MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
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2.002 MECHANICS AND MATERIALS II
SOLUTIONS FOR HOMEWORK NO. 6

Problem 1 (40 points)
Part A:

Results are shown in Figure 1. Matlab scripts for Part A and B are attached. Graphically estimated fitting constants are: $A = 9.5737 \times 10^{-12}[m/cycle(MPa\sqrt{m})^m]$, $m = 3.17144$

Part B:

Least squares fitted constants are: $A = 8.8497 \times 10^{-12}[m/cycle(MPa\sqrt{m})^m]$, $m = 3.21473$

Part C:

With the data for Part B, the"Paris-law" is given as following:

$$\frac{da}{dN} = A(\Delta K_I)^m = 8.8497 \times 10^{-12}(\Delta K)^{3.21473} \tag{1}$$

"Paris-law" can be rewritten as :

$$\frac{da}{dN} = \Delta a_0 (\frac{\Delta K_I}{\Delta K_{I0}})^m \tag{2}$$

where $\Delta a_0 \equiv (\frac{da}{dN})_0$ is the corresponding reference crack growth rate, and ΔK_{I0} is a reference crack driving force. ΔK_{I0} and Δa_0 are the values of any point on the power law growth rate curve. If we choose $\Delta K_{I0} = 6.0 MPa\sqrt{m}$, the corresponding $\Delta a_0 = A \times \Delta K_{I0}^m = 2.8085 \times 10^{-9} m/cycle$, and m does not change.

