Quiz 3 Review: Circuits and Bode plots.

Circuits and the impedance method.

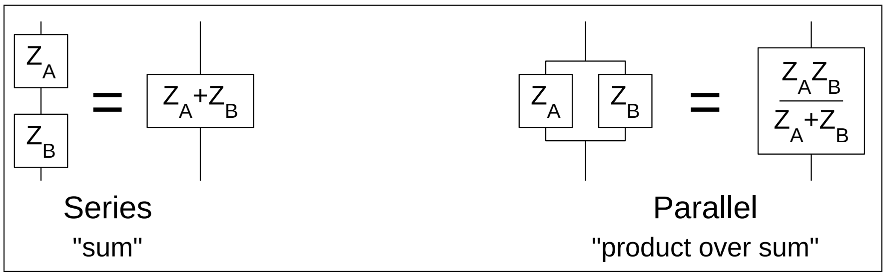
$$Z(s) \equiv \frac{E(s)}{I(s)}$$

capacitor (C) $\boxed{Z_C = \frac{1}{Cs}}$

resistor (R) $\boxed{Z_R = R}$

inductor (L) $\boxed{Z_L = Ls}$

2 impedances in series, vs. 2 impedances in parallel.



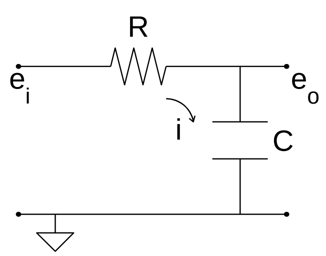
3 impedances all in series with one another.

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

3 impedances all in parallel with one another.

$$Z_{eq} = (Z_1|Z_2)\,|\,Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2}\,|\,Z_3 = \frac{\left(\frac{Z_1 Z_2}{Z_1+Z_2}\right) Z_3}{\left(\frac{Z_1 Z_2}{Z_1+Z_2}\right) + Z_3} = \frac{Z_1 Z_2 Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}$$

Ex. 1

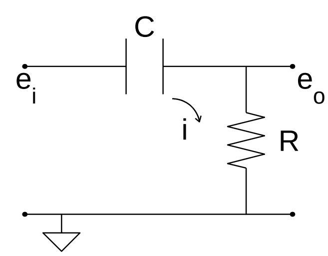


a) $\frac{I(s)}{E_i(s)} = ?$

b) $\frac{E_o(s)}{E_i(s)} = ?$

c) Bode plot for $\frac{E_o(s)}{E_i(s)}$?

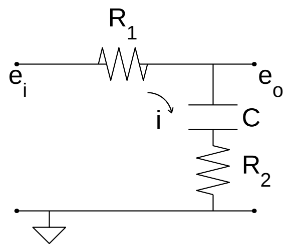
Ex. 2



a) $\frac{E_o(s)}{E_i(s)} = ?$

b) Bode plot for $\frac{E_o(s)}{E_i(s)}$?

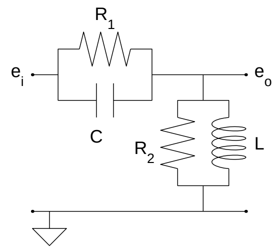
Ex. 3



a) $\frac{E_o(s)}{E_i(s)} = ?$

b) Bode plot for $\frac{E_o(s)}{E_i(s)}$?

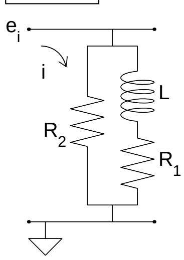
Ex. 4



a) $\frac{E_o(s)}{E_i(s)} = ?$

b) $\frac{E_o(s)}{E_i(s)}$ can be written in the form $\frac{E_o(s)}{E_i(s)} = \frac{Ks(\tau s+1)}{(as^2+bs+1)}$. What are K, τ, a, and b in term of the electrical components (R_1, C, R_2, L)?

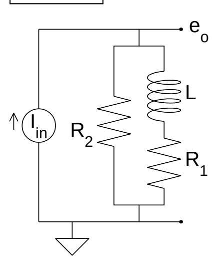
Ex. 5



a) $\frac{I(s)}{E_i(s)} = ?$

b) Sketch the Bode plot for $\frac{I(s)}{E_i(s)}$ ($L = 1\,H,\ R_1 = 100\,\Omega,\ R_2 = 900\,\Omega$).

Ex. 6



a) $\frac{E_o(s)}{I_{in}(s)} = ?$

b) Sketch the Bode plot for $\frac{I(s)}{E_i(s)}$ ($L = 1\,H,\ R_1 = 100\,\Omega,\ R_2 = 900\,\Omega$).

