2.001 - MECHANICS AND MATERIALS I
Lecture #8
10/4/2006
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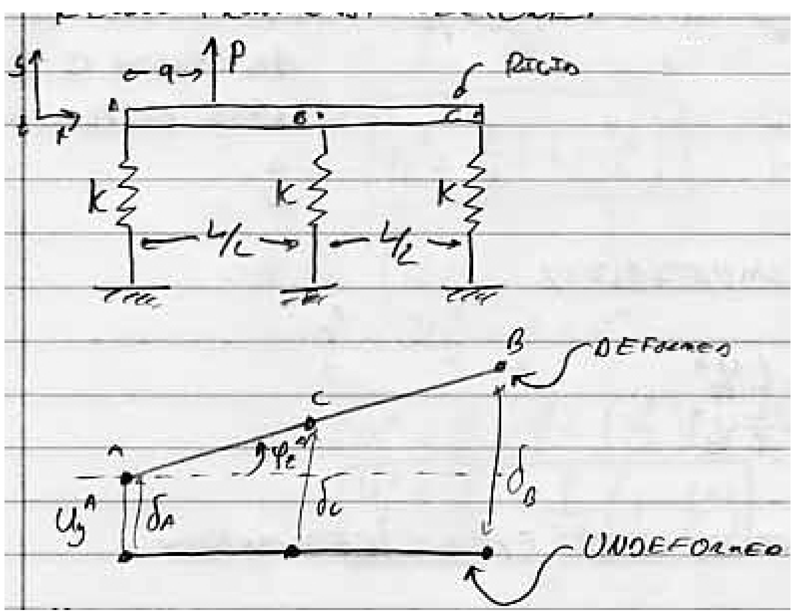
Recall from last lecture:



Find: $u(x), F_A, F_B, F_C$

1. Equilibrium

$$\sum F_u = 0$$

$$P - F_A - F_B - F_C = 0$$

$$\sum M_A = 0$$

$$Pa - \frac{F_c L}{2} - F_B L = 0$$

2. Force-Deformation

$$F_A = k\delta_A$$

$$F_B = k\delta_B$$

$$F_C = k\delta_C$$

3. Compatibility

$$\delta_A = u_y^A$$

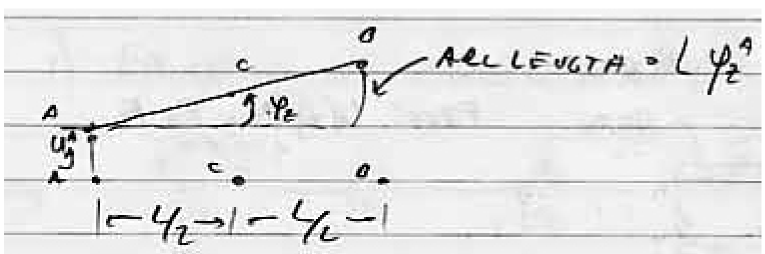
$$\delta_B = u_y^A + L\tan\varphi_z^A$$

$$\delta_C = u_y^A + \frac{L}{2}\tan\varphi_z^A$$

New this lecture:

Small Angle Assumption:

$$\tan\varphi_z^A = \frac{\sin\varphi_z^A}{\cos\varphi_z^A} \approx \frac{\varphi_z^A}{1} = \varphi_z^A$$



For small φ_z^A: Arc length $\approx$ a straight line displacemtn in y.

Rewrite compatibility.

$$\delta_A = u_y^A$$

$$\delta_B = u_y^A + L\varphi_z^A$$

$$\delta_A = u_y^A + \frac{L}{2}\varphi_z^A$$

Substitute compatibility into force-deformation

$$F_A = ku_y^A$$

$$F_B = k(u_y^A + L\varphi_z^A)$$

$$F_C = k(u_y^A + \frac{L}{2}\varphi_z^A)$$

Substitute this result into equilibrium equations:

$$P - ku_y^A - k(u_y^A + L\varphi_z^A) - k(u_y^A + \frac{L}{2}\varphi_z^A) = 0$$

$$Pa - \frac{L}{2}\left[k(u_y^A + \frac{L}{2}\varphi_z^A)\right] - L\left[k(u_y^A + L\varphi_z^A)\right] = 0$$

Solve:

$$P = 3ku_y^A + \frac{3}{2}Lk\varphi_z^A$$

$$Pa = \frac{3}{2}Lku_y^A + \frac{5}{4}L^2k\varphi_z^A$$

Divide by k.

$$\frac{P}{k} = 3u_y^A + \frac{3}{2}L\varphi_z^A$$

Divide by $Lk/2$.

