

Massachusetts Institute of Technology  
Department of Mechanical Engineering  
Cambridge, MA 02139

2.002 Mechanics and Materials II  
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Laboratory Module No. 6

**Fracture Toughness Testing  
and  
Residual Load-Carrying Capacity of a Structure.**

## 1 Background

A component has been manufactured from 7075-T6 aluminum alloy. The geometry consists of a plate of width 1.5" and thickness 0.2", containing a centrally located hole of 0.5" diameter, as shown schematically in Fig. ???. In service, the component will be subjected to cyclic tensile loading, and there is concern about the possibility of fatigue cracks emanating from the stress concentration of the hole. The purpose of this laboratory module is the development of a methodology enabling us to estimate the load-carrying capacity of this part in the presence of such fatigue cracks.

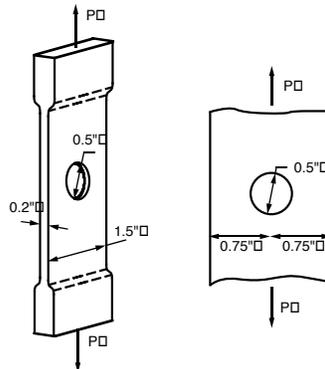


Figure 1: Geometry of the component.

This laboratory module will be concerned with standard test procedures to evaluate material toughness, and will introduce and apply a methodology to evaluate the residual load-carrying capacity of a cracked structure.

## 2 Laboratory Tasks

1. We will conduct a plane strain fracture toughness test, in which we will measure the fracture toughness,  $K_{Ic}$ , of 7075-T6 aluminum, following the ASTM standard (E-399) procedure for determining  $K_{Ic}$ . We will use a compact tension specimen (CTS) geometry to evaluate  $K_{Ic}$ .
2. We will use the material properties of the 7075-T6 Al that quantify resistance to both
  - (a) plastic deformation ( $\sigma_y$ ), and
  - (b) resistance to crack extension ( $K_{Ic}$ ),

in fracture-based and collapse-based analyses of the effects of cracks of various sizes emanating from the equatorial stress concentration of the component in order to predict the **residual load-carrying capacity** (as a function of crack size) for the component.

3. We will conduct load-to-failure tests on specimens of the structural component, containing pre-existing cracks of various lengths in order to obtain experimental values for residual strength, and we will compare these results with our predictions.

### 3 Lab Assignments: Specific Questions to Answer

1. During the lab session, we conducted a  $K_{Ic}$  test on a compact tension specimen for the 7075-T6 aluminum alloy. The specimen was machined from a rolled plate with the crack extension direction parallel to the rolling direction. Describe the protocol of material testing, and calculate the value of  $K_{Ic}$  according to the ASTM standard ( $\sigma_y$  of 7075-T6 is 500 MPa). Use the  $K_{Ic}$  Test - Data Sheet provided in this Handout.
2. Construct the theoretical residual strength curve for the [crack-containing] component in Fig. ?? : evaluate and plot  $P_{res}$  as a function of crack length,  $L$ , for  $0 \leq L \leq b$ , where  $b$  is the semi-width of the plate. The configuration correction factor for the crack geometry in Figure ?? is given in Fig. ?. Assume  $\sigma_y = 500$  MPa, and you should use the appropriate value of the plane strain fracture toughness,  $K_{Ic}$ , as evaluated for this material/crack orientation in the preceding section.
3. Obtain from the web site the experimental crack-size/failure-load data for the component of Figure 6, as evaluated over all of the lab sessions. Compare these experimental values of  $P_{res}$  with your theoretical predictions. Is the agreement satisfactory? Are the predictions conservative?
4. Consider effects of component thickness on crack [fracture] toughness: the so-called “plane stress” fracture toughness value,  $K_c$ , often exceeds the asymptotic plane strain fracture toughness,  $K_{Ic}$ , obtained from thick[er] test specimens. It is customary to use a thickness-dependent toughness value,  $K_c(B)$ , where  $B$  is plate thickness, to better fit fracture data in sheets of reduced thickness,  $B$ . Fig. ?? shows an experimentally-based normalization of the ratio  $F \equiv K_c(B)/K_{Ic}$ , appropriate to the component geometry of Fig. ?. In Fig. ??, the relation between the normalization factor,  $F \equiv K_c(B)/K_{Ic}$ , and the specimen thickness,  $B$ , is expressed in terms of the normalized thickness  $B/B_0$ , where  $B_0$  is a reference length given by

