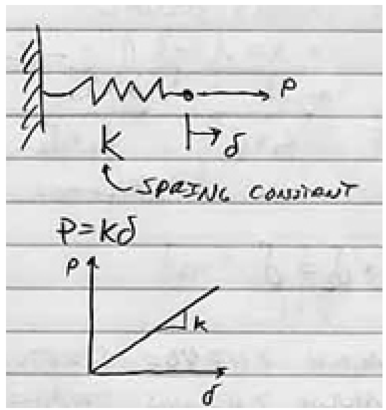
2.001 - MECHANICS AND MATERIALS I
Lecture #7
10/2/2006
Prof. Carol Livermore

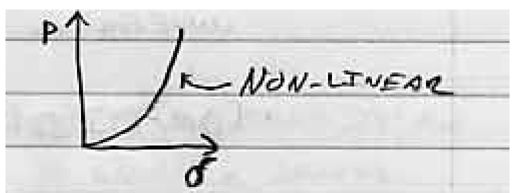
Recall: 3 Basic Ingredients

1. Forces, Moments, and Equilibrium
2. Displacements, Deformations, and Compatibility
3. Forces-Deformation Relationships

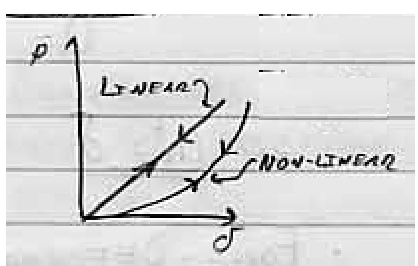
Linear Elastic Springs



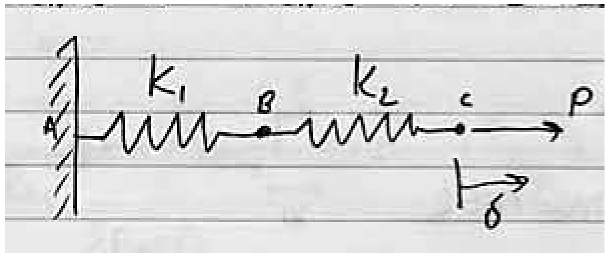
Linear: k is a constant $\rightarrow$ not a function of P or δ.
If it were non-linear:



Elastic: Loading and unloading are along the same curve.

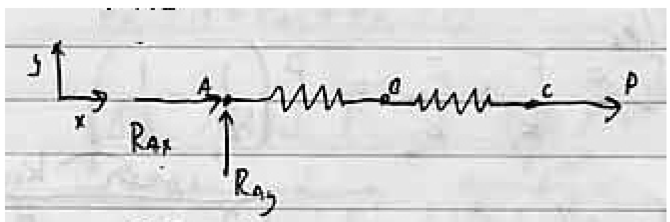


EXAMPLE: Springs in series



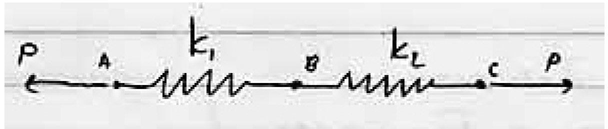
Q: What are the reactions at the supports?
Q: How are P and δ related?

FBD

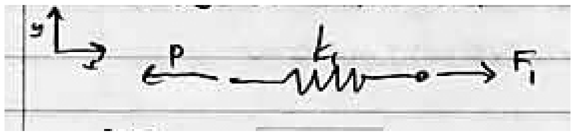


$$\sum F_x = 0$$
$$R_{A_x} + P = 0$$
$$R_{A_x} = -P$$

$$\sum F_y = 0$$
$$R_y = 0$$



FBD of Spring 1



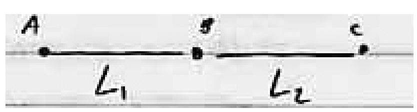
$$\sum F_x = 0$$
$$F_1 = P$$

FBD of Spring 2

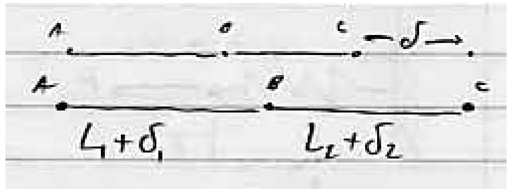
$$\sum F_x = 0$$
$$F_2 = P$$

Look at compatibility:

