

2.001 - MECHANICS AND MATERIALS I

Lecture #5

9/20/2006

Prof. Carol Livermore

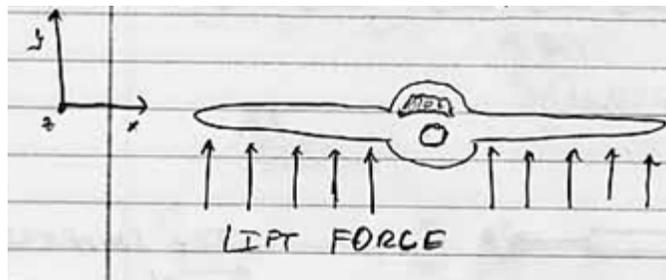
TOPIC: FORCES AND MOMENTS TRANSMITTED BY SLENDER MEMBERS?

When will a structure fail?

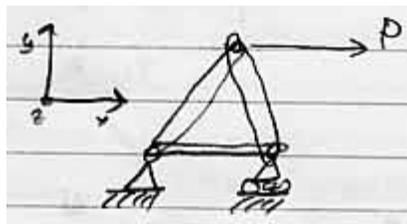
Recall:

1. What can the structural elements tolerate?
2. What forces and moments are experienced in the various parts of the structure?

EXAMPLE: Airplane

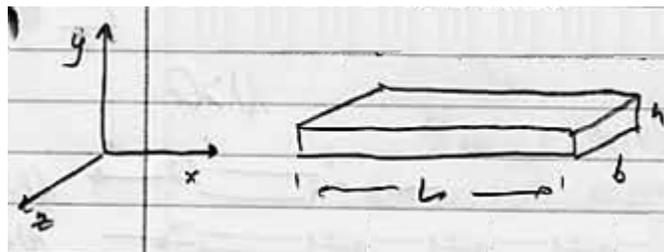


EXAMPLE: Truss



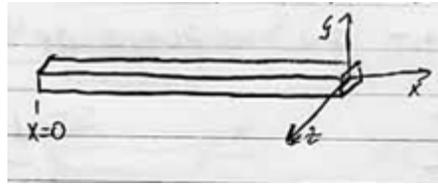
SLENDER MEMBERS

Long, skinny structural elements

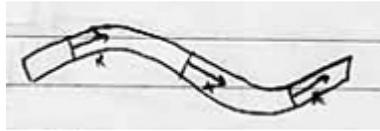


$$L \gg b, L \gg h$$

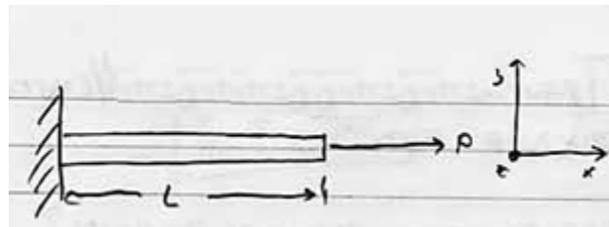
EXAMPLE: Skis, Golf Clubs, I-Beams



Local axes follow the beam.

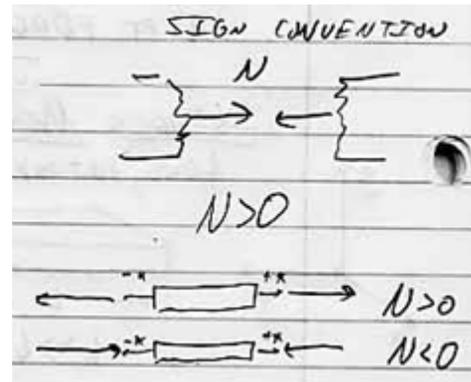
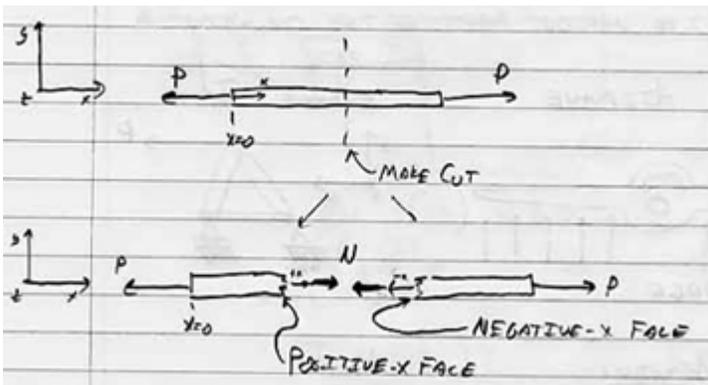


AXIAL FORCES:



Forces that act along the axis of the slender member.