$$B_0 \equiv \frac{1}{3\pi} \left( \frac{K_{Ic}}{\sigma_y} \right)^2.$$

Choose a ‘thickness-corrected’ value of material toughness,  $K_c(B)$  for constructing a modified residual strength curve of the component, using  $K_c(B)$  in place of  $K_{Ic}$ . Can better agreement between the experimental data and the new residual strength curve be obtained between predicted ”fracture load” and that experimentally observed?

**Note:** use of the thickness-based “correction factor for toughness” shown in Fig. ?? can lead to substantial confusion unless it is properly done and interpreted. If your attempts at using the figure to estimate a “better”  $K_c$ -value for the 0.2 *in*

thick component lead you to unreasonable numbers that do not really improve the agreement (compared to the residual strength curve constructed from  $K_{Ic}$ ), then, instead, just try to fit the “fracture load” part of the theoretical residual strength curve with some value of  $K_c \geq K_{Ic}$ . What is your best estimate of  $K_c(B = 0.2 \text{ in})$ , and hence of  $F = K_c/K_{Ic}$ ?

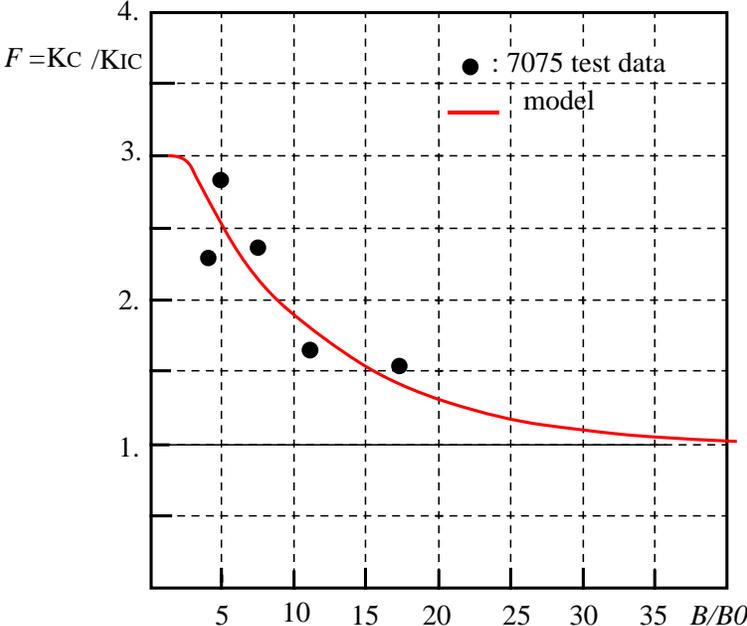


Figure 2: Thickness correction factor for  $K_c(B)$  in terms of plane strain fracture toughness,  $K_{Ic}$ , tensile yield strength,  $\sigma_y$ , and plate thickness  $B$ . The reference thickness  $B_0$  used in normalizing the abscissa depends on the material properties  $\sigma_t$  and  $K_{Ic}$  through  $B_0 = (K_{Ic})^2 / (3\pi\sigma_y^2)$ .

## 4 Plane-Strain Fracture Toughness Test Procedure

Reference: see Dowling sections 8.7 and 8.6

The ASTM standard (E-399) for plane strain fracture toughness testing provides a procedure for experimentally determining values of  $K_{Ic}$  for metallic materials. The test permits three different specimen shapes: a bend specimen, a C-shaped specimen, and a so-called “compact tension” (CT) specimen<sup>1</sup>. The CT specimen, whose geometry is illustrated in Figure ??, will be used in this laboratory.