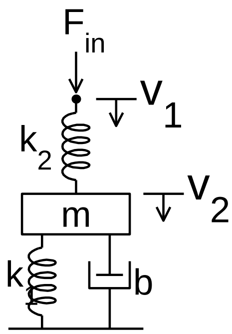
Mechanically equivalent circuits.

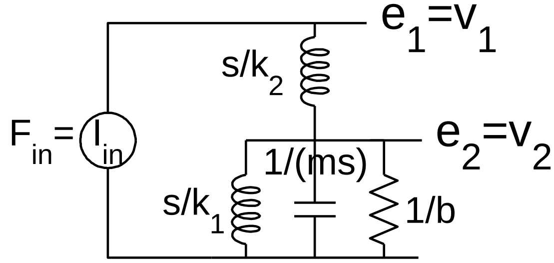
Force-current (velocity-voltage) analogy.

capacitor (C) $\boxed{Z_m = \frac{1}{ms}}$

resistor (R) $\boxed{Z_b = \frac{1}{b}}$

inductor (L) $\boxed{Z_k = \frac{s}{k}}$





$$\begin{aligned}\frac{E_1(s)}{I_{in}} = \frac{V_1}{F_{in}} = \frac{sX_1}{F_{in}} &= Z_{k2} + (Z_{k1}||Z_b||Z_m) \\ &= \frac{s}{k_2} + \left(\frac{s}{k_1}||\left(\frac{\frac{1}{ms}\frac{1}{b}}{\frac{1}{ms}+\frac{1}{b}}\right)\right) \\ &= \frac{s}{k_2} + \frac{\frac{s}{k_1}\frac{1}{ms}\frac{1}{b}}{\frac{s}{k_1}\frac{1}{ms} + \frac{s}{k_1}\frac{1}{b} + \frac{1}{ms}\frac{1}{b}}\end{aligned}$$

Bode plots.

One method to make (or analyze) a Bode plot.

1) Put the transfer function into the form:

$$[Ks^p] \cdot \left[\frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\ldots\left(\frac{s}{z_m}+1\right)}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\ldots\left(\frac{s}{p_n}+1\right)}\right] \tag{1}$$

2) At low freq ($s \approx 0$), the RHS in eqn 1 is unity (1), so we can now draw a low frequency asymptote based on the LHS alone. (Note p is of course just an integer equal to the number of zeros at the origin minus the number of poles at the origin.)

$$Ks^p$$

We can pick *any* value of ω, calculate $K\omega^p$ at this frequency, and draw a line through this point with slope p. $s = 1$ is often a useful point to choose, because $1^p = 1$ for any p! The low frequency phase will be $p \cdot 90°$ (if $K > 0$).

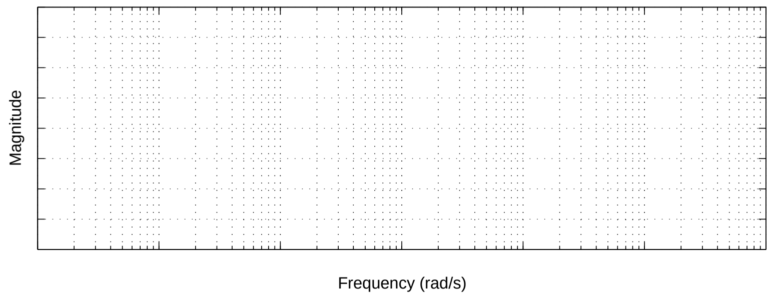
3) Find all break points (poles and zeros) by looking at the RHS and indicate where they will occur on the plot, so we can get ready for step 4. (Remember to use ω_n for any complex pairs.)

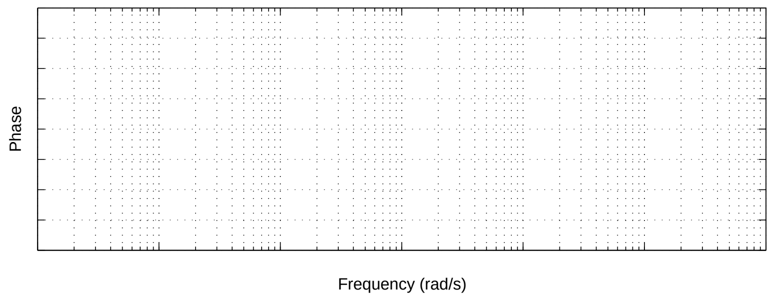
4) Start on the low frequency asymptote at a frequency below any breakpoints. Moving toward higher frequencies, "break" at each breakpoint. The slope will change by $+1$ (20dB/dec) for each zero or by -1 (-20dB/dec) for each pole at a particular breakpoint. (Remember there can be two or more at one particular frequency.) The phase will change by $+90°$ for each zero or by $-90°$ for each pole at a particular frequency (if these poles or zeros are in the left-half plane).