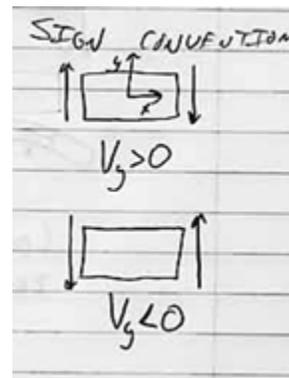
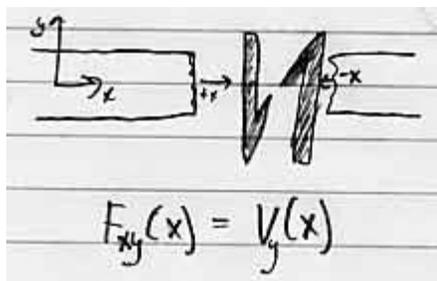
FBD



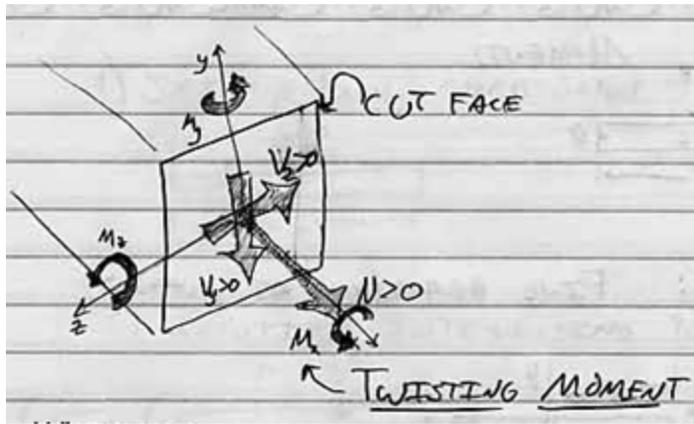
$$F_{ij}(x) = N(x), i: \text{face}, j: \text{direction}, x: \text{location}$$

SHEAR FORCES:

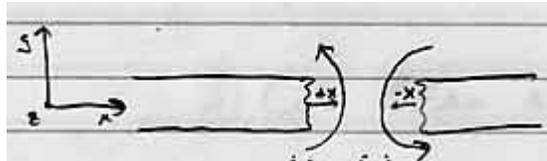
Forces that act in the plane of a face.



Summarize Internal Loads:



Moments:

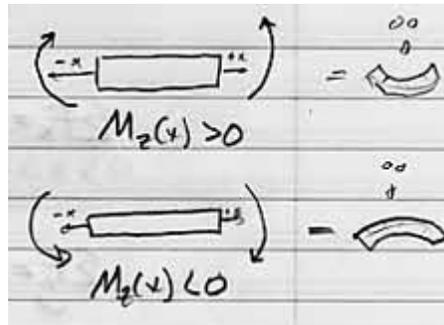


$$M_{xz}(x_2) > 0: \text{face, } z: \text{axis, } x_2: \text{location}$$

$$M_{xz}(x) = M_z(x)$$

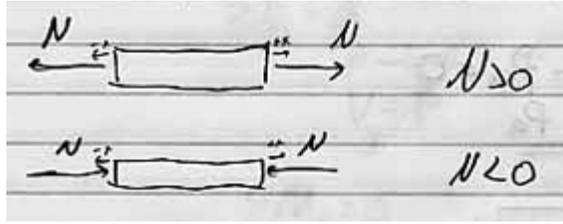
Bending Moment

Sign Convention

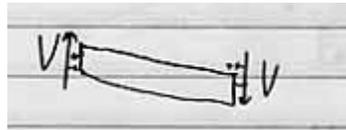


Summarize sign conventions

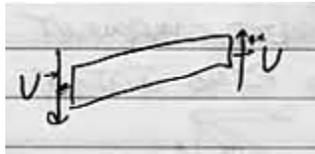
$N > 0$   
 $N < 0$



$V > 0$



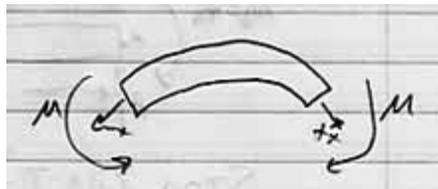
$V < 0$



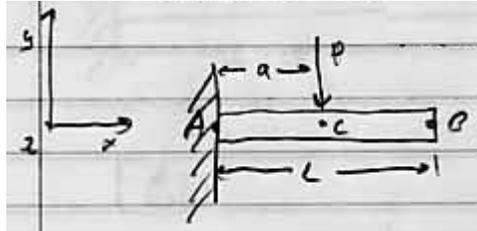
$M > 0$



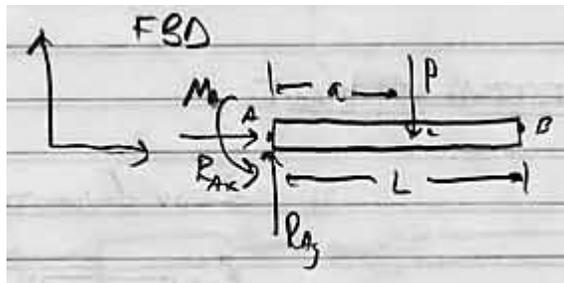
$M < 0$



EXAMPLE: Calculating internal forces and moments



Step 1: Find reactions at supports.