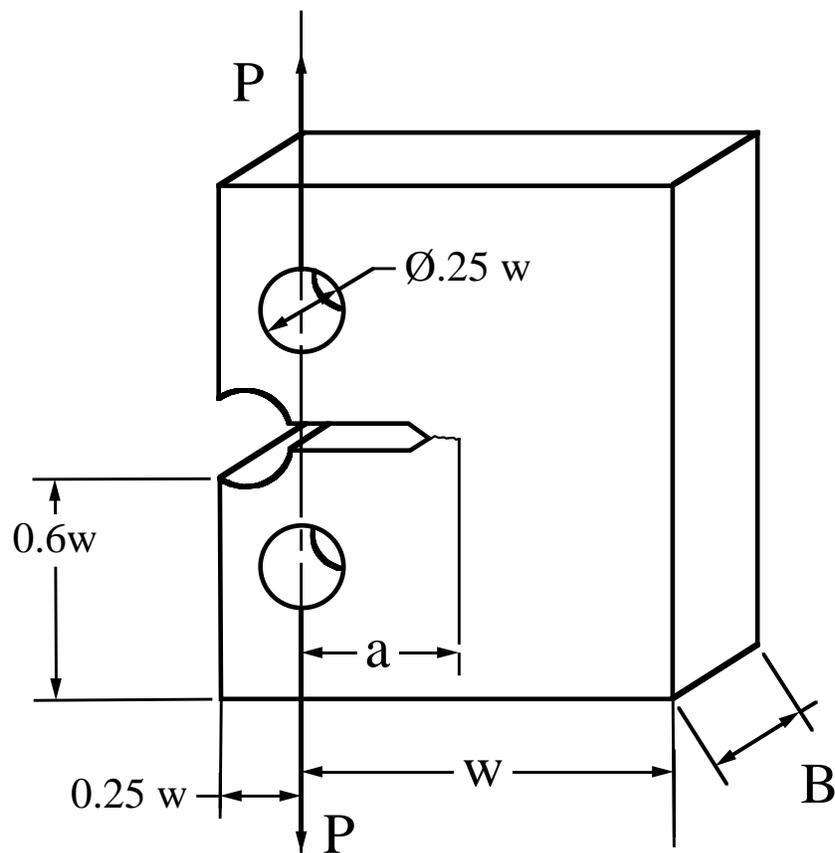


Figure 3: Standard ASTM compact tension (CT) specimen.

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<sup>1</sup>In fact, the loading on the remaining, uncracked ligament of the CT specimen is predominantly bending, so perhaps a more accurate name for this specimen geometry would be “compact-bending”.

The procedure for measuring  $K_{Ic}$  with a CT specimen follows:

1. Make a guess<sup>2</sup> of the expected value of  $K_{Ic}$ . This enables you to calculate an estimated critical plastic zone size

$$r_{Ic} \equiv \frac{1}{2\pi} \left( \frac{K_{Ic}}{\sigma_y} \right)^2.$$

2. To ensure that only small-scale yielding occurs at the crack tip, the length,  $a$ , of the crack and the remaining ligament,  $(w - a)$ , should be greater than or equal to  $15r_{Ic}$ :

$$a, (w - a) \geq 15r_{Ic}.$$

3. To ensure plane strain, the thickness,  $B$ , of the CT specimen should be greater than or equal to  $15r_{Ic}$ :

$$B \geq 15r_{Ic}.$$

4. Once a CT specimen is machined, according to the dimensions calculated above, a sharp crack is introduced at the root of the machined notch. This is accomplished by fatigue pre-cracking the specimen. This procedure involves imposing a time-varying tensile load on the CT specimen to cause a sharp crack to initiate and slowly grow at the root of the machined notch. The maximum load during fatigue pre-cracking ( $P_{f_{\max}}$ ) should be less than 0.6 times the value of the estimated final fracture load ( $P_Q$ ):

