1.053/2.003 Dynamics and Control I
Fall 2007

Problem Set 10

Out: Tuesday, December 4th, 2007
Due: Wednesday, December 12th, 2007

Free and forced system response

Problem 1. Rack and pinion

Consider the rack and the pinion system shown in Fig.1 below as in Problem 1 of Pset 8. The axis of the pinion is fixed in frictionless bearings. A massless rocket is attached to the circular massless pulley of radius a at a point along its edge as shown in the figure. It exerts a thrust $F(t)$ which remains tangential to the pulley at all times. Assume that the pinion can be modeled as a uniform cylinder of mass m_2 and radius b and that the friction between the rack and the horizontal surface can be modeled as viscous damping having a dashpot constant c.

a. Find the equilibrium of the system, and linearize the equations of motion about it.
b. Reduce the system to the canonical form with equivalent mass, spring, dashpot, and generalized force. Identify the values of parameters ω_n and ζ.
c. Determine the conditions on the values of m_1, m_2, k and c for which the system is under-damped, critically damped and over-damped.
d. For $F(t) = F_o \cos(\omega t)$, determine the particular solution to the ODE. Determine the value of the driving frequency at which the resonance occurs (when $\zeta = 0$).

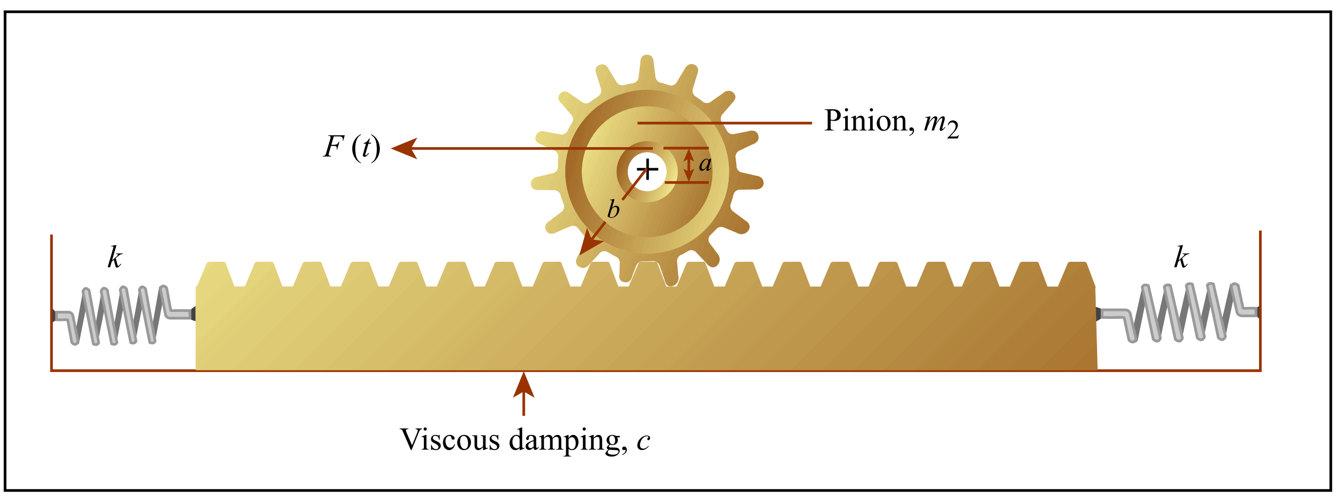

Figure by MIT OpenCourseWare.

Figure 1. Rack and pinion

Problem 2. Design the rack and pinion system

Constraints on the operation of the system in Problem 1 (about equilibrium) require that the system operates in the underdamped mode, with a maximum overshoot of 15% and a peak time of 1 ms. The mass of the rack is m_1 = 50 kg, and that of the pinion is m_2 = 20 kg.

a. Find k and c required to meet the above specifications.

b. Approximately how long will it take the system to settle?

