

2.003J/1.053J Dynamics and Control I  
Fall 2007

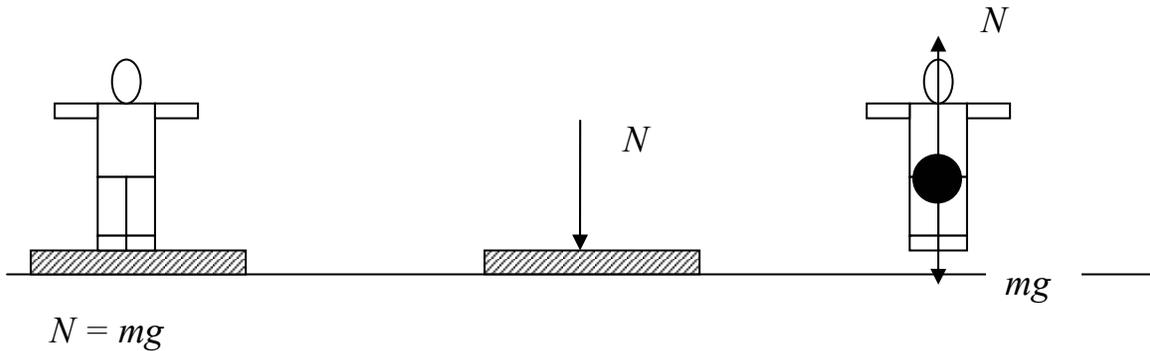
A Benevolent Quiz  
5 September, 2007

**Important Notes:**

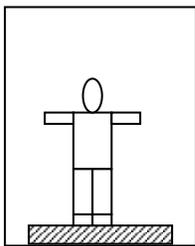
1. This quiz does not count towards the final grade.
2. It consists of two parts. The first part consists of five problems that you should be able to easily solve based on your prerequisites (8.01, 18.03). The second part consists of three problems which give you a glimpse into the kind problems we will be dealing with in this class. We hope that you would be able to solve these problems by the end of the semester.

## Part I: Prerequisites check

1. Suppose that a weighing scale measures the reaction force exerted on it and shows the reading as the corresponding weight value. See the figure below and the corresponding force diagrams.

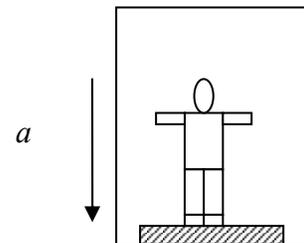


Thus, a person with mass  $m$  exerts a reaction force  $N$  (which is equal to  $mg$  in this case) on the weighing scale. The weighing scale measures the value of  $N$  and divides it by gravitational acceleration  $g$  and shows the corresponding mass. Now the person takes this weighing scale to an elevator and decides to perform a small experiment. In each of the following cases, write down the reading shown by the weighing scale. Clearly draw the corresponding force diagram.

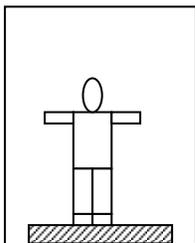


1. Elevator is moving down with uniform velocity  $V$ .

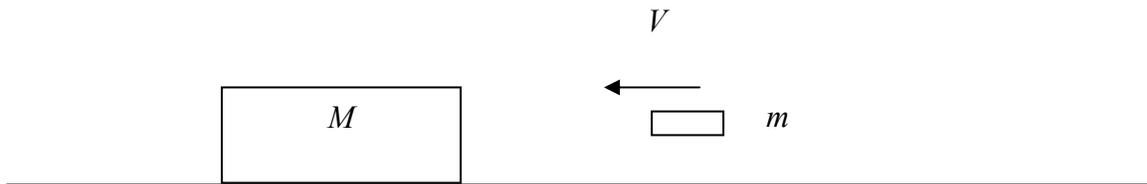
2. Elevator is moving down with uniform acceleration  $a$ , where  $a < g$ .



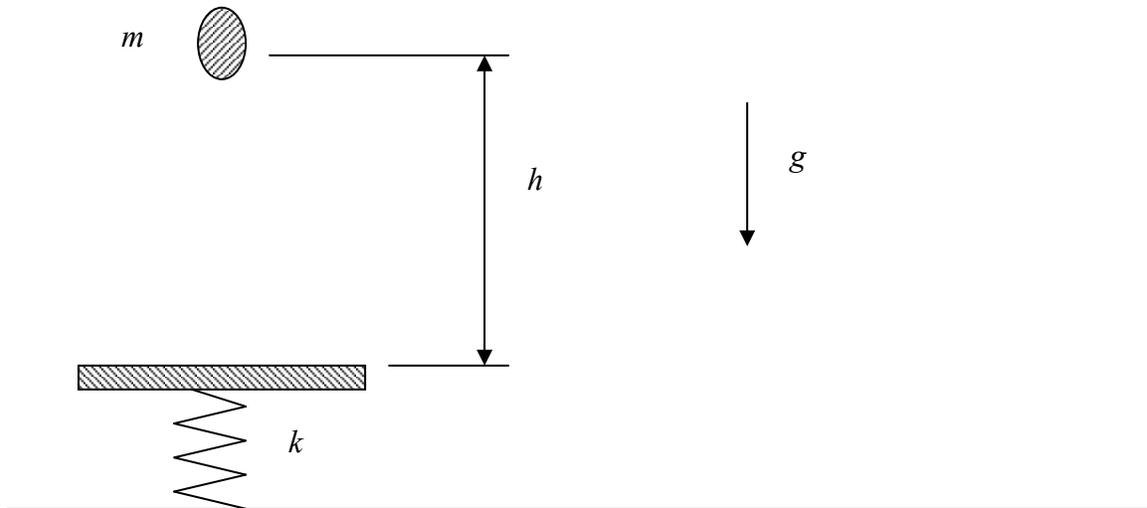
3. Elevator is falling freely.