$$\frac{2Pa}{Lk} = 3u_y^A + \frac{5}{2}L\varphi_z^A$$

So:

$$\varphi_z^A = \frac{-P}{Lk}\left(1 - \frac{2a}{L}\right)$$

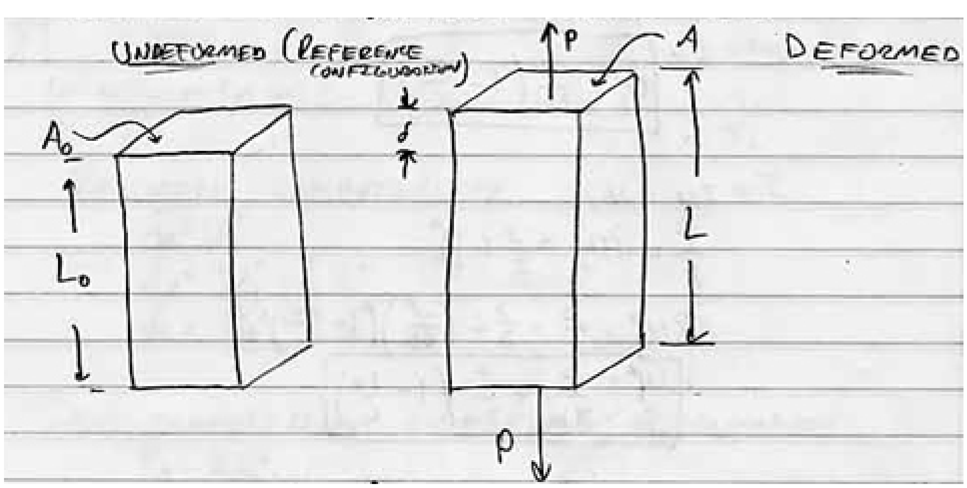
Substitute:

$$u_y^A = \frac{P}{3k} + \frac{P}{2k}\left(1 - \frac{2a}{L}\right)$$

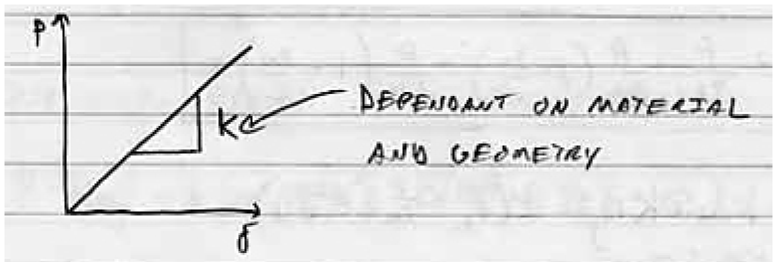
$$u(x) = \frac{P}{3k} + \frac{P}{2k}(1 - \frac{2a}{L}) - \frac{P}{Lk}(1 - \frac{2a}{L})x$$

UNIAXIAL LOADING

Behavior of a uniaxially loaded bar



$$L = L_D + \delta$$



$P = k\delta$ is a property of the bar.

$$\text{STRESS} = \frac{\text{FORCE}}{\text{UNIT AREA}} = \sigma(\text{ Like Pressure})$$

Units: $1\frac{N}{m^2} = 1$ Pa (SI Units)
Units: $1\frac{lbs}{in^2} =$ PSI (English Units)

$$\sigma = \frac{P}{A}$$

Engineering Stress
For small deformation

$$\sigma = \frac{P}{A_0}$$

STRAIN $= \frac{\text{CHANGE IN LENGTH}}{\text{LENGTH}} = \epsilon$

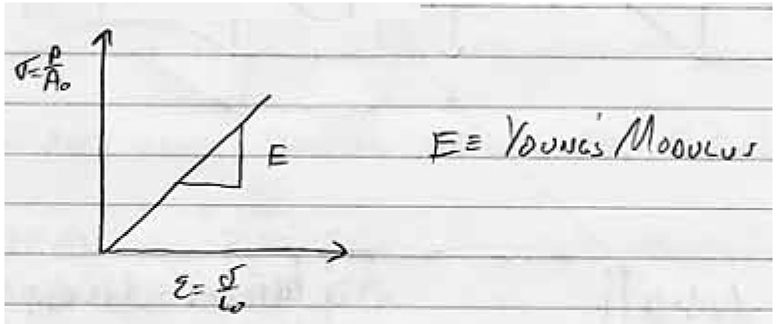
$$\epsilon = \frac{\delta}{L}$$

Strain is dimensionless.

Engineering Strain (For small deformations)

$$\epsilon = \frac{\delta}{L_0}$$

Stress-Strain plot



For Uniaxial Loading:

$$\sigma = E\epsilon$$

Material property is E, Young's Modulus.
Note: Units of E = Pa, E 10^9 Pa (or GPa).

