1.053J/2.003J Dynamics and Control I
Fall 2007

Exam 2

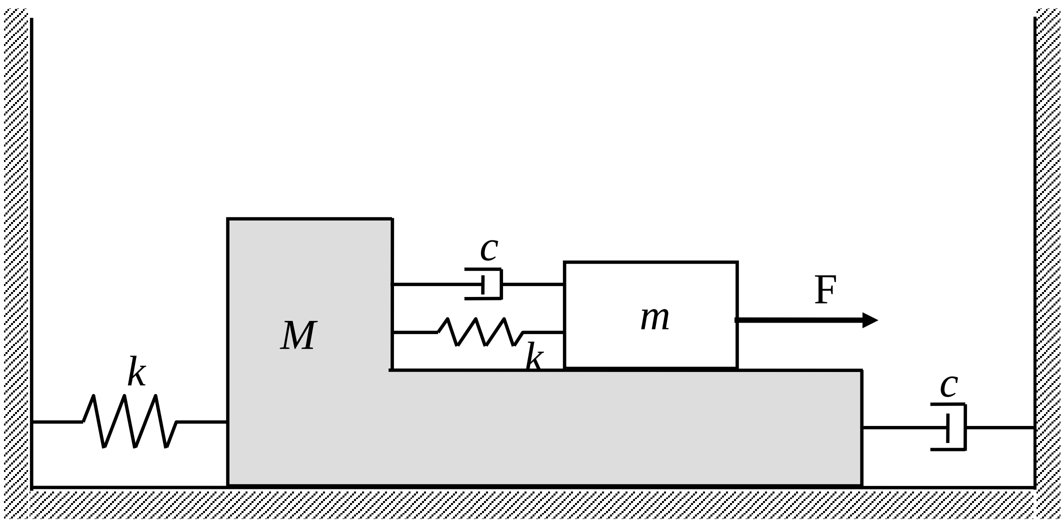
19th November, 2007

Important Notes:

1. You are allowed to use two letter-size sheets (two-sides) of notes.
2. There are three problems totaling to 100 points. You have 80 minutes to solve them.

1. Sliding masses (30 points)

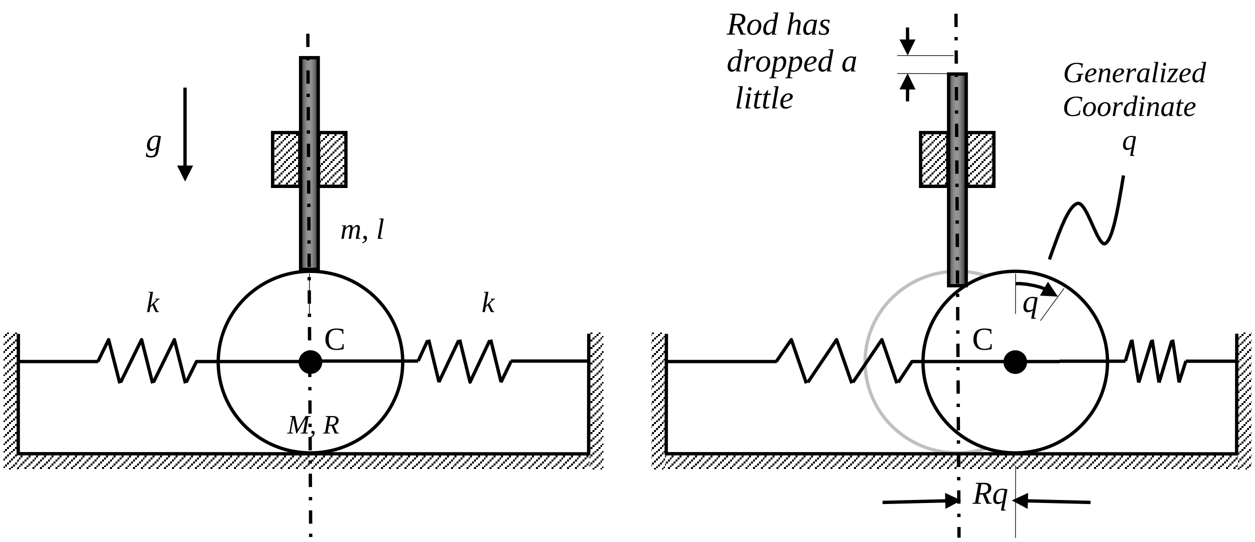
An L-shaped rigid platform of mass M slides along the *frictionless* floor as shown in the figure below. The platform is connected to the fixed side walls by a mass-less spring of spring-constant k and a mass-less dashpot of damping-constant c. A smaller mass m slides along the horizontal edge of the platform without friction. As shown in the figure, the small mass is connected to the vertical edge of the platform by a mass-less spring of spring-constant k and a mass-less dashpot of damping-constant c. Constant *horizontal* force F is applied to the smaller mass. Determine the equation(s) of motion of the system using the Lagrangian approach. Assume that both the springs are initially un-stretched.