2. A bullet of mass  $m$  moving with the uniform velocity  $V$  hits a box of mass  $M$  as shown in the figure below. After penetrating the box, the bullet remains in the box. Find the final velocity of the combined mass. Assume that there is no friction at the contact surface between the box and the ground.



3. A mass  $m$  falls freely under gravity from height  $h$  on a weightless platform attached to a spring with spring constant  $k$  as shown in the figure below. The mass sticks to the platform after it falls on it. Find the maximum distance the mass moves downwards after it sticks to the platform.



4. Solve the following second order ordinary differential equation (ODE)

$$m\ddot{y} + ky = 0$$

with the initial conditions:

$$\dot{y}(0) = v \text{ and } y(0) = 0$$

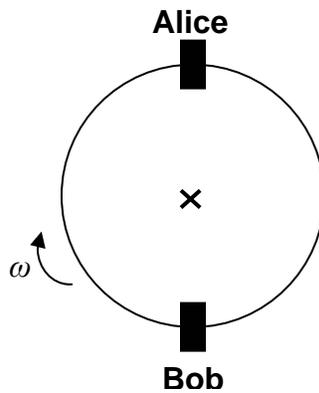
5. Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

## Part II: Sample future problems

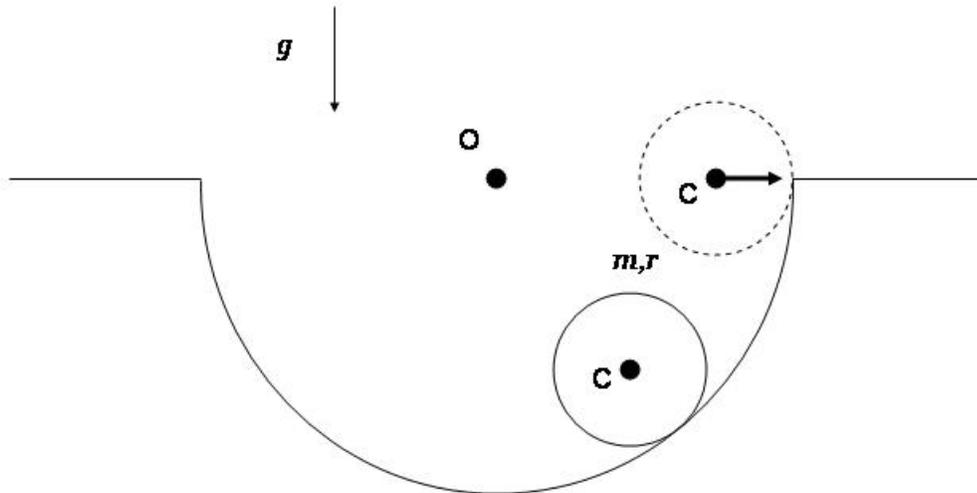
1. Alice and Bob are sitting facing each other on a merry-go-round at two diametrically opposite locations. The merry-go-round is rotating clockwise at a constant angular velocity  $\omega$  as shown in the figure. Alice pushes a hockey-puck (a disk-shaped object) along the floor of the merry-go-round straight towards Bob. Both the puck and the floor are smooth. Do you think that the puck will reach Bob? How does the trajectory of the puck look to:

- Bob?
- a stationary observer seeing from above?



2. A semicircular well of radius  $R$  and a disc of mass  $m$  and radius  $r$  are shown in the figure below. We paint an arrow on the disc from the center  $C$  going radially outwards. Initially the disc is held such that the disc touches the tip of the well and the arrow is exactly horizontal as shown in the figure (the dotted circle). The disc is then suddenly released. Note that gravity acts. Derive the equation that specifies the orientation of the arrow as observed by a stationary observer (looking directly into the page) as a function of time when:

- the disc and the well are smooth.
- the disc *purely rolls* on the surface of the well.



3. An L-shaped groove is shown in the figure below. Two ends of a slender rod of mass  $m$  and length  $L$  are attached to identical springs, each with a spring constant  $k$ . Both the springs are fixed to the L-shaped groove as shown in the figure below. Both the ends of the rod are constrained to move only along the L-shaped groove. Ignore gravity for this problem. Determine the frequency of oscillations of the rod for small perturbations.