$$P_{f_{\max}} \lesssim 0.6P_Q.$$

5. The fatigue-generated portion of the crack should be at least 1.2 *mm* long. The “target” value for the relative crack size is  $a/w \doteq 1/2$ , and provisions are made to permit small variations about this value.
6. Once a suitably long and sharp fatigue crack exists, the actual fracture toughness test can be performed. The test consists of monotonically increasing the tensile load,  $P$ , on the specimen slowly while measuring the opening displacement of the crack mouth,  $\Delta$ . Plotting the  $P$  versus  $\Delta$  produces a curve similar to the one illustrated in Fig. ???. Fast fracture is indicated by a gross nonlinearity in the load-displacement record.

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<sup>2</sup>Note that in order to experimentally determine  $K_{Ic}$ , it is necessary first to specify the specimen dimensions on the basis of a known value of  $K_{Ic}$ ! This paradoxical situation is resolved by making a suitable overestimate of  $K_{Ic}$  on the basis of known values for similar materials, and checking the validity after the test. Subsequent tests may then make use of specimens which are more economically dimensioned.

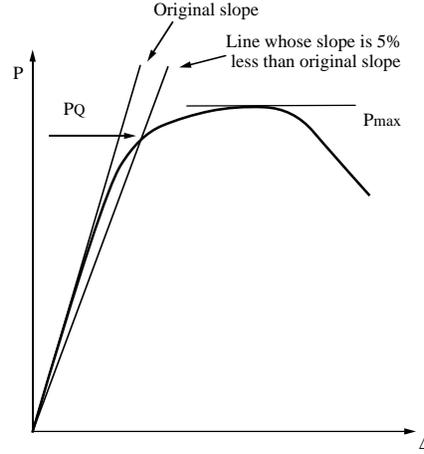


Figure 4: Schematic load ( $P$ ) vs. crack-mouth opening displacement ( $\Delta$ ) curve obtained during fracture toughness test. The load level “ $P_Q$ ” is defined as the load at the intersection of the  $P$ - $\Delta$  curve with a straight line from the origin having a slope 5% less than the initial linear slope of the  $P$ - $\Delta$  curve. More details are contained in the ASTM E-399 standard.

7. Determine the crack length<sup>3</sup>  $a$  by measuring the initial crack length, notch plus fatigue pre-crack, on the fractured specimen. The fracture surface appearance will in general change at the (crack-front) boundary marking the transition between [prior] fatigue cracking and rapid fracturing in the test.
8. Calculate the configuration correction factor,  $Q_{CT}$ , for the CT specimen geometry of relative crack depth,  $a/w$ , as:

$$Q_{CT}(a/w) = 16.7 - 104.7(a/w) + 369.9(a/w)^2 - 573.80(a/w)^3 + 360.5(a/w)^4.$$

A graph of this relation, which has been calibrated from detailed elastic stress analysis of the CT specimen, is given in Fig. ??.

9. To calculate  $K_{Ic}$ , first calculate a “conditional” value, termed  $K_Q$ , according to:

$$K_Q \equiv Q_{CT} \sigma_Q \sqrt{\pi a},$$

where

$$\sigma_Q \equiv P_Q/(Bw).$$

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<sup>3</sup>Note: the length “ $a$ ” is measured from the loading line to the crack-front (see Fig. ??).