Undeformed



Deformed



Compatibility:

$$\delta_1 + \delta_2 = \delta$$

Define:
δ - How much things stretch
u - How much things move

So:

$$u_x = \delta$$

Force-Deformation:

$$F_1 = k_1\delta_1$$
$$F_2 = k_2\delta_2$$

Put it all together:

$$\delta_1 = F_1/k_1$$
$$\delta_2 = F_2/k_2$$
$$\delta = \delta_1 + \delta_2 = \frac{F_1}{k_1} + \frac{F_2}{k_2} = P\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

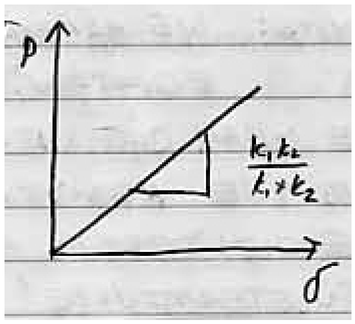
So:

$$\delta = P\left(\frac{k_2 + k_1}{k_2 k_1}\right)$$

$$P = \frac{k_1 k_2}{k_1 + k_2}\delta$$

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$$

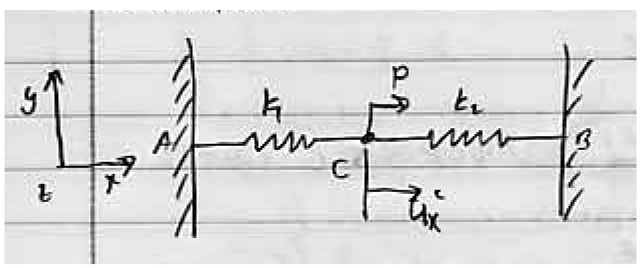
Plot



Sanity Check

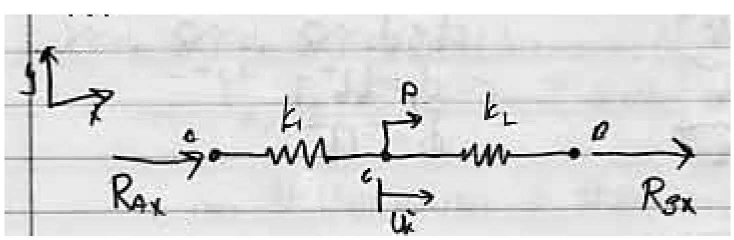
1. $k_1 = k_2 = k$
   $k_{eff} = \frac{k^2}{2k} = \frac{k}{2}$, $P \Rightarrow 2 \times \delta$
2. $k_1 >> k_2$
   $k_{eff} = \frac{k_2}{1+\frac{k_2}{k_1}} \approx k_2 \Rightarrow$ All flexibility is due to weaker spring.

EXAMPLE



Q: How far does C displace?
Q: What are the forces in the spring?

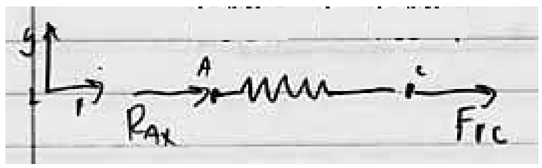
1. FBD



Note: No forces in y.

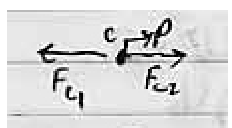
$$\sum F_x = 0$$
$$P + R_{A_x} + R_{B_x} = 0$$

FBD: Spring 1



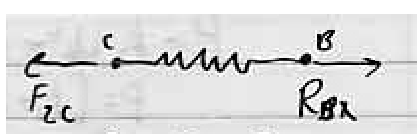
$$R_{A_x} + F_{1c} = 0$$
$$F_{1c} = -R_{A_x}$$

FBD: Pin



$$P - F_{1c} + F_{2c} = 0$$

FBD: Spring 2



$$R_{B_x} - F_{2c} = 0$$

$$F_{2c} = R_{B_x}$$

So:

$$P + R_{A_x} + R_{B_x} = 0$$

Note: We already have this equation.

