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2.002 MECHANICS AND MATERIALS II
FATIGUE CRACK GROWTH EXAMPLE

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A bar of 4340 steel, of thickness $t = 12\,mm$ and width $w = 60\,mm$, is subjected to a cyclic bending moment that ranges from maximum value $M_{(\max)} = 4\,kNm$ to minimum value $M_{(\min)} = 0.8\,kNm$.

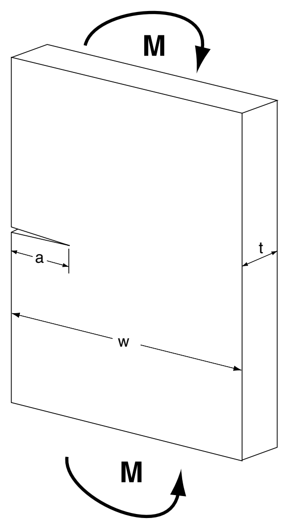

Figure 1: Schematic of edge-cracked specimen under bending.

The steel has Young's modulus $E = 208\,GPa$, Poisson ratio $\nu = 0.3$, tensile yield strength $\sigma_y = 1255\,MPa$, and ultimate tensile strength $UTS = 1295\,MPa$.

Fatigue crack propagation in the alloy is well-represented by a Paris-type relation of the sort

$$\frac{da}{dN} = \Delta a_0 \left(\frac{\Delta K_I}{\Delta K_{I0}} \right)^m ,$$

where the exponent is $m = 3.24$, and the reference constants can be taken as a growth rate of $da/dN = 10^{-3}\,mm/\text{cycle} \equiv \Delta a_0$ when the applied stress intensity range is $\Delta K_I = 100\,MPa\sqrt{m} \equiv \Delta K_{I0}$.

After $60,000$ cycles of the loading described, the bar fractures, with failure due to a through-thickness edge crack of [failure] length $a_f = 14\,mm$, emanating from the "tensile side" of the bending stress field.

The stress intensity factor for a rectangular beam containing an edge crack of length "a" and subjected to bending moment "M" can be expressed as

$$K_I = Q\,\sigma_b\,\sqrt{\pi a},$$

where σ_b is the peak tensile bending stress in the uncracked beam, subjected to bending moment M, and the configuration correction factor can be taken as constant, $Q = 1.12$, providing the relative crack depth $a/w <\sim 0.3$.

Estimate the initial crack size, a_i, that grew to cause the final fracture.