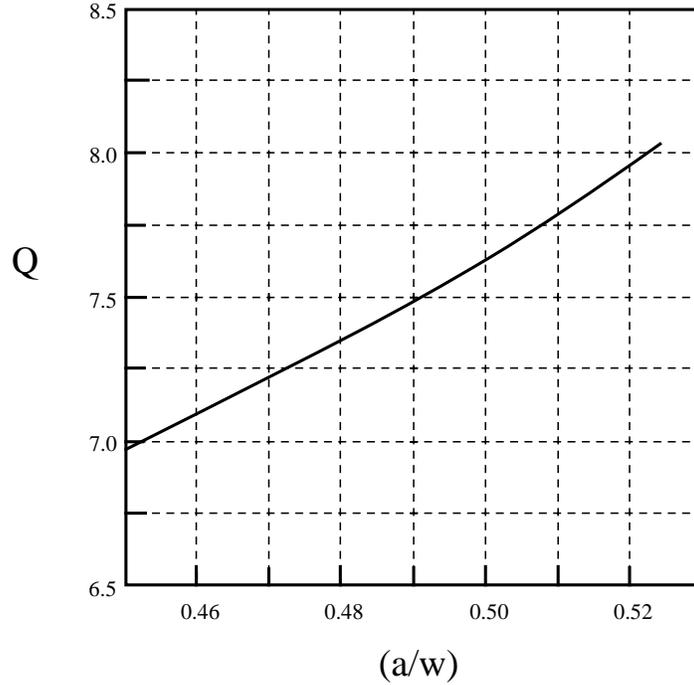


Figure 5: Graph of configuration correction factor,  $Q_{CT}$ , in the CT specimen geometry, vs. relative crack depth,  $a/w$ . The graph is of the fourth-order polynomial fit to the function  $Q_{CT}$  given in point number 8.

As noted above,  $P_Q$  is determined by projecting a line whose slope is five percent less than the original slope of the  $P - \Delta$  curve.  $P_Q$  is the load corresponding to the intersection of this line with the  $P - \Delta$  curve. See Fig. ???. (The subtleties of determining  $P_Q$  from variously-shaped  $P - \Delta$  records are clearly explained in the standard.)

10. The ratio  $P_{\max}/P_Q$  should be less than 1.10, where  $P_{\max}$  is the maximum load encountered in the test:

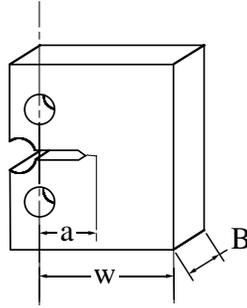
$$\frac{P_{\max}}{P_Q} < 1.10.$$

11. If condition 10 holds, then calculate the length-dimensioned entity

$$L_Q \equiv \frac{15}{2\pi} (K_Q/\sigma_y)^2.$$

If this quantity,  $L_Q$ , is less than the specimen thickness,  $B$ , the crack length,  $a$ , and the remaining ligament ( $w - a$ ), then the fracture toughness,  $K_{Ic}$ , is simply equal to the previously-calculated value,  $K_Q$ . If  $L_Q$  exceeds any of these specimen dimensions, the test is not a “valid”  $K_{Ic}$  test, in the sense of the E-399 standard.

# PLANE STRAIN FRACTURE DATA SHEET



Material 7075-T6 Al. Crack || to R.D.

$\sigma_y =$  500 MPa

B = 0.01285 m

W = 0.05008 m

a = \_\_\_\_\_ m

$P_Q =$  \_\_\_\_\_ N

$\sigma_Q \equiv P_Q / (BW) =$  \_\_\_\_\_ MPa

Configuration correction factor  $Q_{CT} =$  \_\_\_\_\_

$K_Q = Q_{CT} \cdot \sigma_Q \cdot \sqrt{\pi a} =$  \_\_\_\_\_ MPa $\sqrt{m}$

$L_Q \equiv \frac{15}{2\pi} \left( \frac{K_Q}{\sigma_y} \right)^2 =$  \_\_\_\_\_ m

Are  $\{a, (W - a)\} > \frac{15}{2\pi} \left( \frac{K_Q}{\sigma_y} \right)^2$  ? (small-scale yielding?) \_\_\_\_\_

Is  $B > \frac{15}{2\pi} \left( \frac{K_Q}{\sigma_y} \right)^2$  ? (plane strain crack plasticity?) \_\_\_\_\_