We need more than just equilibrium!

We cannot find the reactions at the supports using equilibrium alone. This is *statically indeterminate*.

Test for Static Indeterminancy

Unknowns

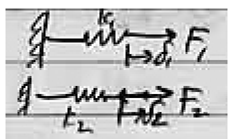
Equations of Equilibrium

If #Unknowns > #Equations ⇒ Static Indeterminancy.

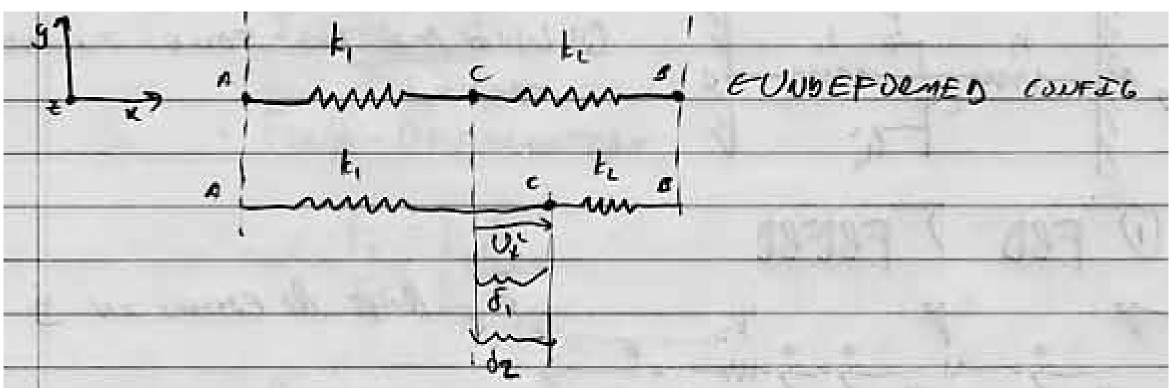
2. Let's try Force Deformation relationships.

$F_1 = k_1\delta_1$

$F_2 = k_2\delta_2$



3. Now add compatibility.



$$\delta_1 + \delta_2 = 0$$
$$\delta_1 = u_x^c$$
$$\delta_2 = -u_x^c$$

4. Solve equations.

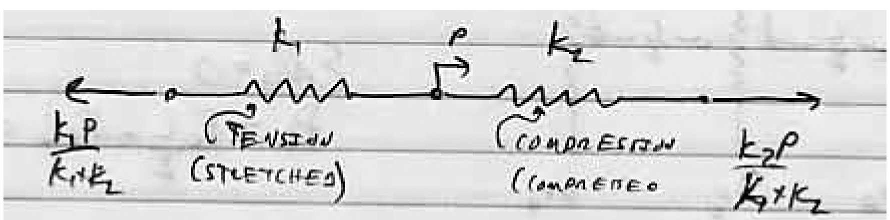
$$F_1 = k_1\delta_1 = k_1 u_x^c$$
$$F_2 = k_2\delta_2 = -k_2 u_x^c$$

$$P - k_2 u_x^c - k_1 u_x^c = 0$$
$$P = (k_1 + k_2)u_x^c$$
$$u_x^c = \frac{P}{k_1 + k_2}$$

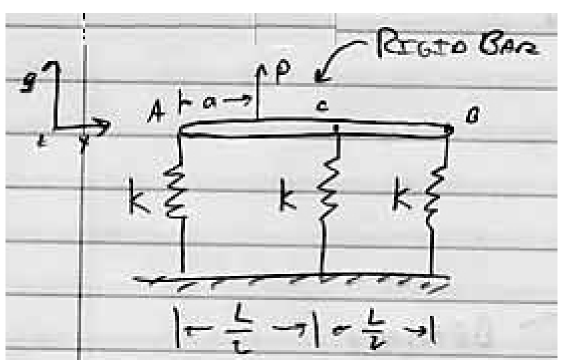
What is the loadsharing?

$$F_1 = \frac{k_1 P}{k_1 + k_2}$$
$$F_2 = -\frac{k_2 P}{k_1 + k_2}$$

Check



EXAMPLE



Q: Forces in springs A, B, C?
Q: Find y(x).