Problem 3. Gear train

Consider the gear train shown in Fig. 2 below, which consists of three gears, G_1, G_2, and G_3, having radii R_1, R_2, and R_3, and moments of inertia I_1, I_2, and I_3 about their centers, respectively. G_1 is connected to a bearing via a shaft having a torsional stiffness k such that, when G_1 rotates by an angle θ_1, an opposing torque $k\theta_1$ develops about its center. G_2 is connected to a bearing which has an angular viscous coefficient c such that, when G_2 rotates by an angle θ_2, an opposing torque $c\theta_2$ develops about its center. G_1 and G_2 mesh without backlash (no slipping), and G_2 and G_3 are connected by a rigid, massless shaft. A driving counterclockwise torque T(t) is applied to G_1.

a. Find the equation of motion of the system.
b. Find the equilibrium of the system, and linearize the equation of motion about it.
c. Reduce the system to the canonical form with equivalent mass, spring, dashpot, and generalized force. Identify the values of parameters ω_n and ζ.
d. Determine the conditions on the values of I_1, I_2, I_3, k and c for which the system is under-damped, critically damped and over-damped.
e. Determine the conditions on the values of I_1, I_2, I_3, k and c for which the system is under-damped with a settling time of 1 s and a maximum overshoot of 10%.
f. For $T(t) = T_o \cos(\omega t)$, determine the particular solution to the ODE. Determine the value of the driving frequency at which the resonance occurs (when $\zeta = 0$).

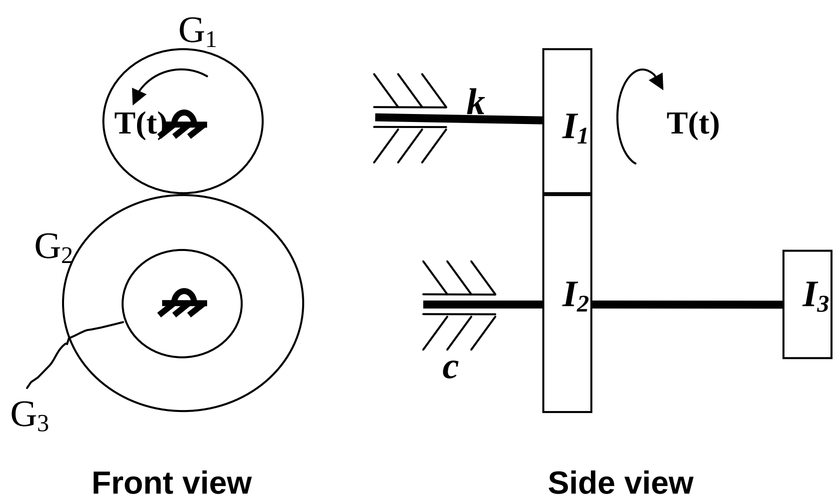

Figure 2. Gear Train

Problem 4. Mass-spring-dashpot in rotating frame

A mass m lies in a frictionless groove, on a disk that lies on the horizontal plane and rotates at a constant angular velocity Ω, as shown in Fig. 3 below. The mass is attached to the center of the disk by a spring and dashpot system, with constants k and c respectively.

a. Find the equation of motion of the system.
b. Find any equilibrium point(s).
c. Reduce the system to the canonical form with equivalent mass, spring, and dashpot. Identify the values of parameters ω_n and ζ.
d. For what values of Ω are the equilibrium points unstable?
e. Assuming Ω is chosen so that the equilibrium is stable, determine the conditions on the values of m, k and c for which the system is under-damped, critically damped and over-damped.

