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2.002 MECHANICS AND MATERIALS II
HOMEWORK # 6

Distributed: Wednesday, April 14, 2004
Due: Wednesday, April 21, 2004

Problem 1

A fatigue crack growth test was done on a compact tension specimen of a hard (HRC=60) tool steel. A table of respective values of stress intensity factor range, ΔK_I, and corresponding fatigue crack growth rates, da/dN, are given in the table.

da/dN (mm/cycle)	ΔK_I ($MPa\sqrt{m}$)
4.26×10^{-6}	6.84
9.12×10^{-6}	8.76
1.75×10^{-5}	10.35
3.51×10^{-5}	13.3

- Plot these points on log-log coordinates (over a suitable range!) and graphically estimate values of the "Paris-law" fitting constants A and m that describe fatigue crack growth in this material according to

$$\frac{da}{dN} = A\,(\Delta K_I)^m\,.$$

  Be sure to give appropriate units!

- Use a least squares fit to the dataset to obtain refined values for A and m.

- For the least squares best fit to the Paris law constants, describe how you would obtain an alternative set of parameters Δa_0, ΔK_{I0}, and m that equivalently describe fatigue crack growth according to

$$\frac{da}{dN} = \Delta a_0 \left(\frac{\Delta K_I}{\Delta K_{I0}}\right)^m,$$

  and give numerical values for all parameters.

(Based on Dowling text, problem 11.3).

Problem 2

An edge crack of initial length $a_i = 3mm$ exists in a large plate that is to be subjected to a remote cyclic stress, σ^{∞}, ranging from $\sigma_{\min} = 0$ to some to-be-determined maximum value of $\sigma_{\max}$ ($R = 0$). You may assume that the plate is sufficiently large that the configuration correction factor $Q = 1.12$ in the expression for K_I for all crack length values, a, to be considered.

The fracture toughness of the material is $K_{Ic} = 115\, MPa\sqrt{m}$ and its yield tensile strength is $\sigma_y = 1045\, MPa$. Fatigue crack growth is described by a Paris law form

$$\frac{da}{dN} = \Delta a_0 \left(\frac{\Delta K_I}{\Delta K_{I0}} \right)^m ,$$

with $m = 4$, $\Delta a_0 = 10^{-5}$ mm/cycle, and $\Delta K_{I0} = 20\, MPa\sqrt{m}$.

It is desired to be able to apply $N = 50,000$ load cycles to the pre-cracked structure while still retaining a factor of safety of at least 2 on the **actual fatigue crack propagation life** until fracture. That is, it is desired that the **predicted fatigue crack propagation life** for the chosen value of $\sigma_{\max}$ be at least $\geq 100,000$ cycles.

- **What is the largest value of $\sigma_{\max}$ that meets this constraint?**

  Note: this problem is a bit "non-standard", in that the upper limit of the fatigue crack length, a_f, depends explicitly on the unknown value of $\sigma_{\max}$ (why is this so?), but so does the number of cycles, $N_{a_i \to a_f}$, required to propagate the crack. You may find it desirable to manipulate the relevant equations(s) in a form suitable for numerical iteration or graphical solution.

- For your chosen value of $\sigma_{\max}$, make a graph of the crack length, $a(N)$, for $0 \leq N \leq 100,000$ cycles. What is the predicted value of crack length at $N = 50,000$ cycles, $a(N = 50,000)$, and what is the corresponding value of K_I? Based on this K_I-value, what is the predicted factor of safety with respect to fracture at this point (i.e., what is the ratio of $K_{Ic}/K_I(\sigma_{\max}, a(N = 50,000))$)?

- What conclusions do you draw about the distinction between factors of safety on fatigue crack propagation life and on fracture itself? Discuss.