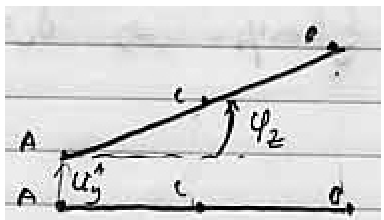
Assumes small deformations.

Is this statically indeterminate?
What are the unconstrained degrees of freedom (D.O.F)?

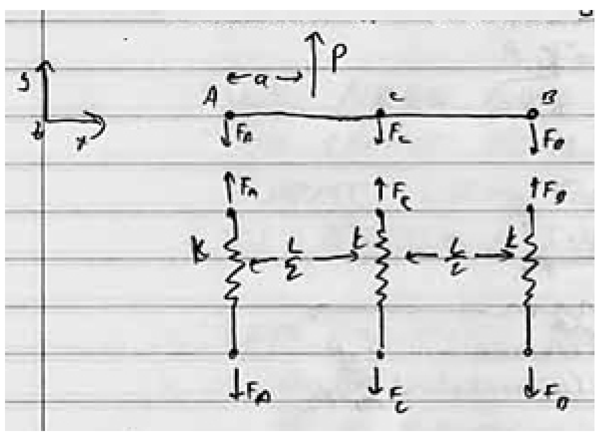
1. Vertical displacements $\Rightarrow \sum F_y = 0$
2. Rotation about z $\Rightarrow \sum M_z = 0$

What are the unknowns?

F_A,F_B,F_C $\Rightarrow$ 3 Unknowns.
#Unknowns > #Equations $\Rightarrow$ This *is* statically indeterminate.



1. Equations of Equilibrium



$$\sum F_y = 0$$

$$P - F_A - F_B - F_C = 0$$

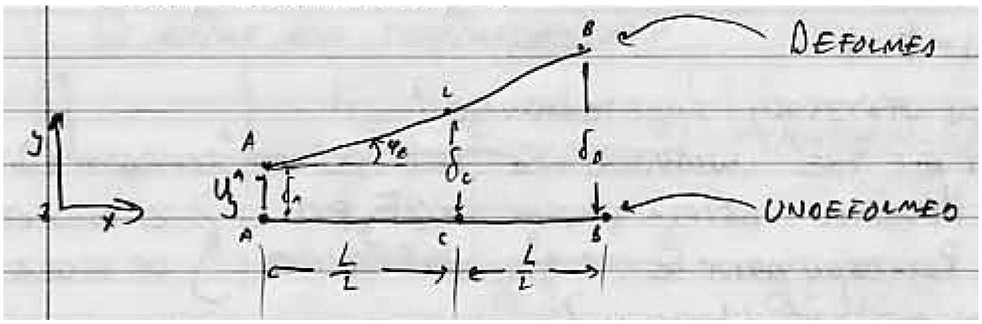
$$\sum M_A = 0$$

$$Pa - F_C\frac{L}{2} - F_B L = 0$$

2. Force Deformation Relationship

$$F_A = k\delta_A$$
$$F_B = k\delta_B$$
$$F_C = k\delta_C$$

3. Compatibility



$$\delta_A = u_Y^A$$
$$\tan\varphi = \frac{\delta_B - \delta_A}{L} \quad \Rightarrow L\tan\varphi_z = \delta_B - \delta_A \Rightarrow \delta_B = \delta_A + L\tan\varphi_z = u_y^A + L\tan\varphi_z$$
$$\tan\varphi = \frac{\delta_C - \delta_A}{L/2} \quad \Rightarrow \frac{L}{2}\tan\varphi_z = \Rightarrow \delta_C = u_y^A + \frac{L}{2}\tan\varphi_z$$