$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum F_y = 0$$

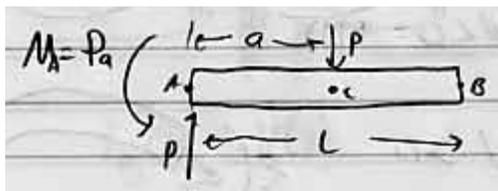
$$R_{Ay} - P = 0$$

$$R_{Ay} = P$$

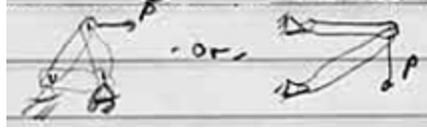
$$\sum M_A = 0$$

$$M_A - Pa = 0$$

$$M_A = Pa$$

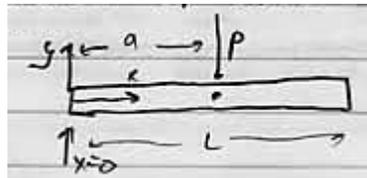


Step 2: If you have a multi-component structure, you will need to find joint forces too.

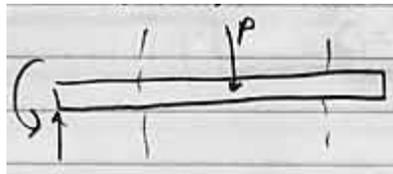


Step 3: Find internal forces and moments.

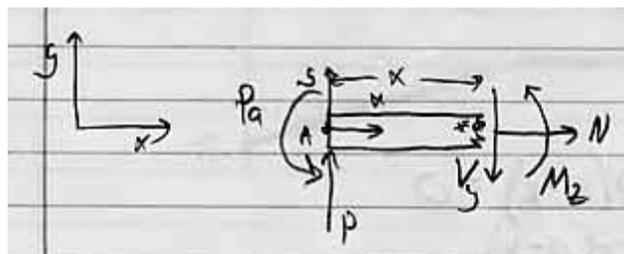
1. Set up local coordinates.



2. Identify interesting points.



3. Cut beam and do FBDs.



For  $0 < x < a$

$$\begin{aligned}\sum F_x &= 0 \\ N &= 0\end{aligned}$$

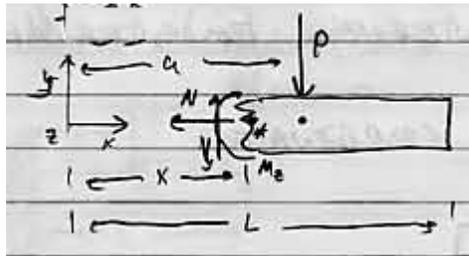
$$\begin{aligned}\sum F_y &= 0 \\ P - V_y &= 0 \\ P &= V_y\end{aligned}$$

$$\begin{aligned}\sum M_A &= 0 \\ Pa + M_z - Px &= 0\end{aligned}$$

$$M_z = Px - Pa = -P(a - x)$$

OR (From the other end)

FBD



$$\begin{aligned}\sum F_x &= 0 \\ -N &= 0 \\ N &= 0\end{aligned}$$

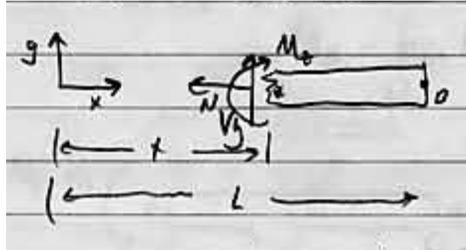
$$\begin{aligned}\sum F_y &= 0 \\ -P + V_y &= 0 \\ V_y &= P\end{aligned}$$

$$\begin{aligned}\sum M_* &= 0 \\ -M_z - P(a - x) &= 0\end{aligned}$$

$$M_z = -P(a - x)$$

Note: This is the same as before.

For  $a < x < L$



$$\sum F_x = 0$$

$$-N = 0$$

$$N = 0$$

$$\sum F_y = 0$$

$$V_y = 0$$

$$\sum M_* = 0$$

$$-M_z = 0$$

$$M_z = 0$$

Step 4: Plot result and sanity check.