If both s.-s.y. and plane strain conditions hold, then we have a valid test, and

$K_{Ic} = K_Q =$  \_\_\_\_\_ MPa $\sqrt{m}$

## 5 Residual Structural Load-Carrying Capacity of a Cracked Component (Residual Structural “Strength”)

The residual load-carrying capacity, (or “*residual strength*”<sup>4</sup>,  $P_{\text{res}}$ ), of a cracked structure may be defined as the maximum *load* which can be applied to the structure without reaching either

- (a) the fracture load ( $P_{\text{fract}}$ ) causing crack extension or
- (b) the collapse ( $P_{\text{collapse}}$ ) causing fully-plastic flow (limit load behavior).

In other words, if the load required to cause fracture by making  $K_I \rightarrow K_{Ic}$  is  $P_{\text{fract}}$  (fracture load), and the load required to cause plastic collapse (by reaching the fully-plastic limit load) is  $P_{\text{collapse}}$ , then one way to estimate residual load-carrying capacity (neglecting any possible interactions between fracture and large-scale yielding) is to take

$$P_{\text{res}} \equiv \min (P_{\text{fract}}; P_{\text{collapse}}).$$

For a fixed structural geometry, both  $P_{\text{fract}}$  and  $P_{\text{collapse}}$  depend on crack size,  $a$ , so that  $P_{\text{res}}$  is a function of crack size as well.

In tensile-loaded structures, one estimate of the structural **collapse load** is obtained by assuming that the collapse load brings the *average* tensile stress on the uncracked ligament to the material’s tensile yield strength,  $\sigma_y$ . Let  $A_{\text{net}}$  denote the net-section area on the crack plane. The average tensile stress (or nominal stress,  $\sigma_{\text{nom}}$ ) acting across the uncracked ligament is  $\sigma_{\text{nom}} = P/A_{\text{net}}$ , and the simple criterion for structural collapse then becomes:

$$\text{Structural collapse when } \sigma_{\text{nom}} = \sigma_y.$$

Equivalently, phrased in terms of load,  $P$ , the estimate of collapse load is given by

$$P = P_{\text{collapse}}(a) = A_{\text{net}}(a) \cdot \sigma_{\text{nom}(\text{collapse})} = A_{\text{net}}(a) \cdot \sigma_y.$$

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<sup>4</sup>In this definition, the term “strength” is used to indicate a “load” level, **not** a “stress” level.

The load required to cause *fracture* of a specimen with a sharp crack is denoted by  $P_{\text{fract}}$ . In tensile-loaded structures, an estimate of the fracture load can be made by stating that it corresponds to raising the mode I stress intensity factor  $K_I$  to the *plane strain fracture toughness*,  $K_{Ic}$ , of the material. Let  $A_\infty$  denote the nominal cross-sectional area of the component far from the crack plane. The far-field tensile stress,  $\sigma^\infty$ , is then:

$$\sigma^\infty \equiv \frac{P}{A_\infty}.$$

The mode I stress intensity factor is

$$K_I = Q \sigma^\infty \sqrt{\pi a},$$

where  $a$  is the crack length, and  $Q$  is the configuration correction factor *applicable to the crack geometry in question*<sup>5</sup>. The simple criterion for structural fracture then becomes:

**Structural fracture when  $K_I = K_{Ic}$ .**

Equivalently, phrased in terms of load, the estimate of fracture load is

$$P = P_{\text{fract}} = \frac{K_{Ic} \cdot A_\infty}{Q(a) \cdot \sqrt{\pi a}}.$$

In this case, the dependence of  $P_{\text{fract}}$  on crack length  $a$  arises from both the square root of crack size itself,  $\sqrt{a}$ , plus the crack-length dependence of the configuration correction factor,  $Q = \hat{Q}(a)$ .

In particular, for the component in Fig. ??, we want to calculate the residual strength (maximum supportable load) of the structure in the presence of two identical equatorial cracks of length  $L$ , emanating from the notch roots, as indicated in Fig. ?. As discussed above, the residual strength,  $P_{\text{res}}$ , of the cracked component will be a function of crack length,  $L$ .