Ex. 1 On the axes provided, sketch the Bode plot for the transfer function

$$H(s) = \frac{(s+10)}{s}$$

Be sure to label the axes as appropriate!

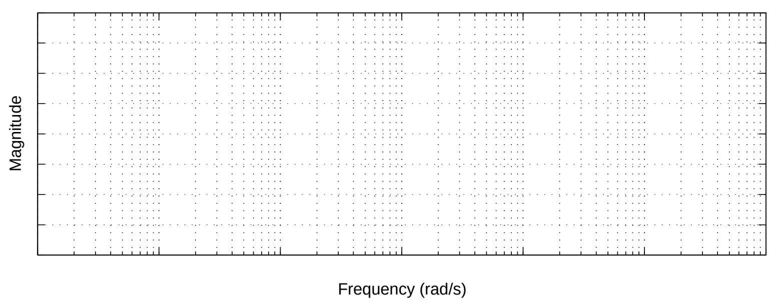


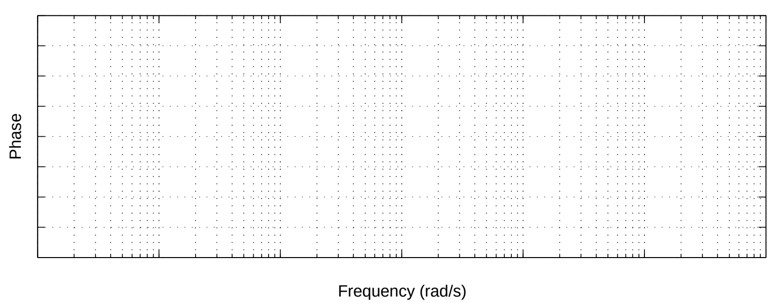


Ex. 2 On the axes provided, sketch the Bode plot for the transfer function

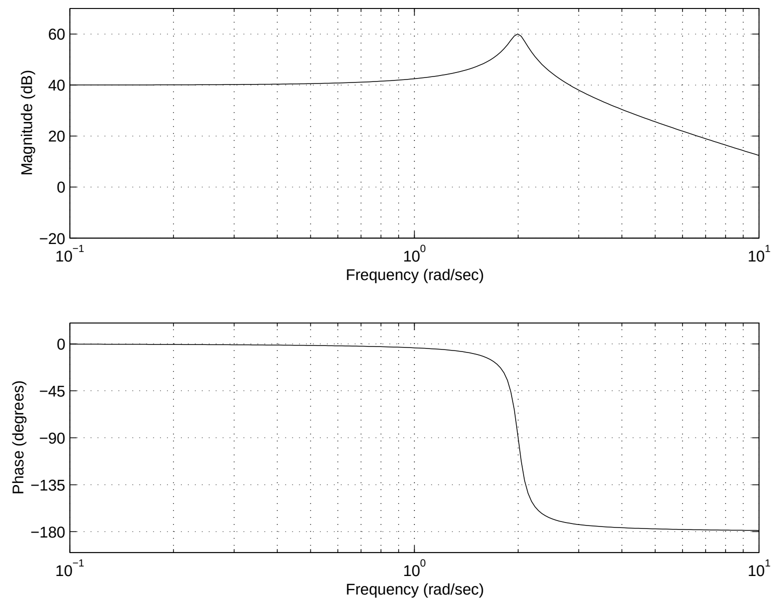
$$H(s) = \frac{20(s+0.1)(s+2)}{s(s+40)}$$

Be sure to label the axes as appropriate!