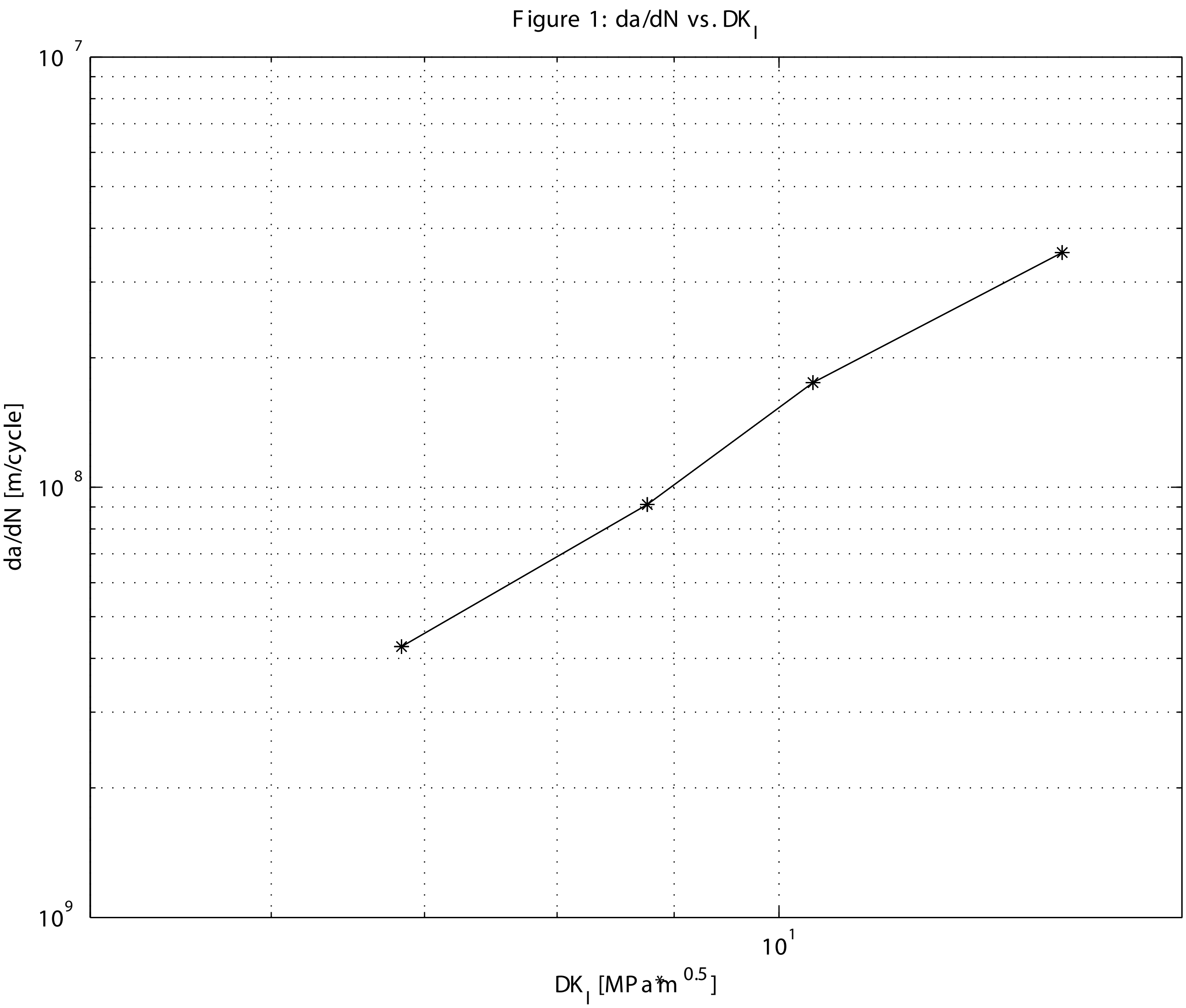
Figure 1: da/dN vs. DK_I
da/dN [m/cycle]
10 7
10 8
10 9
10 1
DK_I [MPa*m 0.5]

```
% Problem 1


da_dN=1e-6*[4.26,9.12,17.5,35.1]';    % unit (mm/cycle)
deltaKI=[6.84,8.76,10.35,13.3]';      % unit (MPa m^0.5)

% ****************************************************
%part 1: Plot these points on    log-log coordinates and graphically estimate
%              values of the "paris-law" fitting constants A and m

loglog(deltaKI,da_dN/1000,'*-')
title('Figure 01: da/dN vs. \DeltaK_I');
xlabel('\DeltaK_I [MPa*m^{0.5}]')
ylabel('da/dN [m/cycle]');
grid on;
hold on;

% estimate of A and m using first and last point

M=log(da_dN(4)/da_dN(1))/log(deltaKI(4)/deltaKI(1));
A=(da_dN(1)/1000)/deltaKI(1)^M;
disp(sprintf('estimate of A*(deltaKI)^M: A=%g [m/cycle/(MPa*m^0.5)^M], M=%g',A,M));

% ****************************************************
%part 2: Use a least squares fit to the dataset to obtain refined values
%              for A and m


p=polyfit(log10(deltaKI),log10(da_dN/1000),1);
disp(sprintf('best fit of A*(deltaKI)^M: A=%g [m/cycle/(MPa*m^0.5)^M], M=%g',...
10^p(2),p(1)));
```

Problem 2 (60 points)
Part A:

Integration of the Crack-Growth equation, we get:

$$N_{a_i \to a_f} \equiv \int_0^{N_{a_i \to a_f}} = \frac{(\Delta K_{Io})^m}{(\frac{da}{dN})_o} \int_{a_i}^{a_f} \frac{da}{(Q(a)\Delta\sigma\sqrt{\pi a})^m} \tag{3}$$

Since Q is a constant, the above equation is reduced to :

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o}(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}})^m \frac{2}{m-2}[1-(\frac{a_i}{a_f})^{\frac{(m-2)}{2}}]; \tag{4}$$

where a_f is a function of the maximum stress:

$$a_f = \frac{1}{\pi}(\frac{K_{IC}}{Q\sigma_{max}})^2 \tag{5}$$

Substitution of a_f and $\Delta\sigma = \sigma_{max}$ into the expression of $N_{a_i \to a_f}$, and rearrange the equation, we get the following expression:

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o}(\frac{\Delta K_{Io}}{\sqrt{\pi a_i}})^m \frac{2}{m-2}[(Q\sigma_{max})^{-m} - (\frac{\pi a_i}{K_{IC}^2})^{\frac{m-2}{2}}(Q\sigma_{max})^{-2}]; \tag{6}$$

In the above equation, $a_i = 3mm$, $\Delta a_o = 10^{-5} mm/cycle$, $\Delta K_{Io} = 20MPa\sqrt{m}$, and $N_{a_i \to a_f} = 100,000$. Substitution of the above numbers, we get the equation to solve for σ_{max}:

$$10^5 = \frac{3}{10^{-5}}(\frac{20}{\sqrt{\pi 0.003}})^4 \frac{2}{4-2}\{1.12^{-4}(\sigma_{max})^{-4} - (\frac{\pi 0.003}{(115)^2} \times 1.12^{-2}(\sigma_{max})^{-2})\} \tag{7}$$

N vs. σ_{max} is plotted in Figure 2. Use Newton's method to solve the above equation, and the Matlab script is attached.The result is $\sigma_{max} = 238.93\ MPa$.

Part B:

$a(n = 50,000) = 5.7[mm]$, the corresponding value of $K_I = 35.8[MPa\sqrt{m}]$ and the predicted factor of safety at this point is $K_{Ic}/K_I(\sigma_{max}, a(N = 50,000)) = 115/35.8 = 3.2$

Part C:

The safety factor on fatigue K_{Ic}/K_I is larger than the factor on fatigue life. That is because the growth of crack length a accelerates as N increase (Figure 3). Thus, when $N = 1/2N_{critical}$, $a < 1/2a_{critical}$ and so does K_I.

