2.003 Spring 2003
Quiz 2 - Sample problem Set 2 Solutions

Problem A - RLC circuit analysis

1.

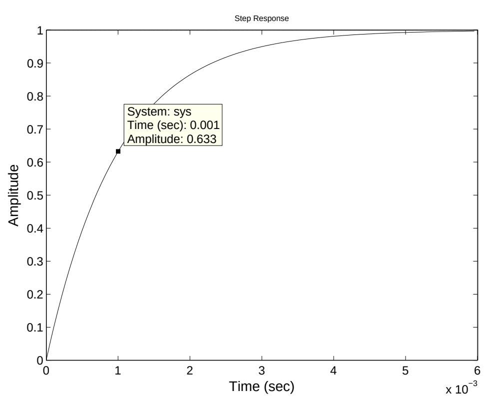
$$\frac{V_o}{V_i} = \frac{1}{LCs^2 + RCs + 1}$$

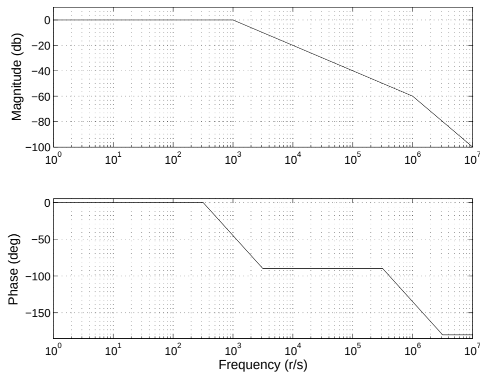
2.

$$\begin{aligned}\omega_n &= 2*\pi*5000 = 31,400\,r/s = \frac{1}{\sqrt{LC}} \\ L &= \frac{1}{\omega_n^2 C} = 0.001\,H = 1\,mH \\ \frac{R}{L} &= 2\zeta\omega_n = 2*0.707*31,400 \\ R &= 44.4\,\Omega\end{aligned}$$

3. There are no zeros, poles at roots of

$$\begin{aligned}s^2 + \frac{1000}{0.001}s + \frac{1}{1e-6*1e-3} &= 0 \\ s_1 &\approx -1e6 \\ s_2 &\approx -1e3 \text{ dominant pole} \\ x_{ss} = 1\end{aligned}$$





4.

5. $s = -1e3 = s_2$

6.

$$\text{For circuit with R\&C in } \parallel \; \frac{v_o}{v_i} = \frac{R_2}{R_2LCs^2 + (R_1R_2C + L)s + R_1 + R_2}$$
$$\text{For circuit with R\&C in series } \frac{v_o}{v_i} = \frac{R_2Cs + 1}{LCs^2 + (R_1 + R_2)Cs + 1}$$

Problem B

1.

$$\begin{aligned} T(s) &= \frac{K}{s^2 + 20s + K} \\ \omega_n &= \sqrt{K} \\ 2\zeta\omega_n &= 2\sqrt{K} = 20 \\ K &= 100 \end{aligned}$$

2.

$$\begin{aligned} \frac{V_{out}}{V_1} &= \frac{(s+3)(6s+1)}{(s+3)(6s+1) + (8s+7)} \\ \frac{V_{out}}{V_2} &= \frac{(6s+1)}{(s+3)(6s+1) + (8s+7)} \end{aligned}$$

3. Solve using superposition

$$\begin{aligned} V_{out}(6s^2 + 27s + 10) &= V_1(6s^2 + 19s + 3) + V_2(6s + 1) \\ 6\ddot{V}_{out} + 27\dot{V}_{out} + 10V_{out} &= 6\ddot{V}_1 + 19\dot{V}_1 + 3V_1 + 6\dot{(V)}_2 + V_2 \end{aligned}$$

Problem C
The transfer function for this system is

$$\frac{x(s)}{f(s)} = \frac{1}{m_{eq}s^2 + cs + k}$$
$$\omega_n = \sqrt{\frac{k}{m_{eq}}}$$
$$2\zeta\omega_n = \frac{c}{m_{eq}}$$

1. From graph, we measure the following

$$T \approx 1.0s \Rightarrow \omega_d = \frac{2\pi}{T} = 6.28r/s$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$
$$M_p \approx 100\frac{0.75-0.5}{0.5} = 50$$
$$\zeta = \frac{A}{\sqrt{\pi^2 + A^2}} = 0.215$$
$$A = ln\frac{100}{M_p} = 0.693$$
$$\omega_n = 6.43r/s$$
$$m_{eq} = \frac{\omega_n^2}{k} = 4.8 \approx 5kg$$
$$c = 2\zeta\omega_n m_{eq} = 13.8Ns/m \approx 14Ns/m$$

Alternately, you could determine ζ using the log decrement method.

2.

$$m_{eq} = m + \frac{I}{r^2}$$
$$I = 0.5\,kg\,m^2$$

Problem D
The transfer function for this system is

$$\frac{\omega(s)}{\phi(s)} = \frac{k}{Js^2 + cs + k}$$
$$\text{Thus } \omega_n = \sqrt{\frac{k}{J}}$$
$$2\zeta\omega_n = \frac{c}{J}$$

1. There are a couple of ways to solve this part of the problem. First, you can read $\omega_r = 9\,r/s$ and $M_p = 5\,dB$ from the bode plot and use the following

relationships

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$
$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

to find $\zeta \approx 0.3$ and $\omega_n \approx 10\, r/s$. Or you can read $\omega_n = 10\, r/s$ directly from the phase plot ($\theta = -90°$)

2. $k = 1500\, Nm/r$, $c = 90 Nms/r$

3.

$$\begin{aligned}
\omega = 1.1\, r/s \;\; \theta(t) &\approx \sin(1.1t + 0) \\
\omega = 10\, r/s \;\; \theta(t) &\approx 1.58 \sin(10t - \pi/2) \\
\omega = 20\, r/s \;\; \theta(t) &\approx 0.3 \sin(20t - 2.75)
\end{aligned}$$