In order to obtain a theoretical prediction for the fracture load,  $P_{\text{fract}}$ , associated with each value of  $L$ , we need to determine the corresponding configuration correction factor,

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<sup>5</sup>Note: A graph containing information for determining the  $Q$ -factor for the component of Fig. ?? appears in Fig. ?? below; for this purpose, do not use the graph of Figure ??, which is the  $Q$ -factor for the CT specimen geometry,  $Q_{CT}$ .

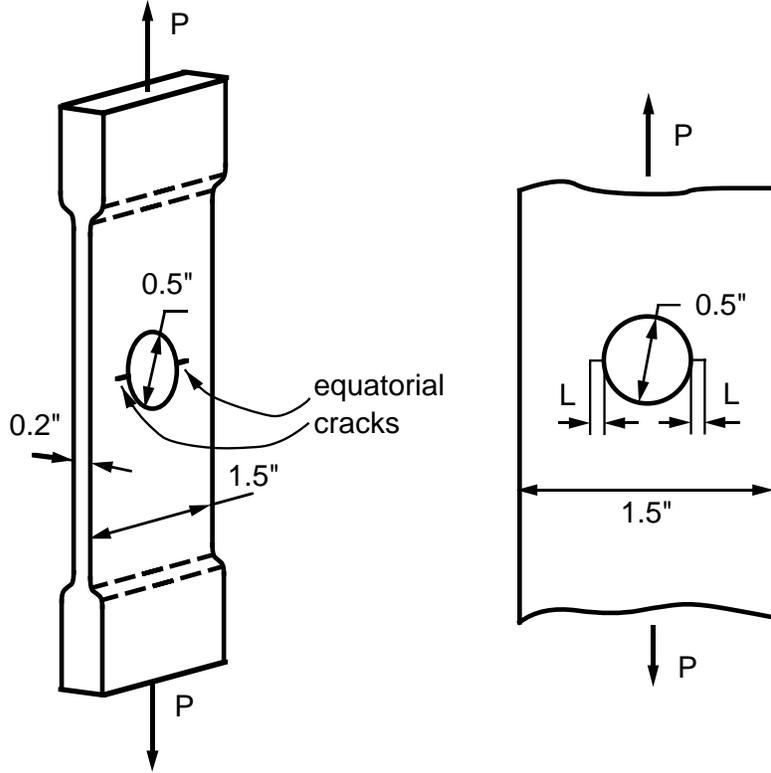


Figure 6: Geometry of the cracked component.

$Q$ , for this structural crack geometry. Values of  $Q$  for this type of component/crack geometry can be obtained by making use of the graph in Figure ??, where  $Q$ -factors are expressed for various combinations of hole sizes, specimen widths, and crack lengths; some interpolation may be needed in order to match the current specimen geometry. (Note that data are expressed in terms of the corrected crack length,  $a = L + R$ , where  $R$  is the radius of the hole.)

In order to assess the accuracy of our predictions, we will experimentally measure the residual strength of two components. First, we will consider an undamaged component, i.e., a component with no cracks ( $L = 0$ ;  $a = R$ ). We will apply increasing levels of tensile load to the component, and record the failure load [ $P_{\text{res}}(L=0)$ ]. We will then load to failure a second component which has been pre-cracked by (i) machining starter notches, and then (ii) propagating a sharp fatigue crack to total length  $L = L^{\text{star}}$  by subjecting the component to cyclic loading. After testing the pre-cracked component to failure, we will measure the initial (fatigue) crack length,  $L^*$ , by examining the fracture surface, and record the experimental failure load [ $P_{\text{res}}(L = L^*)$ ].

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Figure 7: Configuration correction factor  $Q$  for a central hole in a tensile-loaded strip, containing two equal cracks emanating from the stress concentrations of the hole. Source: *The Stress Analysis of Cracks Handbook*, H. Tada, et al., ASME, NY,. 2001