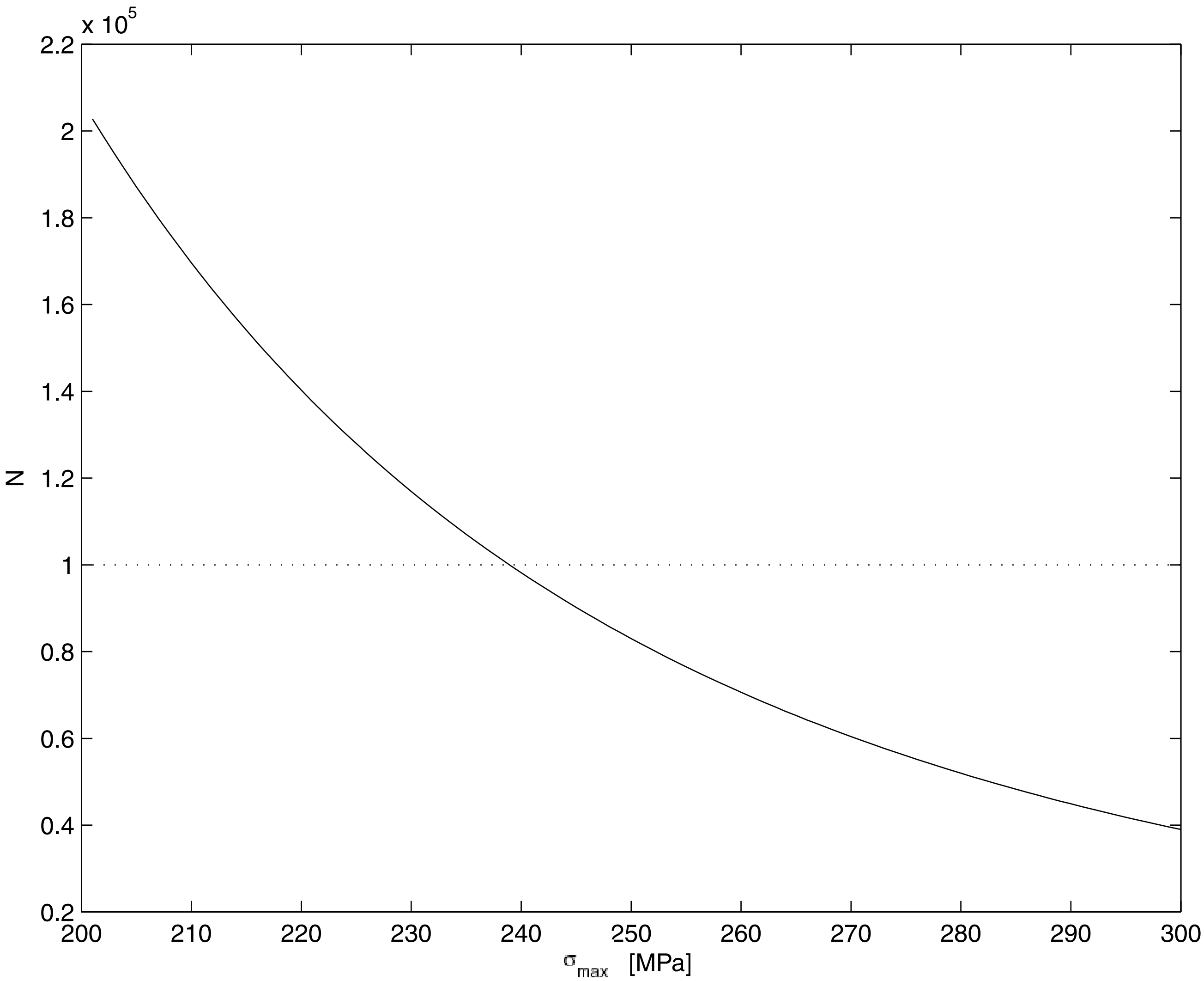

Figure 2 N vs. σ_{max}

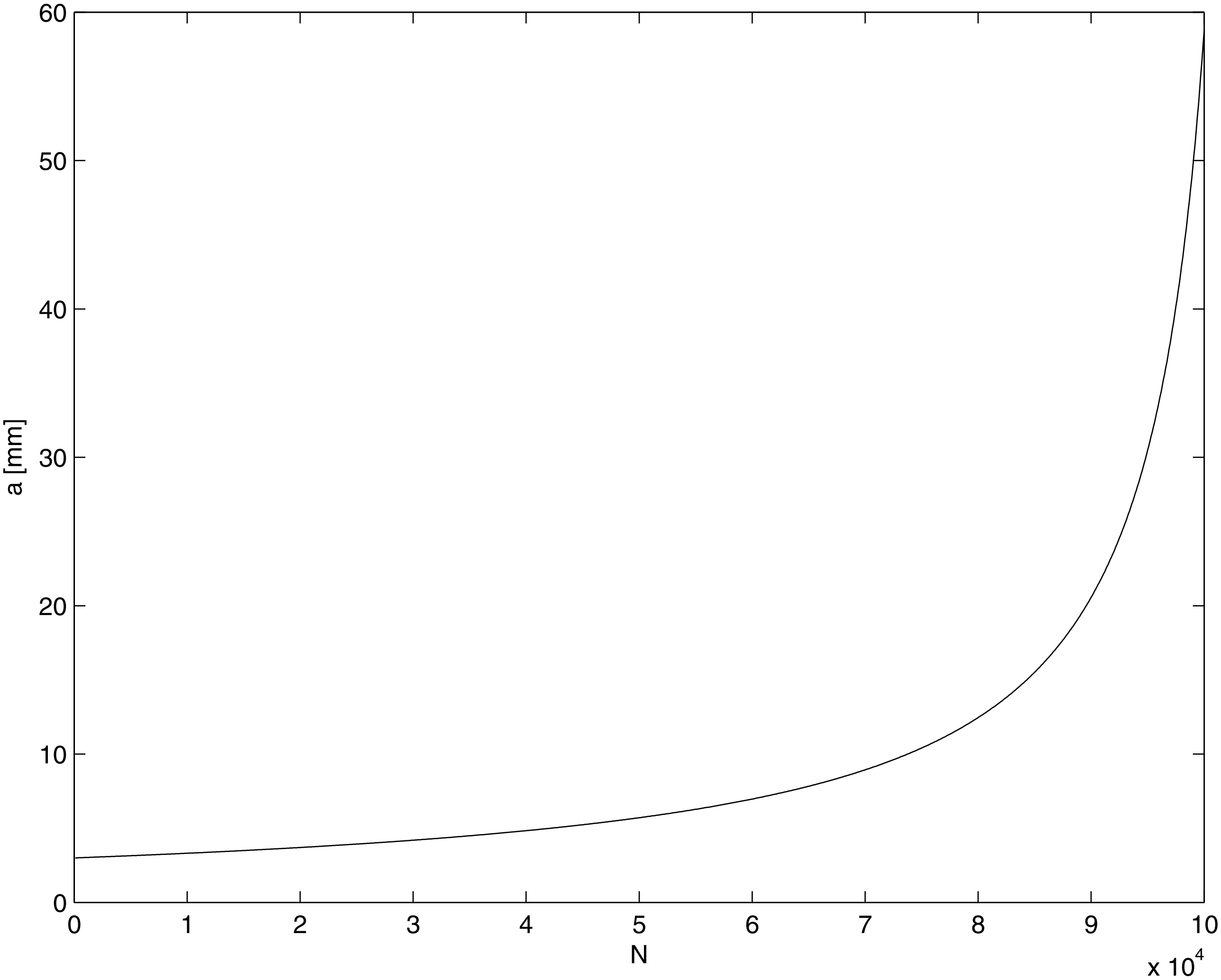

Figure 3   a(N) v.s. N

```
%problem 2 part A:

clear all;
close all;
global N delta_a0    K_1c delta_K_10 a_i m Q


N=100000;                               % [cycle]
delta_a0=1e-8;                          % [m/cycle]
K_1c=115;                               % [MPa m^0.5]
delta_K_10=20;                          % [MPa m^0.5]
a_i=0.003;                              % [m]
m=4;
Q=1.12;

% initial value for the Newton's method
x=220;                                  % innitial value for sigma_max [MPa]
normdx=1.0;
normf=1.0;
% loop of the Newton's method
while(normdx > 1e-6 | normf >1e-6)
    f=fn(x);
    dx=-f(1)/f(2);
    normdx=abs(dx) ;
    normf=abs(f(1));
    x=x+dx;
end


function f=fn(x)
global N delta_a0    K_1c delta_K_10 a_i m Q
% f(1) is the value of f(x)
