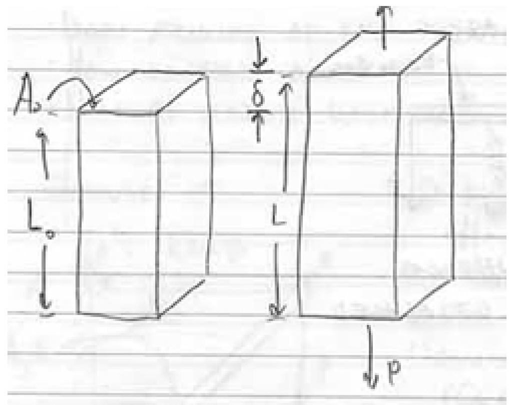
2.001 - MECHANICS AND MATERIALS I
Lecture #9 10/11/2006
Prof. Carol Livermore

Review of Uniaxial Loading:



δ = Change in length (Positive for extension; also called tension)

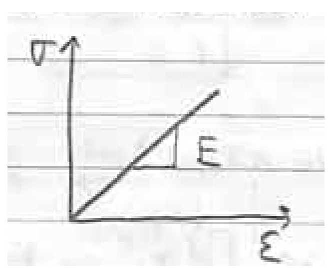
Stress (σ)

$$\sigma = P/A_0$$

Strain (ϵ) at a point

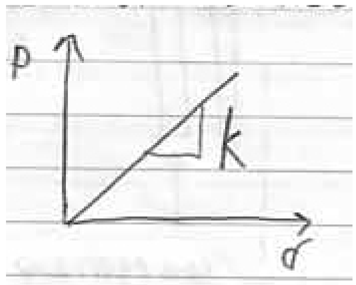
$$\epsilon = \frac{L - L_0}{L_0} = \frac{\delta}{L_0}$$

Stress-Strain Relationship $\Rightarrow$ Material Behavior



$$\sigma = E\epsilon$$

Force-Displacement Relationship



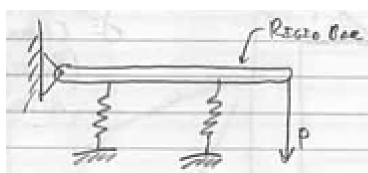
$$P = k\delta$$

For a uniaxial force in a bar:

$$k = \frac{EA_0}{L_0}$$

Deformation and Displacement

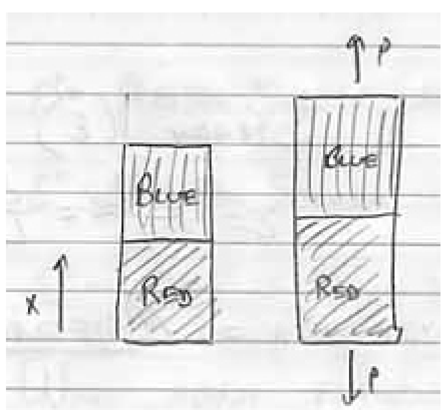
Recall from Lab:



The springs deform.

The bar is displaced.

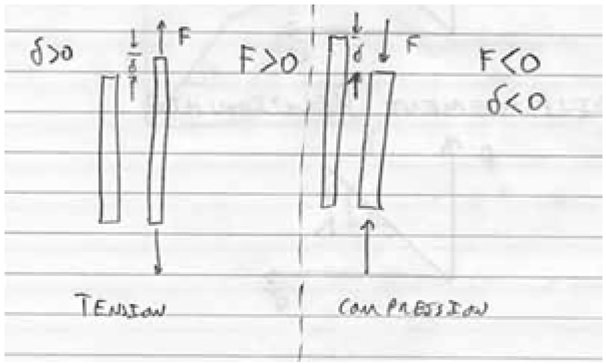
EXAMPLE:



$$\epsilon(x) = \frac{du(x)}{dx}$$

$$\int \epsilon(x)dx = \int du$$

Sign Convention



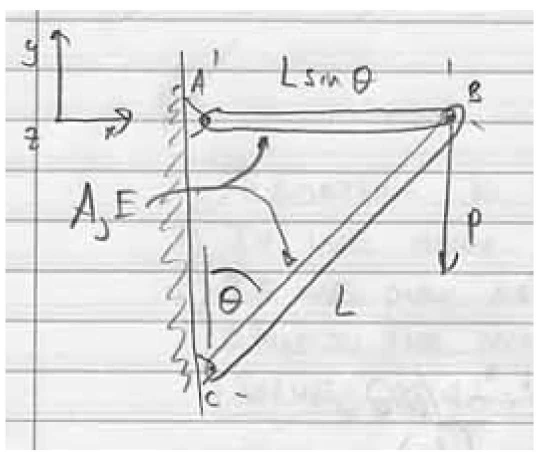
Trusses that deform

Bars pinned at the joints

How do bars deform?

How do joints displace?

EXAMPLE:



Q: Forces in bars? How much does each bar deform? How much does point B displace?

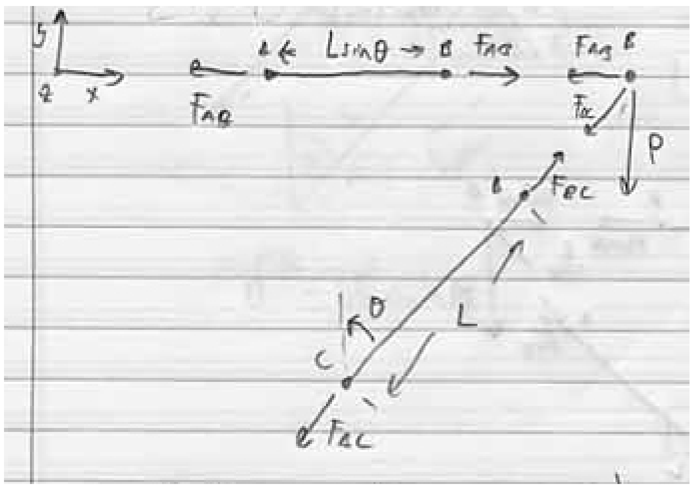
Unconstrained degrees of freedom

1. u_x^B
2. u_y^B

Unknowns

1. F_{AB}
2. F_{BC}

This is statically determinate (Forces can be found using equilibrium)
FBD:



$$\sum F_x = 0 \text{ at Pin B}$$
$$-F_{AB} - F_{BC}\sin\theta = 0$$

$$\sum F_y = 0$$
$$-P - F_{BC}\cos\theta = 0 \Rightarrow F_{BC} = \frac{-P}{\cos\theta}$$

$$-F_{AB} + \frac{P}{\cos\theta}\sin\theta = 0$$

So:

$$F_{AB} = P\tan\theta$$

Force-Deformation Relationship

$$P = k\delta$$
$$k = \frac{EA}{L}$$

$$\delta_{AB} = \frac{F_{AB}}{k_{AB}}$$
$$\delta_{BC} = \frac{F_{BC}}{k_{BC}}$$

$$k_{AB} = \frac{AE}{L \sin\theta}$$

$$k_{BC} = \frac{AE}{L}$$

So:

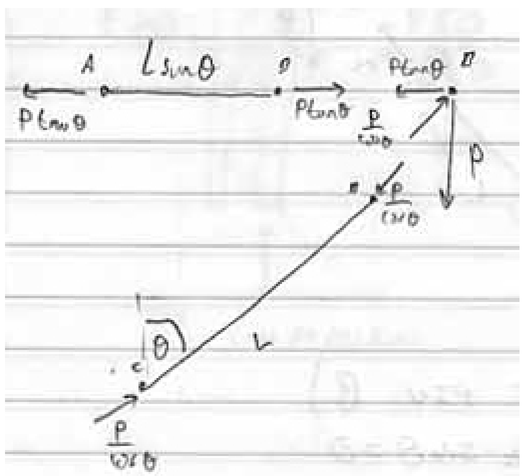
$$\delta_{AB} = \frac{P \tan\theta}{\left(\frac{AE}{L \sin\theta}\right)}$$