So, what is k for uniaxiail loading?

$$\sigma = E\epsilon$$

$$\sigma = \frac{P}{A_0}$$

$$\epsilon = \frac{\delta}{L_0}$$

So:

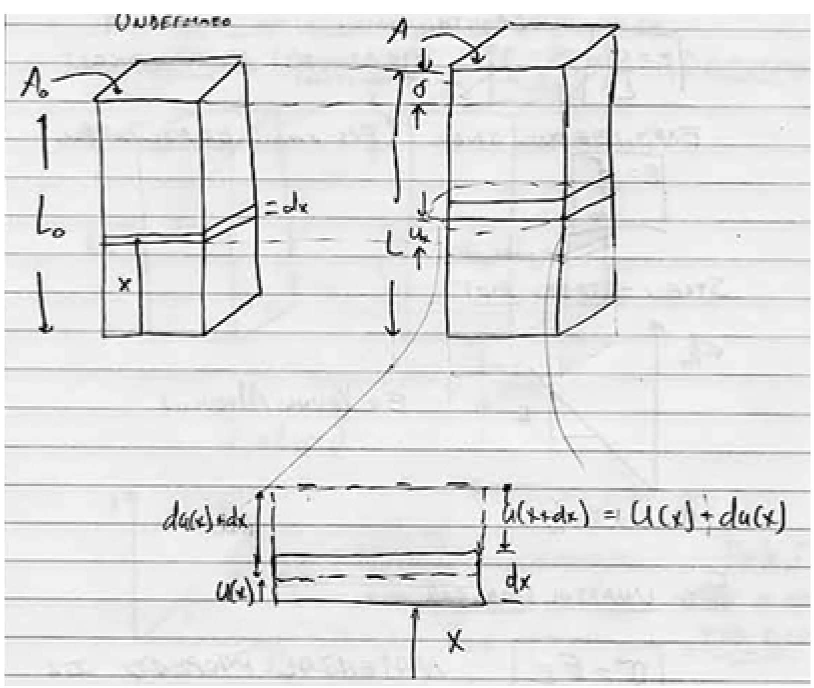
$$\frac{P}{A_0} = \frac{E\delta}{L_0}$$

$$P = \frac{EA_0}{L_0}\delta$$

So:

$$k = \frac{EA_0}{L_0} \text{ for uniaxial loading}$$

Deformation and Displacement



$$\epsilon = \frac{\delta}{L} = \frac{(du(x) + dx) - dx)}{dx}$$

$$\epsilon = \frac{du(x)}{dx}$$

$$u(x) = \text{axial displacement of x}$$

$$\epsilon(x) = \frac{du(x)}{dx}$$

$$\epsilon(x) = \frac{\sigma(x)}{E}$$

So:

$$\frac{du(x)}{dx} = \frac{\sigma(x)}{E} = \frac{P}{AE}$$

$$\int_0^\delta du = \int_0^L \frac{P}{AE} dx$$

$$\Big[u\Big]_0^\delta = \left[\frac{Px}{AE}\right]_0^L$$

$$\delta = \frac{PL}{AE}$$

So:

$$P = \frac{AE\delta}{L}$$

What are some typical values for E?

	E
Steel	200 GPa
Aluminum	70 GPa
Polycarbonate	2.3 GPa
Titanium	150 GPa
Fiber-reinforced Composites	120 GPa

Selection of material? Optimize k for a particular A

Steel: $k_s = k = \frac{A_s E_s}{L}$

Al: $k_A = k = \frac{A_A E_A}{L}$

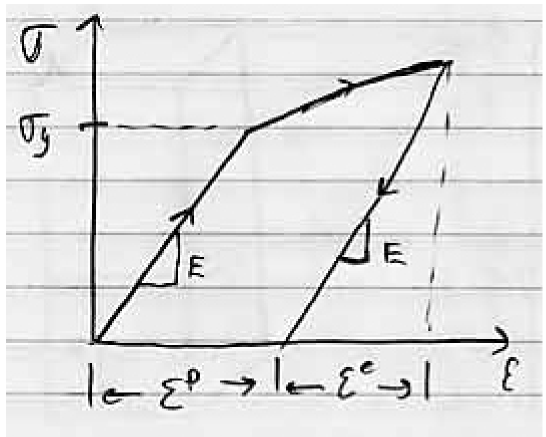
Same $k \Rightarrow A_s E_s = A_A E_A$

So: $A_A = A_s \frac{E_s}{E_A}$

So: $A_A \approx 3A_s \Rightarrow$ the aluminum is three times bigger

May need to optimize weight (think about airplanes) $\Rightarrow$ need to include density.

What happens if you keep pulling on a material?



$$\sigma_y = \text{ Yield Stress}$$

ϵ^p = Plastic Strain - Not Recoverable.
ϵ^e = Elastic Strain - Fully Recoverable.
$\epsilon^t = \epsilon^e + \epsilon^p$ = Total Strain

What about pulling on a bar in uniaxial tension?