f(1)=(a_i/delta_a0)*(delta_K_10/(pi*a_i)^0.5)^m*(2/(m-2))*((Q*x)^(-m)-(a_i*pi/K_1c^2)^((m-2)
/2)*(Q*x)^(-2))-N;
% f(2) is the value of df/dx
f(2)=(a_i/delta_a0)*(delta_K_10/(pi*a_i)^0.5)^m*(2/(m-2))*(-m*Q^(-m)*x^(-m-1)+2*(a_i*pi/K_
1c^2)^((m-2)/2)*Q^(-2)*x^(-3));
```

```
% problem 2 part B:
clear all;
close all;

N=100000;                              % [cycle]
delta_a0=1e-8;                         % [m/cycle]
K_1c=115;                              % [MPa m^0.5]
delta_K_10=20;                         % [MPa m^0.5]
a_i=0.003;                             % [m]
m=4;
Q=1.12;
d_sigma=238.93;                        % [MPa]

n=[100:100:100000];
n0=(a_i/delta_a0)*(delta_K_10/(Q*d_sigma*(pi*a_i)^0.5))^m*(2/(m-2));
a=a_i./(1-(n./n0)).^(2/(m-2));
K_1=d_sigma*Q*(pi*a(500)/1000)^0.5;

plot(n,a);
disp(sprintf('a(N=50,000)= %g [mm]',a(500)));
disp(sprintf('K_I (N=50,000)= %g [MPa m^0.5]',K_1));
```