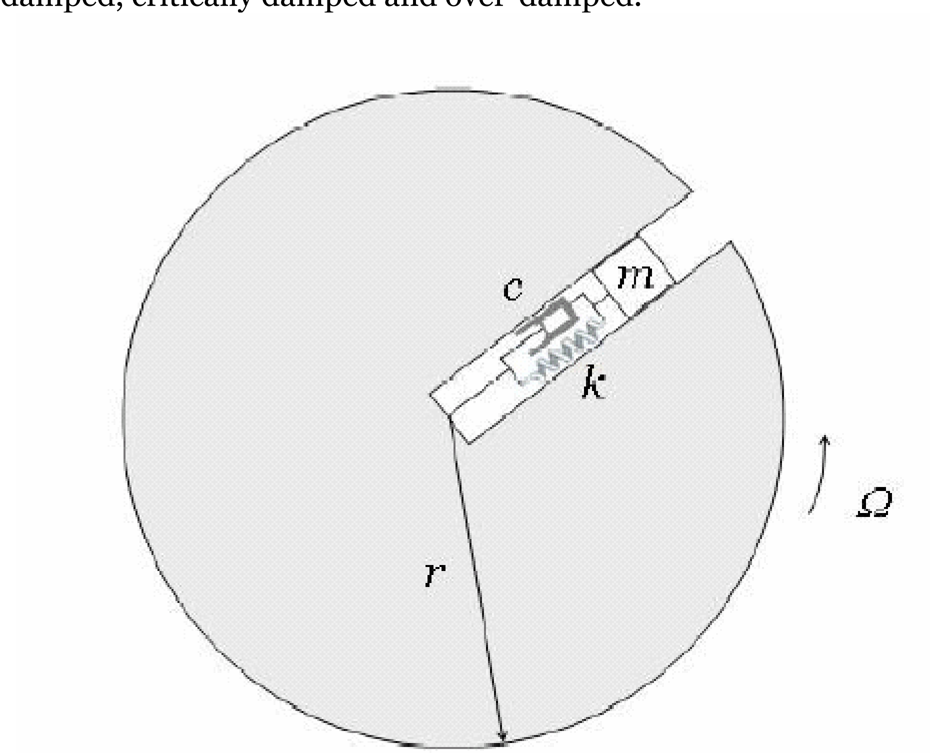

Figure 3. Mass-spring-dashpot in rotating frame

Problem 5. Automobile suspension

A car's suspension system can be modeled as shown in the figure below. Assume the car is made of two equal masses m attached via a spring of constant k and a dashpot of constant c. The vertical position of the lower mass, $y_2(t)$, is a known function determined by the terrain on which the car rides. Note that gravity acts.

a. Find the equation of motion of the system.
b. Find the equilibrium position and examine whether it is stable. Linearize the equation of motion if necessary.
c. Determine the conditions on the values of m, r, k and c for which the system is under-damped, critically damped and over-damped.
d. Assume the vehicle travels horizontally at a constant speed over a sinusoidal terrain, such that $y_2(t) = h \sin(\Omega t)$. Determine the particularsolution to the ODE. For what value of Ω will resonance occur (when $\zeta = 0$) ?

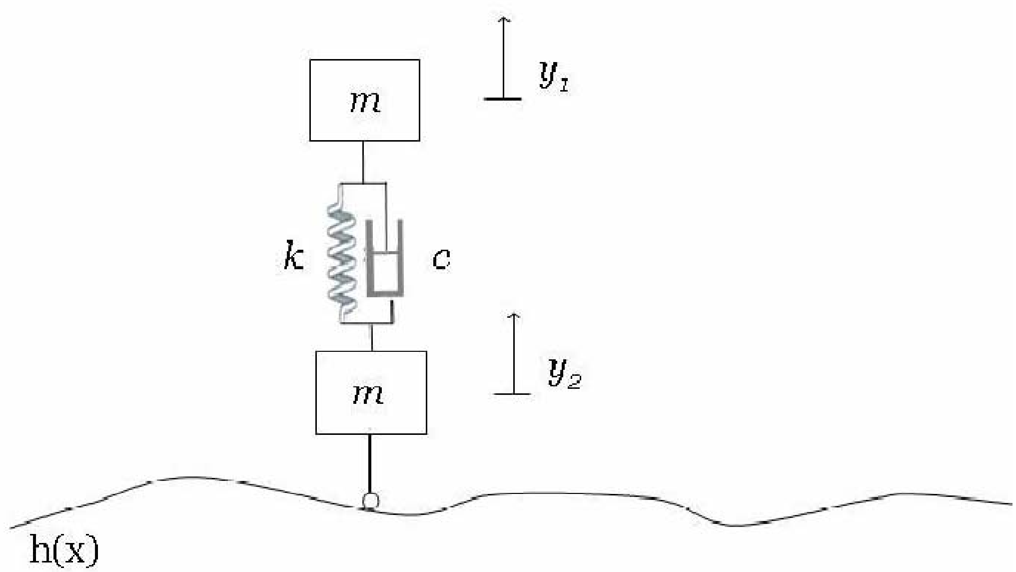

Figure 4. Automobile suspension