Massachusetts Institute of Technology
Department of Mechanical Engineering

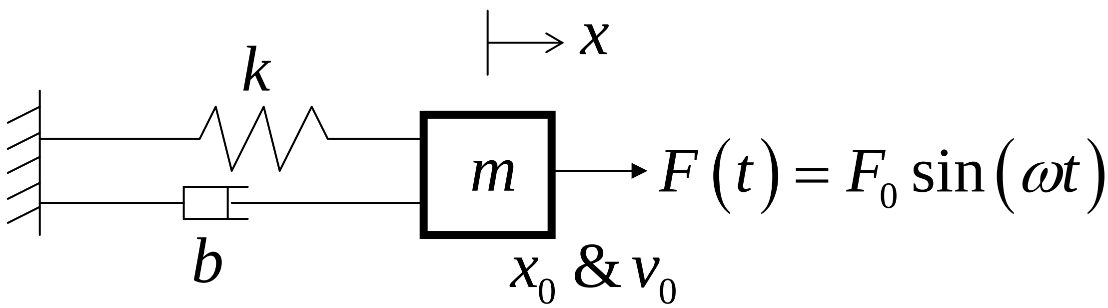
2.003J/1.053J Dynamics & Control I

Fall 2007

Homework 6

Issued: Oct. 19. 2007 **Due**: Nov. 2. 2007

Problem 6.1 : Dynamics of a mass-spring-damper system



A mass-spring-damper system is subject to an external sinusoidal force $F(t)$ with amplitude F_0, and angular frequency ω. The position of particle, x, is zero when the spring is neither compressed or stretched. The mass of the particle is m, the damping coefficient is b and the spring constant is k. Initial condition is expressed as $x(0) = x_0$ and $v(0) = v_0$. Derive the equation of motion for this mass-spring-damper system.

Problem 6.2 : Solver for a mass-spring-damper system using Euler method

You are familiar with Euler method used to obtain the trajectory of a ball dropping and bouncing in Homework #3. In this homework, you will create a function that use **Euler** method to derive the particle trajectory (two column matrix for time and position each) in mass-spring-damper system from t=0 to 50 sec. The input parameters of the function are coefficients (m, b, k, F_0 and ω) and initial conditions (x_0 and v_0). Use time increment $\Delta t = 0.1\text{sec}$. **Function name (and m-file name) should be '`MSDSE_your_kerberos_name`' and upload it to 2.003 MIT Server site. You also submit the hardcopy of your code with appropriate comments.**

Problem 6.3 : Solver for a mass-spring-damper system with using Runge-Kutta method

Using **Runge-Kutta** algorithm to solve the differential equation of this system, generate a m-file function with input parameters (m , b , k , F_0 , ω , x_0 and v_0) to calculate particle trajectory (two column matrix for time and position each) from t=0 to 50 sec. You may use either Runge-Kutta 23 or 45 algorithm (See `ode23` or `ode45` in the MATLAB help, and use one of them in your m-code.). Use $\Delta t = 0.1\text{sec}$ for both the time step size for the solver (See also `odeset` in the MATLAB help) and the time step for the output time from the solver (See input argument for `tspan` in `ode23` or `ode45`). **Function name (and m-file name) should be '`MSDSRK_your_kerberos_name`' and upload it to 2.003 MIT Server site. You also submit the hardcopy of your code with appropriate comments**.

Problem 6.4 : Trajectory of a mass-spring-damper system with different parameters and initial conditions

For the following cases, plot the time response of the particle. Plot the trajectories with Euler and Runge-Kutta solvers on a single graph with appropriate legends. Compare the results for these two approaches. **Submit a hardcopy of your trajectory plots.**

i) $m = 1\,\text{kg}$, $b = 0.5\,\text{N}\cdot\text{sec/m}$, $k = 1\,\text{N/m}$, $F_0 = 1\,\text{N}$, $\omega = 3\,\text{rad/sec}$, $x(0) = 0m$ and $v(0) = 0m/\text{sec}$

ii) $m = 1\,\text{kg}$, $b = 0.5\,\text{N}\cdot\text{sec/m}$, $k = 1\,\text{N/m}$, $F_0 = 0\,\text{N}$, $\omega = 0\,\text{rad/sec}$, $x(0) = 1m$ and $v(0) = 0m/\text{sec}$

iii) $m = 1\,\text{kg}$, $b = 0\,\text{N}\cdot\text{sec/m}$, $k = 1\,\text{N/m}$, $F_0 = 1\,\text{N}$, $\omega = 1\,\text{rad/sec}$, $x(0) = 0m$ and $v(0) = 0m/\text{sec}$