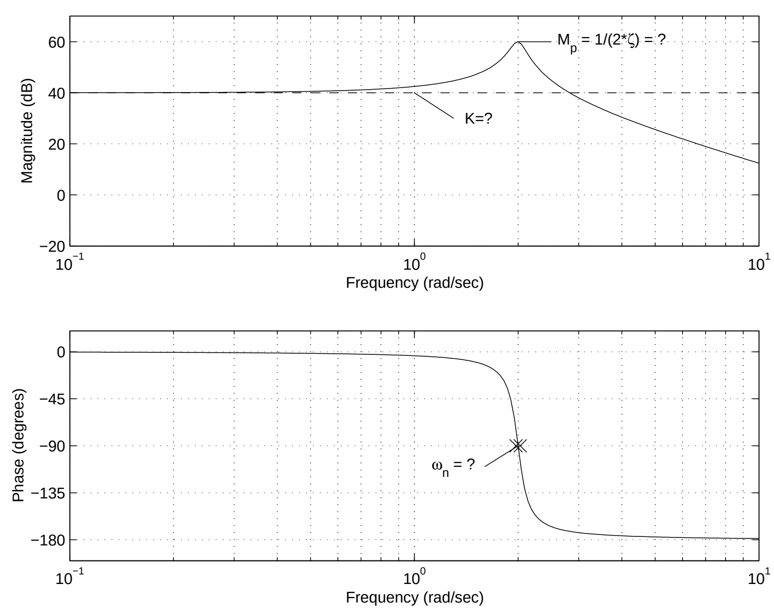


Ex. 3 What is the transfer function $H(s)$ represented by the Bode plot below?

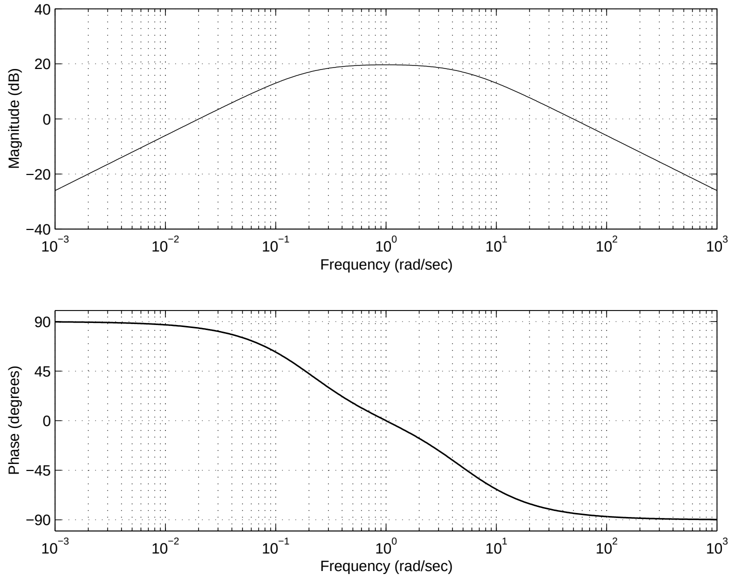


The plot below shows a good starting point. You should recognize this transfer function is of the form $H(s) = K_{dc}\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$. We can find ω_n where the phase is $-90°$: $\omega_n = 2$ rad/s. K_{dc} is the DC (low frequency) of the magnitude: $K_{dc} = 10^{(40/20)} = 100$. At ω_n, the magnitude is at $M(\omega_n) = 10^{(60/20)} = 1000$. $M(\omega_n)/K_{dc} = \frac{1}{2\zeta} = 1000/100$. So $\zeta = 1/(2 \cdot 10) = 0.05$.

$$\begin{aligned} H(s) &= \left[100s^0\right] \cdot \left[\frac{2^2}{s^2 + 2 \cdot .05 \cdot 2 + 2^2}\right] \\ &= \frac{400}{s^2 + 0.2s + 4} \end{aligned}$$



Ex. 4 What is the transfer function $H(s)$ represented by the Bode plot below?



We can assume there is a zero at the origin, because the slope is +20 dB/dec and the phase is $+90°$ at low frequency. Therefore the LHS of the TF will be $Ks^1 = Ks$. To find K, we can extend the low frequency asymptote, and its value at 1 rad/sec *is* K. This is difficult to estimate, so we can look at *any* other frequency that looks more convenient. At $\omega = 0.02$ rad/s, the magnitude is 0dB=1, so $K \cdot 0.02 = 1.0$, $K = 1/0.2 = 50$. There are then two breakpoints, both poles, so the phase goes to $-90°$ and the slope is -20 dB/dec at high frequency. We can use either phase (at -45 and -135) or magnitude (estimating where asymptotes intersect) to find that the poles are at approximately 0.2 and 5.0 rad/s.

$$
\begin{aligned}
H(s) &= [50s] \cdot \left[\frac{1}{(\frac{s}{.2}+1)(\frac{s}{5}+1)} \right] \\
&= \frac{50s}{(s+.2)(s+5)}
\end{aligned}
$$