$$\delta_{BC} = \frac{-P \cos\theta}{\left(\frac{AE}{L}\right)}$$

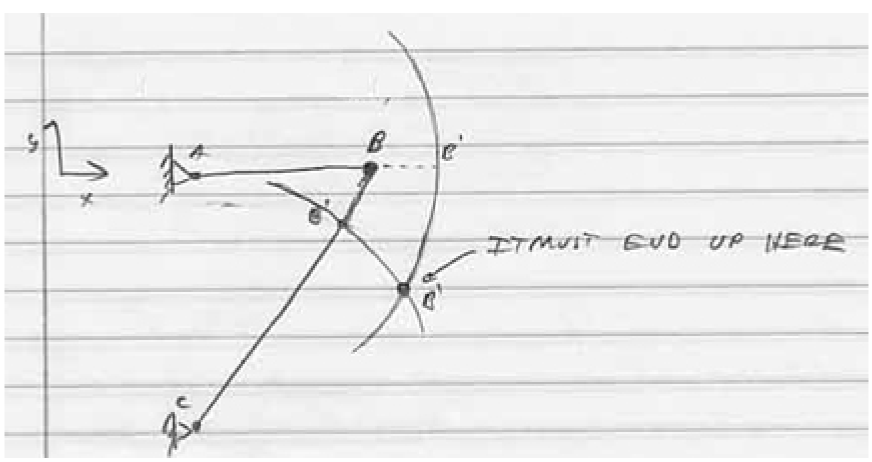
$$\delta_{AB} = \frac{PL \sin\theta \tan\theta}{AE}$$

$$\delta_{BC} = \frac{-PL}{AE \cos\theta}$$

Check:



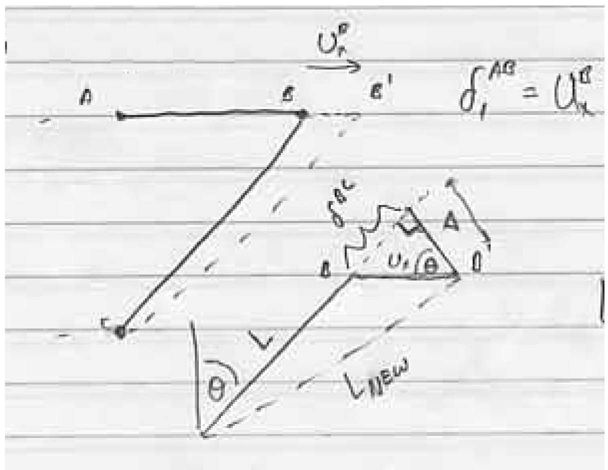
Compatibility



Algorithm to find B'

1. If we only have u_x^B, what δ^{AB} and δ^{BC} would result?
2. If we only have u_y^B, what δ^{AB} and δ^{BC} would result?
3. What is the total δ^{AB} and δ^{BC} is I have both u_x^B and u_y^B? 4. Solve for u_x^B and u_y^B from known δ^{AB} and δ^{BC}.

Step 1



$$\delta^{BC} = u_x^B \sin\theta$$

$$\begin{aligned} L_{NEW} &= \sqrt{(L+\delta_1^{BC})^2+\Delta^2} \\ &= L\sqrt{(1+\frac{2\delta_1^{BC}}{L}+\frac{\delta_1^{BC^2}}{L^2})+\frac{\Delta^2}{L^2}} \end{aligned}$$

$$\sqrt{1+x} \approx 1+\frac{x}{L}$$

$$L_{NEW} = L\left(1+\frac{\delta_1^B C}{L}+\frac{\delta_1^{BC^2}}{2L^2}+\frac{\Delta^2}{2L^2}\right)$$

$$\frac{\delta_1^{BC^2}}{2L^2} \to 0$$

$$\frac{\Delta^2}{2L^2} \to 0$$

For $\delta << L$:

$$\delta^{BC} \approx D$$

So:

$$L_{new} = L + \delta_1^{BC}$$