2. **Cam follower – a rolling disc and an oscillating slender rod (35 points)**

A rigid disc of mass M and radius R *rolls* along the horizontal surface *without slippage*. The axle through the center of the disc is connected to each side-wall by a spring of spring-constant k. The springs are un-stretched in the center position shown in the left of the figure. A slender rod of mass m and length l rides on top of the roller without friction. The rod is constrained to move only vertically, and assume that it remains in contact with the disc at all times. The disc is disturbed to the right or the left and the whole assembly starts motion – the disc to the right and left, and the rod up and down. Clearly, the system has just one degree of freedom as shown in the figure to the right. Hint: Use the rotation of the disc as your (obvious) generalized coordinate as shown in the figure.

a. Define appropriate frames and determine the velocity of the rod with respect to a frame attached to the ground in terms of q and its derivatives. (10 points)
b. Determine the equation of motion of the system using the Lagrangian approach. (20 points)
c. You have assumed (because we asked you to) that the rod remains in contact with the disc at all times. How would you determine later whether this assumption was valid? Don't actually solve for it, just say it in English (or French). (5 points)



The disc rolls to the right a distance equal to R q

Help: You might find the following derivative rules useful while solving this problem.

$$\frac{d}{dq}\left(\frac{q^2}{(1-q^2)}\right)=\frac{2q}{(1-q^2)^2}$$

$$\frac{d}{dq}\left(\sqrt{1-q^2}\right)=-\frac{q}{\sqrt{1-q^2}}$$

$$\frac{d}{dq}\left(\arcsin q\right)=\frac{1}{\sqrt{1-q^2}}$$

3. Ski-board on an incline (35 points)

In this problem we will study dynamics of a ski-board sliding down an incline via a simple model. The ski-board is modeled as a slender rod of mass *m* and length *2l* with two end-points P and Q as shown in the figure below. Let C be the center of the rod. The moment of inertia of the rod about the axis coming out of the page through C is $\frac{1}{3}ml^2$. The incline makes 45^o angle with the horizontal. Each of the end-points P and Q is connected to a separate mass-less platform that rolls along the inclined surface by a spring of spring-constant *k*. The center of the rod is connected to yet another mass-less platform that rolls along the inclined surface by a dashpot of damping-constant *c*. The mass-less trucks always maintain contact with the incline and ensure that the springs and the dashpot always remain perpendicular to the inclined surface. In this simple setting, we sum up the effect of a person standing on the ski-board as an external force F that *always acts vertically downwards* at point Q as shown in the figure. Assume that both the springs are initially un-stretched. Determine the equation(s) of motion of the ski-board using the Lagrangian approach following the six steps we studied in the class: define frames and generalized coordinate(s), find kinetic energy, find potential energy, define Lagrangian of the system, take derivatives and find the generalized force(s).