2.003J/1.053J Dynamics and Control I
Fall 2007

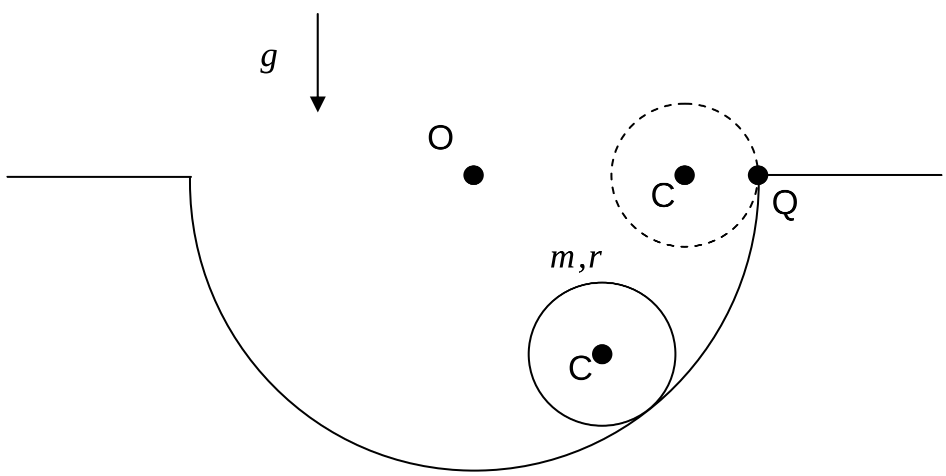
Problem Set 6

Out: Monday, 15 October, 2007
Due: Monday, 22 October, 2007

1. A disc sliding in a circular well

A circular disc of radius r and mass m is sliding in a semi-circular well of radius R as shown in the figure below. The contact between the disc and the well is *frictionless*. Let O be the center of the semi-circular well. Let C represent the center of the disc and let Q be some fixed point on the edge of the disc. At t = 0, the disc is held such that Q is at the tip of the well as shown in the figure. The disc is then released. Note that gravity acts.

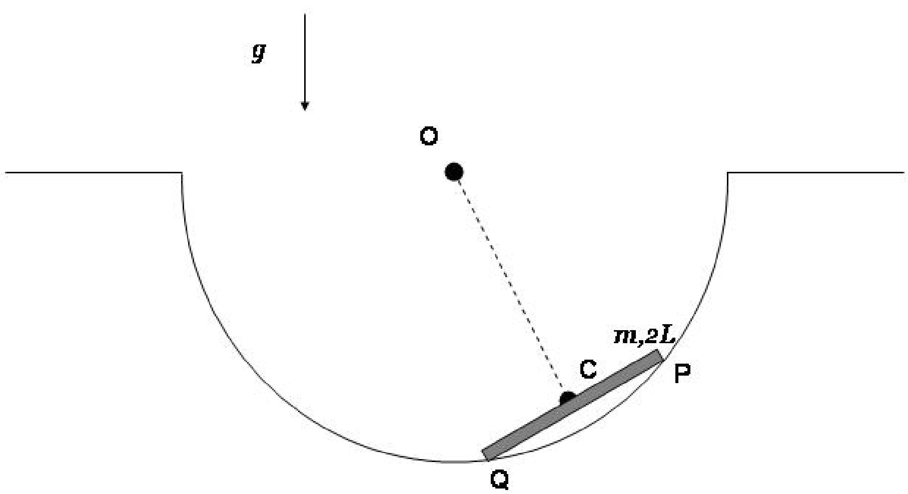
a. Where is the instantaneous center of rotation for the disc, if it exists?
b. Find the equation of motion of the disc.
c. Find the equation of motion of the disc using the work-energy principle (Take the derivative to verify whether the answer matches with the answers in the above part).



2. A slender rod sliding in a circular well

A slender rod of length $2L$ and mass m is sliding in a semi-circular well of radius R as shown in the figure below. The contact between the rod and the well is *frictionless*. Let O be the center of the semi-circular well. Let C represent the center of the rod. Let P and Q represent endpoints of the rod. At $t = 0$, the rod is held such that P is at one tip of the well and Q is in contact with the surface of the well. The rod is then released. Note that gravity acts.

a. Use the torque equation about the center of mass of the rod and the force equations to find the equation of motion of the rod.
b. Where is the instantaneous center of rotation for the rod? Use the torque equation about the instantaneous center of rotation to find the equation of motion of the rod.
c. Find the kinetic energy of the rod using two methods, once about the center of mass and once about the ICR. Find the equation of motion of the rod using the work-energy principle (Take the derivative to verify whether the answer matches with the answers in the above parts).



3. A disc purely rolling on a circular well

A circular disc of radius r and mass m is *rolling* in a semi-circular well of radius R as shown in the figure below. Let O be the center of the semi-circular well. Let C represent the center of the disc and let Q be some fixed point on the edge of the disc. At $t = 0$, the disc is held such that Q is at the tip of the well as shown in the figure. The disc is then released. Note that gravity acts.

a. What is the *kinematics constraint* for the rolling motion of the disc in the circular well?
b. Use the torque equation about the center of mass of the rod and the force equations to find the equation of motion of the rod.
c. What is the instantaneous center of rotation (ICR) for the disc? Use the torque equation about the instantaneous center of rotation to find the equation of motion of the rod.
d. Use the torque equation about point O and the force equations to find the equation of motion of the rod.
e. Find the kinetic energy of the disc using two methods, once about the center of mass and once about the ICR. Find the equation of motion of the disc using the work-energy principle (Take the derivative to verify whether the answer matches with the answers in the above parts).