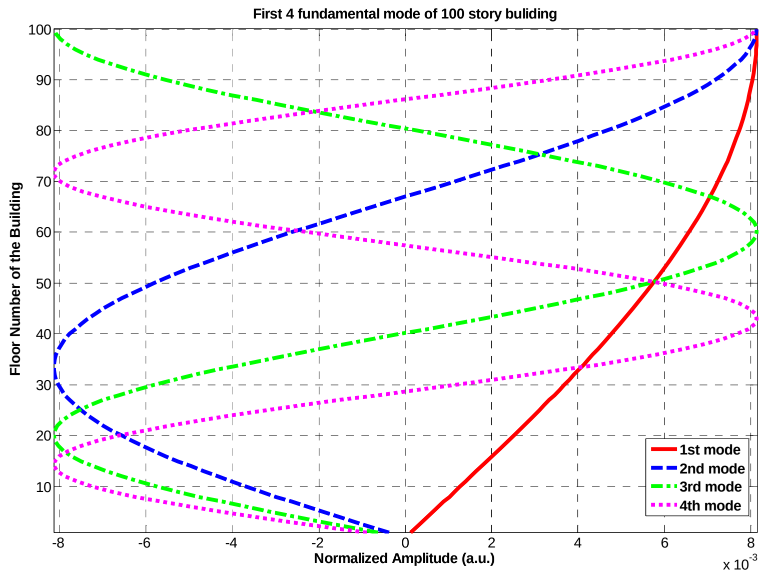
```
    K(i,i-1:i+1)=2*k*[-1 1 0]+2*k*[0 1 -1];
end
% [-2k_99 2k_100]
K(n,n-1:n)=2*k*[-1 0]+2*k*[0 1];
% Obtain eigen value and eigen vector matrix
[V,D]=eig(K,M);
% Extract eigen vector for nvm-th vibration mode
v=V(:,nvm);
end
```

Problem 9.3 : Plotting some vibration modes of a 100 story building

i)

Following in-line commands generate plots for the first 4 fundamental modes.

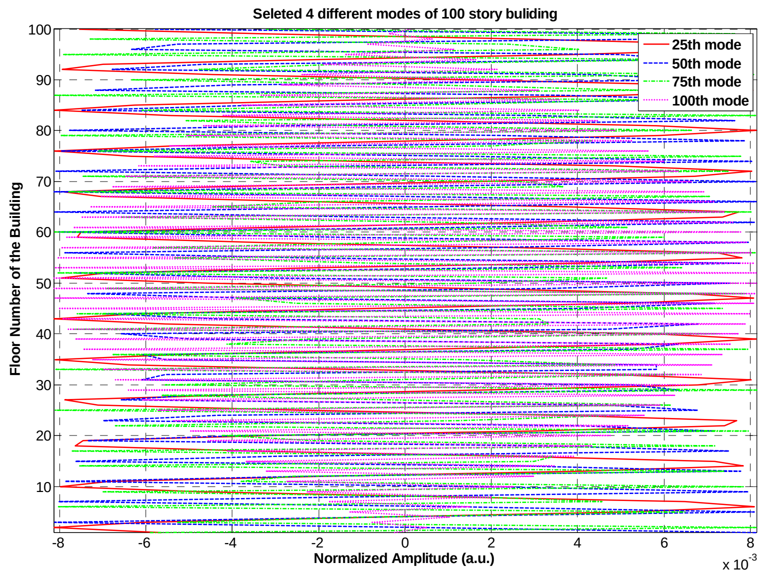
```
>> h=plot(building(1),1:100,'r-',building(2),1:100,'b--', …
        building(3),1:100,'g-.',building(4),1:100,'m:');
>> axis tight; grid on;
>> set(h,'LineWidth',5)
>> set(gca,'FontSize',16);
>> xlabel('\bfNormalized Amplitude (a.u.)');
>> ylabel('\bfFloor Number of the Building');
>> title('\bfFirst 4 fundamental mode of 100 story buliding')
>> legend('\bf1st mode','\bf2nd mode','\bf3rd mode','\bf4th mode');
```



ii)

Following in-line commands generate plots for 25th, 50th, 70th, and 100th fundamental modes.

```
>> h=plot(building(25),1:100,'r-',building(50),1:100,'b--', …
            building(75),1:100,'g-.',building(100),1:100,'m:');
>> axis tight; grid on;
>> set(h,'LineWidth',2)
>> set(gca,'FontSize',16);
>> xlabel('\bfNormalized Amplitude (a.u.)');
>> ylabel('\bfFloor Number of the Building');
>> title('\bfSelected 4 different modes of 100 story buliding')
>> legend('\bf25th mode','\bf50th mode','\bf75th mode','\bf100th mode');
```


Seleted 4 different modes of 100 story buliding
25th mode
50th mode
75th mode
100th mode
Floor Number of the Building
Normalized Amplitude (a.u.)
x 10^-3
100
90
80
70
60
50
40
30
20
10
-8
-6
-4
-2
0
2
4
6
8